lecture 11

Harris corners

image registration

Recall from last lecture

$$\sum_{(x,y)\in Ngd(x_0,y_0)} \left(I(x,y) - I(x+\Delta x,\ y+\Delta y)\right)^2$$

$$\approx \sum_{(x,y)\in Ngd(x_0,y_0)} \left(\frac{\partial I(x,y)}{\partial x}\Delta x + \frac{\partial I(x,y)}{\partial y}\Delta y\right)^2$$

$$= (\Delta x, \Delta y) \underbrace{\sum_{(x,y)\in Ngd(x_0,y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}}_{M} \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$$

$$(\Delta x \ \ \Delta y)\, M \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} \geq 0$$

If $\begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$ is an unit length eigenvector of M,

then

$$(\Delta x \ \ \Delta y)\, M \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix} = \underbrace{\lambda}_{\text{eigenvalue}} \geq 0$$

M is symmetric

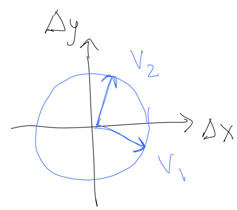
$\therefore$ M has 2 orthogonal eigenvectors.

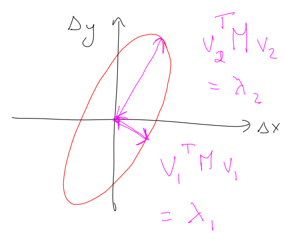
$$v_1^T M v_2 = v_1^T M^T v_2$$

$$= \lambda_2 v_1^T v_2 \qquad\qquad = v_2^T M v_1 = \lambda_1 v_2^T v_1$$

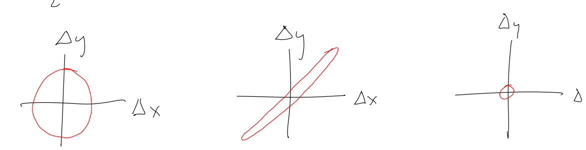
$$\Longrightarrow \quad \lambda_1 = \lambda_2 \quad \text{or} \quad v_1^T v_2 = 0$$

How does quadratic form $(\Delta x \ \ \Delta y)\, M \begin{pmatrix}\Delta x \\ \Delta y\end{pmatrix}$ vary with $(\Delta x, \Delta y)$ on unit circle?





"corner"	"edge"	"uniform"
$\lambda_1 > 0$	$\lambda_1 > 0$	$\lambda_1 \approx 0$
$\lambda_2 > 0$	$\lambda_2 \approx 0$	$\lambda_2 \approx 0$



To distinguish "corner" vs. "edge" vs. "uniform" at each (x,y):

- compute $M = \sum_{(x,y)\in Ngd(x_0,y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$
- Compute $\lambda_1, \lambda_2, v_1, v_2$

$$\det M = M_{11}M_{22} - M_{12}M_{21} = \lambda_1\lambda_2$$

$$\operatorname{tr} M = M_{11} + M_{22} = \lambda_1 + \lambda_2$$

Interesting Cases?

$\det M \neq 0$

$\det M = 0$ and $\operatorname{tr} M \neq 0$

$\det M = 0$ and $\operatorname{tr} M = 0$

Harris (and Stevens) corner detector

$$\lambda_1\lambda_2 - k(\lambda_1+\lambda_2)^2 \qquad k = \frac{1}{10}$$

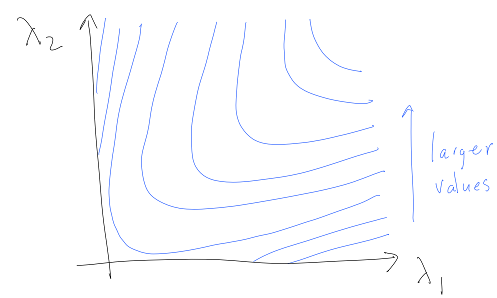
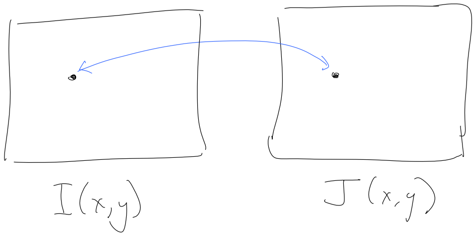
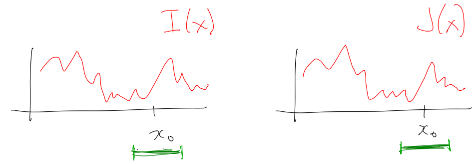


Image Registration



Find corresponding points in two images.

