Questions

1. A point light source can either be local (at a finite distance) or at infinity. For each of these two possibilities, what projection model would be suitable for computing a shadow map? What can you say about the view volume used to compute the shadow map?

2. Suppose the scene is illuminated by a hemispheric uniform source which is centered in the y direction. Give a qualitative formula for $I_{\text{diffuse}}(x)$ which describes attached shadow effects, that is shadows that are due to the surface normal facing away from parts of the sky. (This effect is similar to the surface being a back-face, but now it has its backface to part of the light).

3. Ambient occlusion is better suited to static scenes than dynamics scenes. Why?

4. I mentioned in the lecture that surfaces can reflect light to each other and the amount can be represented by a function $F(x, y)$ which serves a matrix element, where $x$ and $y$ are vertices in the scene. How does $F(x, y)$ depend on the properties at $x$ and $y$?

5. [Added April 19]
   In photography (and in image science), the term “image contrast” refers to the range of intensity values, relative to the average value. A typical way to define contrast is:
   $$\text{contrast} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{ave}}}.$$
   For example a scene with lots of black and white painted surfaces would have a greater contrast than a scene with the same geometry and lighting, but in which all the surface were painted grey.
   Suppose you have an indoor scene that is illuminated by a light bulb. Would the contrast of the scene be different if all the surfaces were painted black versus if all the surface were painted white? Note that real black paint has reflectance slightly greater than 0 and real white paint has reflectance slightly less than 1.
Solutions

1. For a point source at infinity, you would use orthographic projection since all light rays from the source to the scene are parallel. The view volume used for the shadow map should contain the view volume for the camera, since for any point in the camera’s view volume you want to know if that point is in shadow.

For a local point source (a finite 3D point), you would use perspective projection. The view volume aspect of the question is more subtle. If the point source is outside the view volume of the camera, then we can choose a shadow map view volume that contains the camera’s view volume. However, if the point source is inside the view volume, then no since shadow map view volume can contain the camera’s view volume. In this case, we need to compute multiple shadow maps. To be safe and simple, we could consider six shadow maps, covering the six faces of a cube. This is similar to the cube map representation of environment mapping.

2. In the lecture, I gave the formula

\[ I_{\text{diffuse}}(x) = \frac{1 + n(x) \cdot \hat{y}}{2}. \]

The idea is that point with \( n = y \) will face overhead and will be maximally illumination and so \( I_{\text{diffuse}}(x) = 1 \) for these points. For points where \( n \) is in the \( xz \) plane, half the sky is visible and \( I_{\text{diffuse}}(x) = \frac{1}{2} \), and for points such as on the bottom of the sphere where \( n = -y \) we would have \( I_{\text{diffuse}}(x) = 0 \).

3. Since objects cast shadows, any movement of objects will change the shadow relationships between surfaces.

The point of ambient occlusion is to precompute how much of the sky (or diffuse illumination) of the scene is visible and “bake” it into the scene representation e.g. attach a scalar value to each vertex. This allows you to move the camera through the baked-in scene at runtime. If there is motion of objects while you are moving the camera, however, then the precomputing ambient occlusion scalars will change over time. This can be a problem. An example I mentioned in class (in response to a question) was a car. The underside of the car will be dark, but as the car moves, the points that are underside will change. This cannot be baked in.

4. There are three factors:

- distance between \( x \) and \( y \). If you think of \( y \) as a light source for \( x \) then as the distance between them increases, the effect of \( y \)’s reflected light on \( x \)’s illumination decreases. This has a \( d^{-2} \) effect just like any small source.

- \( n(x) \) and \( n(y) \): if \( x \) and \( y \) face each other directly, then they have a bigger effect on each other than if they are turned obliquely away from each other. So, let \( v \) be a unit vector point from \( x \) to \( y \) or viceversa. \( F(x,y) \) will be proportional to both \( |n(x) \cdot v| \) and \( |n(y) \cdot v| \).

- whether \( x \) and \( y \) see each other. If they don’t see each other (either because they are not facing each other or because there is an intervening surface) then \( F(x,y) \) will be zero.
5. [UPDATED April 28]

First, note that if there are shadows in the scene then the contrast can be quite high. Naively, shadows act like black painted surfaces in that they give very low image intensities.

Second, if you understand the definition of contrast, then you might have answered that the contrast would be the same for the black painted scene versus the white painted scene. You might have reasoned that the reflectance (white versus black paint) merely scales the intensity of reflected light, and since contrast is defined by a ratio of intensities, you might have thought that the reflectance scale factor would cancel out – regardless of whether the paint is white or black. If you reasoned this way, then that is good: you understand the definition of contrast.

However, that reasoning is not quite correct since it does not consider interreflections. Interreflections add a component to the illumination at each scene point. In particular, shadows do not have zero intensity since there is still some 'ambient' light that enters shadows and gets reflected. This ambient light comes from indirect illumination i.e. from other surfaces. (In OpenGL, one assumes that the ambient component is constant, but this is not the case in real scenes.)

What is the effect of interreflections on contrast? The answer is a bit subtle, but see if you can understand. The interreflection component will be relatively weak for the black painted scene, and relatively strong for the white painted scene. To see this, compare a black painted scene until a bright light source versus a white painted scene under dim light source. Suppose that the reflected light intensity that is due to the direct illumination is the same in the two cases. Now consider the shadows. In the white painted scene, most of the indirect light that enters the shadow region will be reflected because that is what white surfaces do. But in the black painted scene, most of the light that enters the shadows will NOT be reflected because that is what black surfaces do. So the shadows will have much greater reflected intensity in the white painted scene than in the black painted scene. And thus, the contrast will be much less in the white painted scene than in the black painted scene (because the contrast is basically the ratio of the intensities in the illuminated vs shadowed regions).