Questions

1. The following three terms are used often when discussing binocular vision: "disparity", "Panum’s fusional area", "eye vergence".

What are the analogous terms used when discussing optical focusing? (Hint: for "disparity", the answer is "blur").

2. The optic nerve from each eye splits and half of it crosses the middle of the brain to the other side of the brain. The crossing happens at a place called the optic chiasm. What would happen if the optic chiasm were cut? Would the person suffer any visual loss?

3. For this question, recall Exercises 8 Q2.

   (a) What is the 2D Fourier transform of a 2D cosine Gabor function?

   \[ \text{cosGabor}(x, y, \sigma, k_0, k_1) = \cos\left(\frac{2\pi}{N}(k_0x + k_1y)\right) \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \]

   (b) What is the 2D Fourier transform of a 2D sine Gabor function?

   \[ \text{sinGabor}(x, y, \sigma, k_0, k_1) = \cos\left(\frac{2\pi}{N}(k_0x + k_1y)\right) \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \]

   (c) What is the 2D Fourier transform of a 2D complex Gabor function?

   \[ \text{complexGabor}(x, y, \sigma, k_0, k_1) = (\cos\left(\frac{2\pi}{N}(k_0x+k_1y)\right) + i \sin\left(\frac{2\pi}{N}(k_0x+k_1y)\right)) \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2} \]
Solutions

1. Panum’s fusional area ↔ depth of field, eye vergence ↔ accommodation

2. Half the information in the optic nerve from each eye would not reach the brain, namely the inside (called ”nasal”) half of each retina. However, the outside (called ”temporal”) halves would still carry their information to their respective halves of the brain (V1). But there would be no binocular cells in V1 for such a person/animal. (So there would be no depth perception from binocular stereo.)

3. Here we will use the fact that the Fourier transform of the product of two functions is the convolution of the Fourier transforms of these two functions (the 2D version of Exer. 8 Q2).

   (a) First, show that the 2D Fourier transform of the cosine function is a 2D delta function:
   \[
   \mathbf{F}\cos\left(\frac{2\pi}{N}(k_0 x + k_1 y)\right) = \left(\frac{N}{2}\right)^2 (\delta(k_x - k_0)\delta(k_y - k_1) + \delta(k_x + k_0)\delta(k_y + k_1))
   \]
   that is, it has the value 1 when \((k_x, k_y) = (k_0, k_1)\) or \((k_x, k_y) = (-k_0, -k_1)\). We rewrite each product of two 1D delta functions as a single 2D delta function,
   \[
   \mathbf{F}\cos\left(\frac{2\pi}{N}(k_0 x + k_1 y)\right) = \left(\frac{N}{2}\right)^2 (\delta(k_x - k_0, k_y - k_1) + \delta(k_x + k_0, k_y + k_1)).
   \]
   Again, if you are unable to understand the above, then please ask for clarification.

   Next, since a 2D Gaussian is the product of two 1D Gaussians, the Fourier transform of a 2D Gaussian is a 2D Gaussian. (Recall midterm 1 Question 6. Here we have the special case that \(\sigma\) is the same in \(x\) as in \(y\).) So,
   \[
   \mathbf{F} \quad G(x, y, \sigma) = e^{-\frac{1}{2} \left(\frac{2\pi}{N}\right)^2 (k_x^2 + k_y^2)\sigma^2}
   \]

   Now we take the 2D Fourier transform of the product of the cos() and Gaussian:
   \[
   \mathbf{F} \quad cosGabor(x, y, \sigma) = \frac{1}{2^2} \left( e^{-\frac{1}{2} \left(\frac{2\pi}{N}\right)^2 ((k_x-k_0)^2+(k_y-k_1)^2)\sigma^2} + e^{-\frac{1}{2} \left(\frac{2\pi}{N}\right)^2 ((k_x+k_0)^2+(k_y+k_1)^2)\sigma^2} \right)
   \]
   that is, two Gaussians in the frequency domain, at frequencies \((k_0, k_1)\) and \((-k_0, -k_1)\).

   (b) Same reasoning gives:
   \[
   \mathbf{F} \quad sinGabor(x, y, \sigma) = \frac{1}{4} \left( e^{-\frac{1}{2} \left(\frac{2\pi}{N}\right)^2 ((k_x-k_0)^2+(k_y-k_1)^2)\sigma^2} - e^{-\frac{1}{2} \left(\frac{2\pi}{N}\right)^2 ((k_x+k_0)^2+(k_y+k_1)^2)\sigma^2} \right)
   \]

   (c) Same reasoning gives:
   \[
   \mathbf{F} \quad sinGabor(x, y, \sigma) = \frac{1}{4} e^{-\frac{1}{2} \left(\frac{2\pi}{N}\right)^2 ((k_x-k_0)^2+(k_y-k_1)^2)\sigma^2}
   \]