Image Registration in 1D



For each x_0, find h that minimizes

$$\sum_{x \in N_{gd}(x_0)} \left(I(x+h) - J(x) \right)^2$$

$$\approx \sum_{x \in N_{gd}(x_0)} \left(I(x) + \frac{\partial I}{\partial x} h - J(x) \right)^2$$

$$\sum_{x \in N_{gd}(x_0)} \left(I(x+h) - J(x) \right)^2 \approx \sum_{x \in N_{gd}(x_0)} \left(I(x) + \frac{\partial I}{\partial x} h - J(x) \right)^2$$

To minimize this, take derivative w.r.t. h and set to 0.

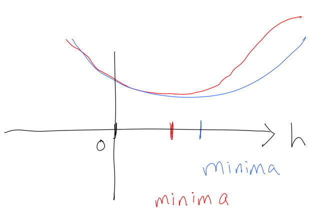
$$0 = \frac{d}{dh} \sum_{x \in N_{gd}(x_0)} \left(I(x) + \frac{\partial I}{\partial x} h - J(x) \right)^2$$

$$= 2 \sum_{x \in N_{gd}(x_0)} \left(I(x) - J(x) + \frac{\partial I}{\partial x} h \right) \frac{\partial I}{\partial x}$$

$$\therefore \quad h = - \frac{\sum_{x \in N_{gd}(x_0)} \left(I(x) - J(x) \right) \frac{\partial I}{\partial x}}{\sum_{x \in N_{gd}(x_0)} \left(\frac{\partial I}{\partial x} \right)^2}$$

$$\sum_{x \in N_{gd}(x_0)} \left(I(x+h) - J(x) \right)^2$$

$$\sum_{x \in N_{gd}(x_0)} \left(I(x) - J(x) + \frac{\partial I}{\partial x} h \right)^2$$



Linear model is good fit near $h \approx 0$ but may be bad fit for $h \not\approx 0$.

Iterative Method

Suppose we have estimated h_k such that

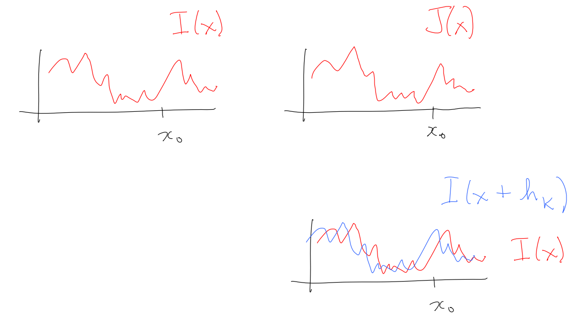
$$\sum_{x \in N_{gd}(x_0)} \left(I(x) + \frac{\partial I}{\partial x}\Big|_{x_0} h_k - J(x) \right)^2 \text{ is minimized.}$$

We want to find h_{k+1} such that

$$\sum_{x \in N_{gd}(x_0)} \left(I(x + h_{k+1}) - J(x) \right)^2 \text{ is minimized.}$$

Given h_k, find h_{k+1} that minimizes

$$\sum_{x \in N_{gd}(x_0)} \left(I(x + h_{k+1}) - J(x) \right)^2$$



Given h_k, find h_{k+1} that minimizes

$$\sum_{x \in N_{gd}(x_0)} \left(I(x + h_{k+1}) - J(x) \right)^2 \equiv \sum_{x \in N_{gd}(x_0)} \left(I(x + h_k + h) - J(x) \right)^2$$

where $h_{k+1} \equiv h_k + h$

$$h = - \frac{\sum_{x \in N_{gd}(x_0)} \left(I(x+h_k) - J(x) \right) \frac{\partial I}{\partial x}}{\sum_{x \in N_{gd}(x_0)} \left(\frac{\partial I}{\partial x} \right)^2}$$

evaluated at $x + h_K$

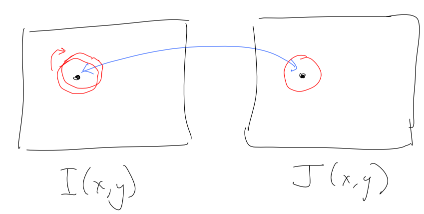
Iterative Method

Repeat until convergence:

$$\sum_{x \in N_{gd}(x_0)} \left(I(x + h_k) - J(x) \right)^2$$

$$\approx \sum_{x \in N_{gd}(x_0)} \left(I(x + h_{k+1}) - J(x) \right)^2$$

Image Registration



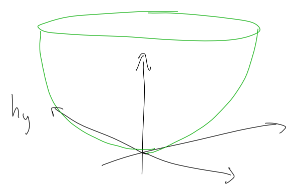
For each (x_0, y_0), find (h_x, h_y) that minimizes

