

1. (a) Since  $(s, t) = (0, 0)$  maps to  $\mathbf{p}_1$ , we have  $(x_0, y_0, z_0) = \mathbf{p}_1 = (-5, 2, -7)$ .

To get  $\mathbf{p}_2$ , we note:

$$\mathbf{p}_2 = \mathbf{p}_1 + (\mathbf{p}_2 - \mathbf{p}_1)$$

and so

$$(a_x, a_y, a_z) = \mathbf{p}_2 - \mathbf{p}_1 = (-6, 4, -8) - (-5, 2, -7) = (-1, 2, -1).$$

Similarly,

$$(b_x, b_y, b_z) = \mathbf{p}_3 - \mathbf{p}_1 = (-7, 5, -3) - (-5, 2, -7) = (-2, 3, 4).$$

Thus,

$$\begin{bmatrix} a_x & b_x & x_0 \\ a_y & b_y & y_0 \\ a_z & b_z & z_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -5 \\ 2 & 3 & 2 \\ -1 & 4 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

We project the triangle onto the  $z = -2$  plane, via perspective projection, and so the projection matrix is:

$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Multiplying these two matrices ( $3 \times 4$  and  $4 \times 3$ )

$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -2 & -5 \\ 2 & 3 & 2 \\ -1 & 4 & -7 \\ 0 & 0 & 1 \end{bmatrix}$$

would give the homography ( $3 \times 3$ ).

- (b) If we were to consider *orthographic projection* in the  $z$  direction, rather than perspective projection, then the projection matrix we would use would be

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which amounts simply to dropping the  $z$  value. The homography would be:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 & -5 \\ 2 & 3 & 2 \\ -1 & 4 & -7 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -5 \\ 2 & 3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$