1. Let $\mathbf{p} = (1, 3, 4)$ and $\mathbf{q} = (5, 2, 7)$ be two vectors in $\Re^3$.

   Calculate $\mathbf{p} \times \mathbf{q}$. Calculate $\mathbf{p} \cdot \mathbf{q}$. Verify for yourself that $\mathbf{p} \times \mathbf{q}$ is perpendicular to both $\mathbf{p}$ and $\mathbf{q}$. The purpose here is to make sure that you know how to do cross and inner products!

2. Let $\mathbf{p}$ and $\mathbf{q}$ be two vectors in $\Re^3$ and let $\mathbf{p}' = (2, 7, 4, 3)$ and $\mathbf{q}' = (1, 3, 5, 2)$ be a representation of these vectors in homogeneous coordinates. Using the $\Re^3$ representation of these vectors, calculate $\mathbf{p}' \times \mathbf{q}'$ and $\mathbf{p}' \cdot \mathbf{q}'$.

3. Write each of the following transformations as a sequence of 4×4 matrices:

   (a) Scale the scene by a factor 3 in the y direction, while leaving the position of the point $(x, y, z) = (4, -1, 1)$ unchanged.

   (b) Rotate the scene θ degrees, such that the axis of rotation is the vector $(1, 1, 0)$ and the origin $(0, 0, 0)$ stays fixed. Define the direction of positive rotation to be clockwise when one is looking towards the origin in the direction opposite to the vector, namely one is looking in direction $(-1, -1, 0)$. Assume (as always) a right handed coordinate system.

4. Rewrite the following matrix as a product of a scaling, a rotation, and a translation (not necessarily in that order) :

$$\begin{bmatrix} 0 & -3 & 0 & 10 \\ 2 & 0 & 0 & 11 \\ 0 & 0 & 4 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

   Hint: Note that the rotation matrix should have determinant 1, and the scaling matrix should have non-negative elements only.

5. Consider the following perspective projection,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}.$$

   What is the projection plane?

6. Give a transformation that maps the plane $y = f_0$ to the plane $y = 0$ and that maps the plane $y = f_1$ to the plane $y = 1$. Hint: use a translation and a scaling.

7. Give the 4×4 projective transformation matrix that is defined by

   `gluPerspective(90.0, 1.0, 4.0, 20.0)` .

   This transformation maps the view volume to a normalized view volume (see lecture 6), in particular, it maps the near plane to $z = -1$ and the far plane to $z = 1$ and in doing so, it switches from a right handed to a left handed coordinates!

8. How would one choose the first four arguments of `glFrustum` to achieve the same view volume as in the previous example ?

   Hint: $\tan(30) = \frac{1}{\sqrt{3}}$.