$$\sum_{(x,y) \in N_{gd}(x_0 y_0)} \left(I(x + h_x, y + h_y) - J(x,y) \right)^2$$

$$\sum_{(x,y)\in Ngd(x_0y_0)} \left(I(x+h_x, y+h_y) - J(x,y)\right)^2$$

$$\approx \sum_{(x,y)\in Ngd(x_0y_0)} \left(I(x,y) + \frac{\partial I}{\partial x}h_x + \frac{\partial I}{\partial y}h_y - J(x,y)\right)^2$$

evaluated at $(x_0 y_0)$



To minimize, take $\frac{\partial}{\partial h_x}$, $\frac{\partial}{\partial h_y}$ of

$$\sum_{(x,y)\in Ngd(x_0y_0)} \left(I(x,y) + \frac{\partial I}{\partial x}h_x + \frac{\partial I}{\partial y}h_y - J(x,y)\right)^2$$

and set to zero.

$$\sum 2\left(I(x,y) + \frac{\partial I}{\partial x}h_x + \frac{\partial I}{\partial y}h_y - J(x,y)\right)\frac{\partial I}{\partial x} = 0$$

$$\sum \left(\qquad '' \qquad \right)\frac{\partial I}{\partial y} = 0$$

$$\sum 2\left(I(x,y) + \frac{\partial I}{\partial x}h_x + \frac{\partial I}{\partial y}h_y - J(x,y)\right)\frac{\partial I}{\partial x} = 0$$

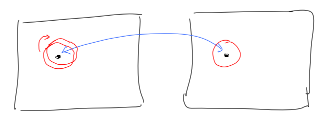
$$\sum \left(\qquad '' \qquad \right)\frac{\partial I}{\partial y} = 0$$

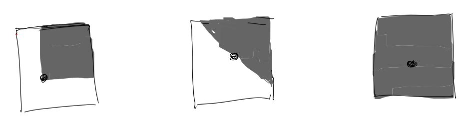
$$\sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix} \begin{bmatrix} h_x \\ h_y \end{bmatrix} = -\sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} (I-J)\frac{\partial I}{\partial x} \\ (I-J)\frac{\partial I}{\partial y} \end{bmatrix}$$

Solve for $(h_x h_y)$.

$$\sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix} \begin{bmatrix} h_x \\ h_y \end{bmatrix} = -\sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} (I-J)\frac{\partial I}{\partial x} \\ (I-J)\frac{\partial I}{\partial y} \end{bmatrix}$$

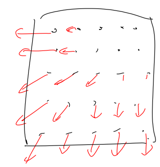
We can solve (uniquely) for $(h_x h_y)$ if M is invertible, namely λ_1, λ_2 both non-zero.





"corner"	"edge"	"uniform"
$\lambda_1 > 0$	$\lambda_1 > 0$	$\lambda_1 \approx 0$
$\lambda_2 > 0$	$\lambda_2 \approx 0$	$\lambda_2 \approx 0$

Notation



$Ngd(x_0, y_0)$

$$A = \begin{bmatrix} \uparrow & \uparrow \\ \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \\ \downarrow & \downarrow \end{bmatrix}_{n\times 2}$$

$$M = A^T A = \sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

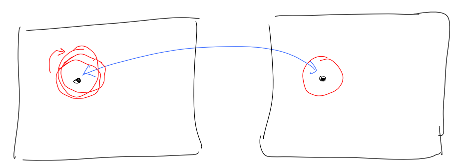
$$\sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix} \begin{bmatrix} h_x \\ h_y \end{bmatrix} = -\sum_{(x,y)\in Ngd(x_0y_0)} \begin{bmatrix} (I-J)\frac{\partial I}{\partial x} \\ (I-J)\frac{\partial I}{\partial y} \end{bmatrix}$$

$$A^T A \begin{bmatrix} h_x \\ h_y \end{bmatrix} = -A^T(\vec{I} - \vec{J})$$

Iterative Method

$$(h_x^0, h_y^0) = (0, 0)$$

$$(h_x^{k+1}, h_y^{k+1}) = (h_x^k, h_y^k) + (h_x, h_y)$$



Iterative Method

Repeat until convergence:

$$\sum_{x\in Ngd(x_0)} \left(I(x+h_x^k, y+h_y^k) - J(x,y)\right)^2$$

$$\approx \sum_{x\in Ngd(x_0)} \left(I(x+h_x^{k+1}, y+h_y^{k+1}) - J(x,y)\right)^2$$