1. Bump mapping is based on the following model of a surface $\mathbf{p}_{bump}(s,t)$,

$$\mathbf{p}_{bump}(s,t) = \mathbf{p}(s,t) + b(s,t)\mathbf{n}(s,t)$$

where $\mathbf{n}(s,t)$ is the normal vector on $\mathbf{p}(s,t)$ and $b(s,t)$ is the height of the bumps.

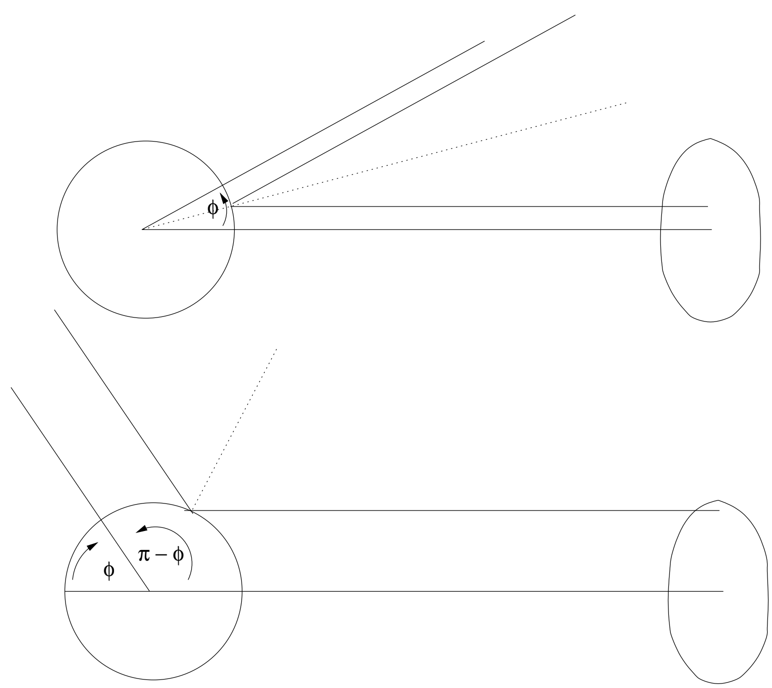
Consider the specific case of adding a bump map to a rectangle, lying in the plane $y = y_0$. Assume the normal of the rectangle is the unit y vector.

   (a) Write the above equation for this specific case. Be sure to specify the 3D coordinates of points on the rectangle.

   (b) How you would compute the surface normal on this bump mapped surface?

2. The disk map represents the environment as the image seen in small mirror sphere. It uses orthographic projection of the mirror sphere onto an image plane. Assuming the disk is sampled by a square grid of pixels, show that the disk map representation uses as many samples of the environment map for 3D points behind[1] the sphere as it does for 3D points in front of the sphere.

More generally, consider any cone of radius say $\phi < \frac{\pi}{2}$ centered in the direction of the camera (z). What is the percentage of the disk map occupied by the directions that are bounded by this cone? Give a similar expression for the case the $\phi > \frac{\pi}{2}$.



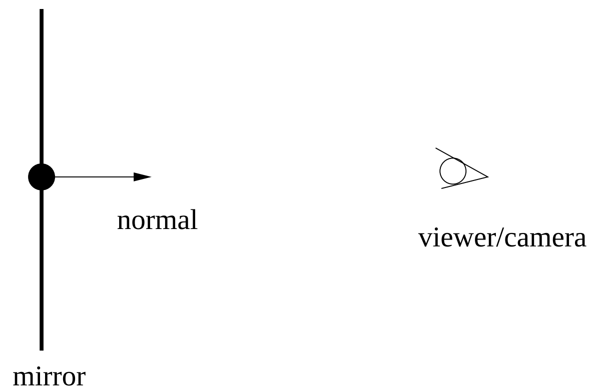
3. Consider the figure below. Indicate the scene volume that is visible to the viewer, as a reflection in the mirror. The viewer position lies in this volume, for example.

