1. In lecture 15, I gave the basic setup for constructing a homography matrix, which involved a 3×3 invertible transformation:

$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \mathbf{H} \begin{bmatrix} s \\ t \\ 1 \end{bmatrix}$$

where

$$\mathbf{H} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_x & b_x & x_0 \\ a_y & b_y & y_0 \\ a_z & b_z & z_0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The matrix

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

is just the matrix

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & f & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

without the third row i.e. we take the perspective projection onto the $z = f$ plane, and we are only interested in the x, y position within this projection plane.

A few students have asked me for clarification on how this works. Here is another example (question) which I hope will help.

(a) Consider a 3D triangle that is defined by vertices:

$$\mathbf{p}_1 = (-5, 2, -7), \quad \mathbf{p}_2 = (-6, 4, -8), \quad \mathbf{p}_3 = (-7, 5, -3)$$

and suppose that we project this triangle onto the $z = -2$ plane, using perspective projection. We also texture map this triangle, and choose texture coordinates such that

$$(s, t) = (0, 0) \rightarrow \mathbf{p}_1$$

$$(s, t) = (1, 0) \rightarrow \mathbf{p}_2$$

$$(s, t) = (0, 1) \rightarrow \mathbf{p}_3.$$

Give the homography that corresponds to the mapping from (s, t) to the projection plane coordinates (x, y).

(b) Suppose we were to consider *orthographic projection* in the z direction, rather than perspective projection. What would the homography be then?