Midterm Exam (Solutions)

Fundamentals of Computer Graphics (COMP 557)
Tues. Oct. 14, 2008
Professor Michael Langer

1. **(4 points)**

   This question was meant to check whether you understand some of the elementary linear algebra that you need for this course. (The specific application to ray casting was described in lecture 7 page 4. Also see the last paragraph of lecture 7 page 6.)

   (a) The vector (A, B, C) is the normal of the plane. The normal can be computed with a cross product
   $$p_0p_1 \times p_1p_2 = (2, 0, 6) \times (-2, 3, 0) = (-18, -12, 6)$$
   Substituting p_0 gives $D = -24$, and so $(A, B, C, D) = (-18, -12, 6, -24)$.

   (b) Substituting $(1, 1, 7)t$ into the equation of the plane gives: $-18t - 12t + 6(7t) - 24 = 0$, and so $t = 2$, and so the point of intersection is $(2, 2, 14)$.

   Marking scheme: 1 point for the normal (A,B,C), 1 point for D, 1 point for the parametric equation of the line, and 1 point for correct substitution.

2. **(4 points)**

   A similar question as this was Exercises 1 Q3.

   (a)
   $$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{9}{2} & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

   Marking scheme: 2 points for each.

3. **(2 points)**

   See lecture 6 pages 1-3.

4. **(2 points)**

   See lecture 7.

   Marking scheme: I expected you to say that the z buffer is a 2D array whose elements correspond to the pixels in the image. The z value stored at each element of the z buffer is the z value of the nearest surface at that pixel that has been examined thus far.

   The z buffer does *not* store polygons or colors.

5. **(3 points)**

See lecture 8 page 1-2.

(a) Expanding the equation, we get

$$x^2 - 6x - 11 + y^2 + 4z^2 = 0,$$

which can be written in the form $\mathbf{x}^T\mathbf{Q}\mathbf{x} = 0$ with:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ -3 & 0 & 0 & -11 \end{bmatrix}.$$

(b) The projective transformation for near and far clipping planes at $z = -1$ and $z = -8$, is:

$$\mathbf{M} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -9 & -8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and so we can rewrite $\mathbf{x}^T\mathbf{Q}\mathbf{x} = 0$ as follows:

$$(\mathbf{x}^T\mathbf{M}^T)(\mathbf{M}^{-T}\mathbf{Q}\mathbf{M}^{-1})(\mathbf{M}\mathbf{x}) = 0.$$

The product $\mathbf{M}^{-T}\mathbf{Q}\mathbf{M}^{-1}$ is the coefficient matrix of the quadric in the projected view volume.

6. **(3 points)**

(a) The main idea is that if a ray doesn't intersect the bounding volume of a (complicated) object, then it doesn't intersect the object. So, you first test the intersection of the ray with the bounding object; if no intersection is found, then you do not need to check all the parts of the object. The bounding box has a simpler shape than the object, so its simpler to compute the intersection of the ray with the bounding box.

(b) You would change the bounding volume of the moving object from frame to frame.

7. **(2 points)**

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 16 & 8 & 4 & 2 & 1 \\ 81 & 27 & 9 & 3 & 1 \\ 256 & 64 & 16 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$