Compare two cases:

[1]namely, at a depth greater than that of the center of the sphere

(a) if ray tracing were used;

(b) if environment mapping were used. (Assume that the environment map is computed from the black dot at the base of the surface normal in the figure.)



4. Show that the "over" operative is associative, when premultiplied $rgb\alpha$ is used,

$$F \text{ over } (M \text{ over } B) \;=\; (F \text{ over } M) \text{ over } B.$$

where F is foreground, M is middle, B is background.

5. To define the illumination reaching a general surface, one needs to define a function of 4 variables. Why?

6. Suppose we have a function $g(x, y, z)$ that is defined at the eight corners of a unit cube. Give a formula for interpolating these corner values to some interior value at (x, y, z).

7. The typical display model for a CRT says that if the brightness and contrast knobs are correctly set, then when the voltage V_{rgb} is set to $(0, 0, 0)$ the display will produce a zero spectrum i.e. no light. In practice this model doesn't hold exactly. For example, displays reflect at least *some* light from the surrounding scene so even if the power of the display is turned off (!) there will be some light coming off the display.

   One way to model the *reflected* component of the light from the display is to let it have some ambient spectrum $a(\lambda)$. How could you model (in LMS space) the spectrum of light coming off the display, given that it is the sum of an emitted and reflected component?

8. About ten percent of human males are missing one of the three types of visual cones needed for seeing in color. Such people are said to be *color blind.*

   Consider the following statement. "Let $I_1(\lambda)$ and $I_2(\lambda)$ be any two color spectra. If a color blind person who is missing the L cones can distinguish these two spectra, then a normal color vision person can distinguish these spectra."

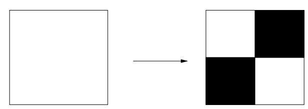
   Is the statement true? If so, then justify it using a mathematical argument. If not, then explain why.

9. Suppose we have five pixels ($x = 1, \ldots, 5$) that are contained in two images F and B. Let the F_α values for these five pixels be $(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1)$ and let the B_α values for these five pixels be $(1, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, 0)$. What are the α values of F over B ?

10. "Blue screen matting" uses a blue opaque screen for the background, i.e. $B_{rgb\alpha} = (0, 0, 1, 1)$. Suppose we were to restrict our foreground scenes so that they had zero blue component, i.e. $F_{rgb} \approx (F_r, F_g, 0)$. Show that these conditions provides sufficient information for us to estimate $F_{rgb\alpha}$.

11. In class, we examined a set of objects called "fractals" which were defined by a scale factor s and a number of copies N. e.g. For the Koch curve, we had $s = 3$ and $N = 4$.

    Consider an object that is defined recursively as follows: Begin with a unit square. Partition this square into four equal subsquares. Delete the two off-diagonal subsquares, as shown below. Repeat recursively on the remaining subsquares.



    Sketch the object that one obtains after the third iteration (the first iteration is shown above), and calculate the dimension D of the object that one obtains in the limit. Use the formula:

$$D = \frac{\log N}{\log s}.$$

12. Suppose that a shiny ground plane $y = 0$ is illuminated by sunlight. Let the direction of the sun be $(1, 2, 2)$. If a viewer is at $(x, y, z) = (4, 6, 7)$, determine the position on the ground plane at which the peak of the highlight occurs.

13. Suppose that a shiny plane, $2x + y + 2z + 17 = 0$, is illuminated by sunlight. Let the direction of the sun be $\mathbf{L} = (0, 1, 0)$. Assuming the viewer is at the origin, $(x, y, z) = (0, 0, 0)$, determine the position on this shiny plane at which the peak of the highlight occurs.

14. Consider a triangle in the image plane. The intensities $I(x, y)$ at the three vertices are:

$$I_1 = I(40, 14) = 10, \quad I_2 = I(60, 34) = 90, \quad I_3 = I(80, 24) = 140$$

    Using Gouraud shading, calculate the intensity $I(50, 19)$.