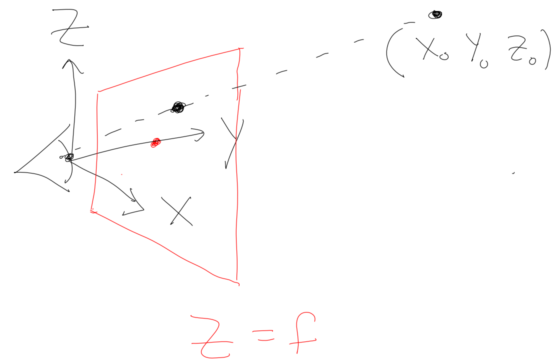
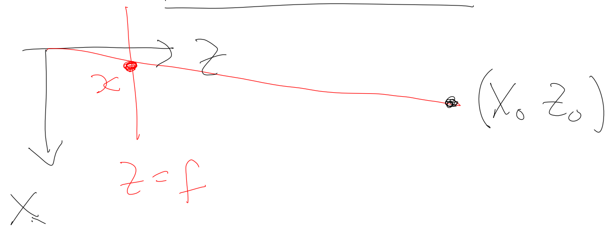
Lecture 1

Image Projection

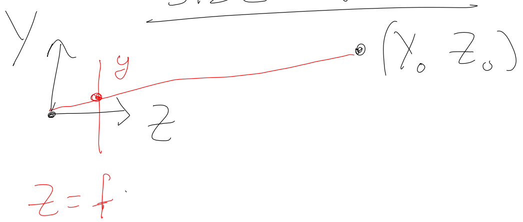


TOP VIEW

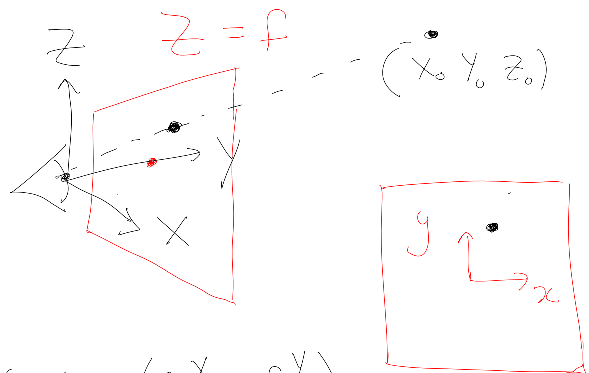


$$\frac{x}{f} = \frac{X_0}{Z_0}$$

SIDE VIEW



$$\frac{y}{f} = \frac{Y_0}{Z_0}$$



$$(x,y) = \left(f\frac{X}{Z},\ f\frac{Y}{Z}\right)$$

Example: Planar Surfaces

Example: a plane

$$aX + bY + cZ = d$$

Multiply by $\frac{f}{Z}$.

$$a\frac{fX}{Z} + \frac{bfY}{Z} + \frac{cfZ}{Z} = \frac{fd}{Z}$$

$$ax + by + cf = \frac{fd}{Z}$$

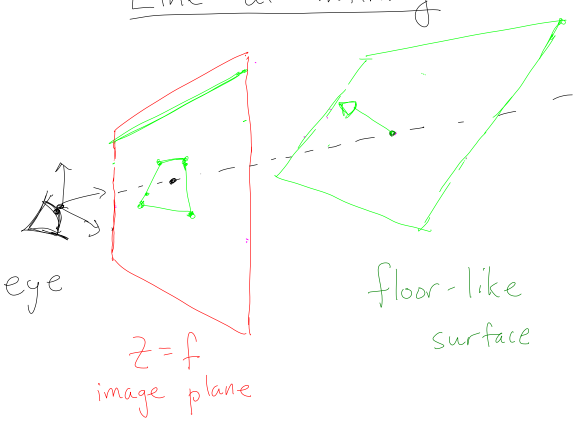
$$ax + by + cf = \frac{fd}{Z}$$

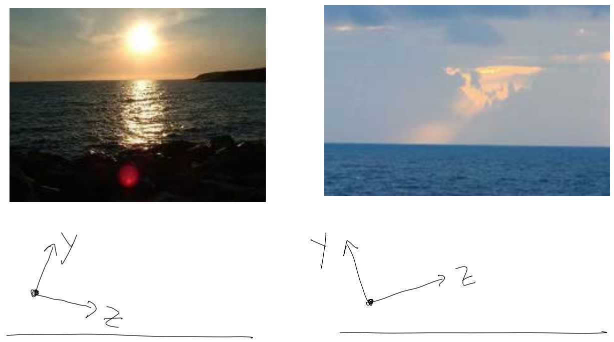
Let $Z \to \infty$ gives

$$ax + by + cf = 0$$

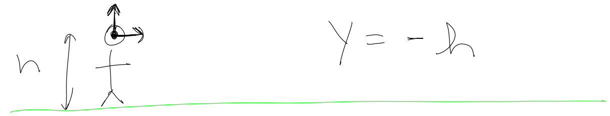
"line at infinity"

