Instructions

- This is a closed book exam. You are allowed one crib sheet.
- If your answer does not fit on a page, then use the reverse side.
- There are 19 points available. However, for any score over 15, you will only half the extra points. Thus the maximum score you can receive is 17/15.
1. (3 points)

(a) Sketch an example of a spike train produced by an ON-CENTER retinal ganglion cell when a bright spot of light moves across its receptive field.
Your answer should be a function of time, not position. Explain your sketch and state your assumptions.

**SOLUTION:**
There should be some random sparse spikes, followed by a quiet period (in the surround), followed by a dense set of spikes (in the center), followed by a quiet period (in the surround), followed by sparse random spikes. Assume the spot is the size of central region, and passes through the center.

(b) How would you answer change if you greatly increased the speed of the motion? Explain.

**SOLUTION:**
If the speed is high, then the cell would not have time to give a strong response e.g. the spot of light might only be in central region for a spike or two. Similarly it would only be in the inhibitory surround for a small time. In this case, you would not see any difference in the spike train between this case and the case in which there is no stimulus.
2. (4 points)

(a) Sketch the result of a convolution of a 1D sine Gabor with a step edge image \( I(x) \):

\[
I(x) = \begin{cases} 
2, & x \geq x_0 \\
1, & x < x_0.
\end{cases}
\]

Mark \( x = x_0 \) in your sketch. Also, sketch the Gabor.

**Hint:** use a small envelope for the Gaussian component of your Gabor, so that you can approximate the Gabor with just one positive and one negative region.

**SOLUTION:**

(1 point) If the Gabor has only one positive and one negative region, then convolving with the Gabor is very similar (qualitatively) to taking the derivative. \( I \ast \text{Gabor}(x) \) has value 0 except near the \( x_0 \). It will have a peak at the value \( x_0 \), but because the convolution flips the Gabor relative to \( I(x) \), it will be a negative peak.

(b) Suppose the image from (a) were replaced by a raised sine function,

\[
I(x) = I_0 + a \sin\left(\frac{2\pi}{N}k_1x\right).
\]

Assume that the frequency \( k_1 \) of the image is equal to the peak frequency \( k_0 \) of the sine Gabor.

Sketch the result of convolving the \( I(x) \) with the Gabor. Mark \( x = 0 \) in your sketch.

**SOLUTION:**

(2 points: 1 for disappearance of \( I_0 \), 1 for cosine of same freq.)

The answer is a (minus) cosine function with frequency \( k_1 \). It has negative peak at \( x = 0 \), which is due to the flipping of the Gabor in the convolution. To get this answer, note that for different positions of the Gabor, the maxes and mins of the Gabor line up with the maxes and mins of \( I(x) \) with the same period as \( I(x) \).

How could this answer be derived formally using Fourier transforms? The sine Gabor has zero response to a constant \( I_0 \), so we may ignore that component. Otherwise, \( I(x) \) has just one frequency component in \( 0 < k < \frac{N}{2} \). By the convolution theorem, convolving the image with a Gabor will just give that same frequency component, with possibly a different amplitude and phase. For the phase, the sine Gabor has a purely imaginary Fourier transform, namely from Exercises 8 Q3, and the phase spectrum of a sine Gabor is -90 degrees for all frequencies. Thus, each sine component becomes a -cosine component.

(c) Same question as (b), but now suppose the frequency \( k_1 \) of the image \( I(x) \) is different from the peak frequency \( k_0 \) of the sine Gabor. In what way(s) does the answer here differ from the answer to (b)?

**SOLUTION:**

(1 point) Here you really have to use the Fourier transform theory to get it right. For the same reason as (b), you again get a minus cosine of frequency \( k_1 \) i.e. a negative peak at \( x = 0 \). The amplitude of the response will be smaller than in (b) because the Gabor has a smaller gain (magnitude of amplitude spectrum) because now \( k_0 \neq k_1 \).
3. (3 points)

(a) For any \((x, y)\) on the retina, briefly describe how different orientations of lines or edges in the image are represented in area V1 of the cortex. Include a sketch in your answer. Consider just one eye’s input for this part of the question.

**SOLUTION:**

(1.5 points)

Cells of each orientation are found in "columns" which run perpendicular to the surface of the cortex. For any \((x, y)\), these columns lie adjacent to each other in a "hypercolumn". Orientation changes continuously from one column to the next.

For the sketch see lecture 9, slide 20.

Grading: Several students just said that there are even and odd cells that response to lines and edges respectively and draw a picture of Gabors. I gave 0.5 points for this. I gave 1.5 points for the sketch with hypercolumns, with a description.

