1. Show that
   - scaling matrices commute, $\mathbf{S}_1\mathbf{S}_2 = \mathbf{S}_2\mathbf{S}_1$ (trivial)
   - translation matrices commute, $\mathbf{T}_1\mathbf{T}_2 = \mathbf{T}_2\mathbf{T}_1$ (trivial)
   - rotation matrices typically do *not* commute, $\mathbf{R}_1\mathbf{R}_2 = \mathbf{R}_2\mathbf{R}_1$ (though they do commuate always, if rotation is about a common axis)
   - rotation and translation typically do *not* commute, $\mathbf{RT} \neq \mathbf{TR}$
   - scaling and translation typically do *not* commute, $\mathbf{ST} \neq \mathbf{TS}$
   - scaling and rotation typically do *not* commute, $\mathbf{SR} \neq \mathbf{RS}$

2. One can define an ellipsoid surface in $\Re^3$ by scaling, rotating, and translating a unit sphere. Define an ellipsoid whose:
   - center is $(8, 1, 3)$ in world coordinates,
   - x, y, z axes are of length $1, 1, 4$, respectively.
   - z axis is in direction $(1, 1, 0)$ in world coordinates,

   For your answer, it is sufficient that you specify suitable matrices **S, R, T**, and then use them to define a quadric matrix **Q** as in (a).

3. Consider a camera at $(3, 4, 3)$ which lies in the plane

   $$x - 2y + z + 2 = 0.$$

   Define the VUP direction to be $(1, -2, 0)$. Give a transformation that projects the scene *orthographically* onto this plane, i.e. the projection is in the direction normal to the plane, all points in $\Re^3$ are projected onto the plane.

4. (a) Given three points $(x(0), y(0))$, $(x(1), y(1))$, $(x(2), y(2))$ in the (x, y) plane, show how to compute a matrix $\mathbf{C}$ such that

   $$\mathbf{q}(t) \;=\; (x(t), y(t)) \;=\; \mathbf{C} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix}$$

   and the curve $\mathbf{q}(t)$ passes through the three given points at $t = 0, 1, 2$. Note that the curve $\mathbf{q}(t)$ is *quadratic*, rather than cubic. Be sure to specify the size of the matrix $\mathbf{C}$ in your answer.

   [Hint: Use a similar method to what we saw in class for cubic curves.]

   (b) Give a formula for the tangent vector $\mathbf{q}'(t)$ to this curve (not necessily of unit length). Your formula should be similar to the one above.

   (c) Suppose we are given only two points $\mathbf{q}(0)$ and $\mathbf{q}(1)$ in the plane. How could one define a *quadratic* curve through these points by choosing a suitable tangent vector, or a set of tangent vectors?