Line at infinity





Example : ground plane



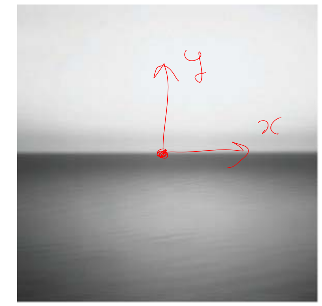
$Y = -h$

$$y = f\,\frac{Y}{Z} = -\frac{f\,h}{Z}$$

when $Z \to \infty$, $y \to 0$

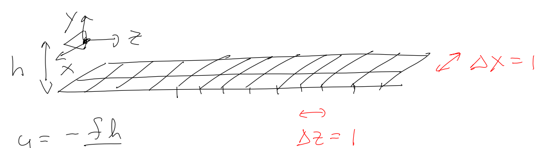
" horizon "

$$y = \frac{-fh}{Z}$$



Ground plane

$Y = -h$



$$y = -\frac{fh}{Z}$$

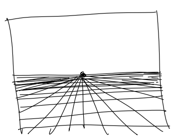
$$\Delta y = \frac{fh}{Z^2}\,\cancel{\Delta Z}\ 1$$

$$x = \frac{fX}{Z}$$

$$\Delta x = \frac{f}{Z}\,\cancel{\Delta X}\ 1$$

$$\frac{\Delta y}{\Delta x} = \frac{h}{Z_0}$$

} foreshortening of tiles depends on position



Depth Gradient

$$aX + bY + cZ = 1$$

$$\frac{\partial Z}{\partial x} = -\frac{a}{c} \qquad \frac{\partial Z}{\partial y} = -\frac{b}{c}$$

$$\nabla Z = \left(-\frac{a}{c},\ -\frac{b}{c}\right)$$

$$|\nabla Z| = \sqrt{\frac{a^2 + b^2}{c^2}}$$

Depth Gradient

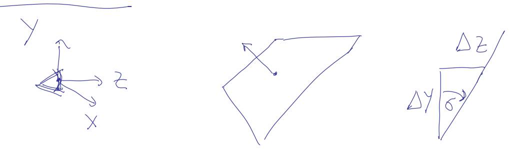
Define angles σ, τ such that

$$\nabla Z = \tan\sigma\,(\cos\tau,\ \sin\tau)$$

i.e. τ is the direction of ∇Z

and $|\nabla Z| = \tan\sigma$

EXAMPLE : $a = 0,\ b \neq 0,\ c \neq 0$

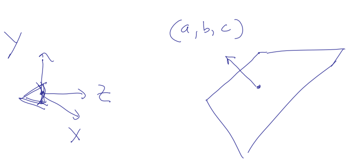


Note: The following few slides were changed to conform better to the lecture notes.

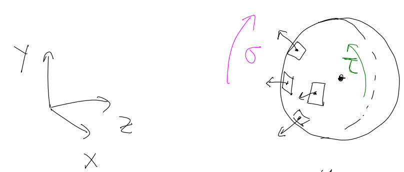
Surface Normal

$$aX + bY + cZ = d$$

normal vector to plane is (a, b, c)



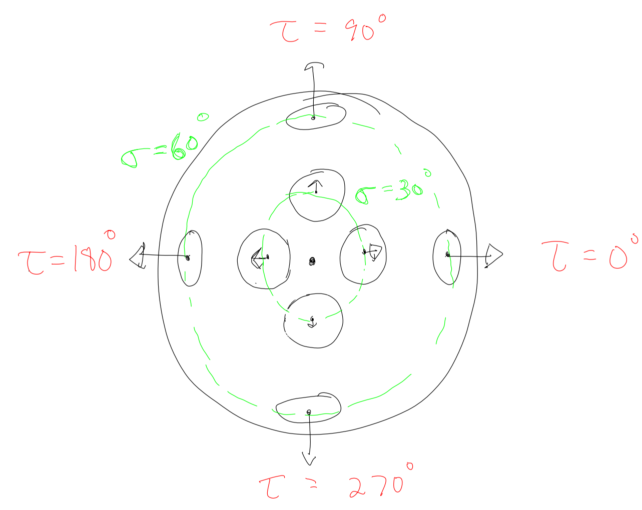
Spherical Coordinates



σ, latitude (north pole is $-Z$ axis)

τ, longitude

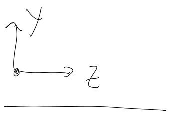
$$\tan\sigma = |\nabla Z| = \sqrt{\frac{a^2 + b^2}{c^2}}$$



slant & tilt

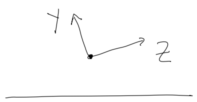
$\sigma \approx 70°$ (roughly)

$\tau = 90°$



$\sigma = 90°$

$\tau = 90°$



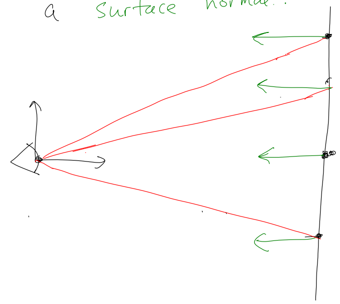
$\sigma \approx 70°$ (roughly)

$\tau = 270°$

very subtle

WARNING (POSSIBLE CONFUSION)

The definition of slant and tilt applies to a (global) plane, not to the angles between the line of sight to a point and a surface normal.

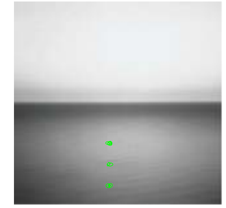


For this example,

$|\nabla z| = 0$

$\sigma = 0$

τ undefined



In this example, (global) slant is constant ($\sigma = \infty$).

But "local slant" varies.

ie angle between line of sight and normal vector.