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(b) Cortical area V1 contains both monocular and binocular cells. Monocular cells exist for all orientations. However, binocular cells typically have orientations that are close to vertical. This makes sense from a computational perspective. Why?

**SOLUTION:**

The eyes are separated horizontally, and so disparities are typically horizontal. If binocular cells were constructed from horizontally oriented left and right components, these cells would have poor sensitivity to horizontal disparities. Why? Because the cells would give large responses only to horizontal lines and edges but these responses wouldn’t change as the lines and edges are shifted horizontally. So horizontal cells would be essentially useless for detecting (horizontal) disparities.

Grading: There was a wide range of answers here. I gave up to 1.5 points.

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1I say "typically" because in fact there can be small vertical disparities too, when the eyes are not looking straight off to infinity, or when the eyes roll slightly because the viewer is looking up or down, relative to the orientation of his head. But I haven’t discussed these vertical disparities in the lectures.
4. (2 points)

Panum’s fusional area refers to the range of depths over which the brain can fuse the left and right eye images of a scene point or surface. In fact, *Panum’s fusional area increases with eccentricity.*

(a) Sketch Panum’s fusional area for 2D "from above" view of a binocular observer, given the statement in italics above.

**SOLUTION:**
See sketch on left.

(b) Draw the corresponding disparity space \((x_l, x_r)\) representation.

**SOLUTION:**
See sketch above right.
5. (4 points)

(a) Suppose a set of vertically oriented lines is travelling to the right at speed \( v_x \) and a set of horizontally oriented lines is travelling upwards at speed \( v_y \).

What is the velocity of the points of intersection of these lines?

**SOLUTION:**

(Grading: 1 point)

The velocity is \((v_x, v_y)\). There is nothing to compute here. (Many students got tangled up in partial derivatives of the image intensities.)

Also, some of you mixed up "speed" and "velocity". Velocity is a vector. Speed is a scalar, namely the length of the velocity vector.

(b) Consider a bright disk on a dark background. Suppose the disk is moving in the \( x \) direction at unit speed. 

What are the normal velocities of points on the boundary of the disk?

For simplicity, assume the disk has radius 1, and answer this question for specific \((\cos \theta, \sin \theta)\) on the disk boundary, namely \(\theta = 0, 45, 90, 135, 180\) degrees.

**SOLUTION:**

- For \(\theta = 0\) and 180 deg, the normal velocity is \((1,0)\).
- For \(\theta = 90\), the normal velocity is \((0,0)\).
- The more subtle ones are \(\theta = 45\), where the normal velocity is \((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\) and \(\theta = 135\), where the normal velocity is \((\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})\).

For example, for 45 deg, we have a motion constraint line that passes through \((1,0)\) but has slope -1, so it also passes through \((0,1)\). By inspection the normal velocity is \((\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})\). Note the speed in the normal direction is \(\frac{1}{\sqrt{2}}\), not 1.

For 135 deg, we have a motion constraint line that passes through \((1,0)\) and has slope 1, so it also passes through \((0,-1)\). By inspection the normal velocity is \((\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})\). The speed in the normal direction is \(\frac{1}{\sqrt{2}}\), not 1.

(Grading: Gave 0 for totally wrong idea. Took off 0.5 points from 3 for a mistake in each of the five angles listed.)
6. (3 points)

(a) The image below depicts a scene viewed through a car window, *i.e.* the image boundary is the frame of the window. Two black dots are shown on the right. The upper one marks a point on a road sign. The lower one marks a point on the ground.

Sketch the 2D image velocity vectors at the two positions marked by black dots. Assume that you are looking in the direction that the car is driving *i.e.* the horizon point at the tip of the triangle.

(b) Same question, but now suppose you are making a smooth pursuit eye movement to track the point marked by the upper dot. Draw the *retinal* image velocity vectors at these two marked points. Briefly explain.

**SOLUTION:**

For (a), the velocities point away from the direction of heading (the end of the road). The two vectors should be close to parallel. However, the speed varies with inverse depth, so the point on the sign has greater speed (longer vector) than the point on the ground.

For (b), the upper black point has zero retinal velocity since you are tracking it with a pursuit movement which cancels the velocity at that point in (a). For the lower one, you take the lower vector in (a) and subtract roughly the upper vector in (a), since this is the vector that comes from the smooth pursuit.

**Grading:** 2 points for (a), namely 1 for each vector. 1 point for (b), namely 0.5 for each vector.