Questions

1. Suppose an observer is moving directly forward on a ground plane and the observer’s eye is a height $h$ above the ground. What is the image velocity field $(v_x, v_y)$? Assume for simplicity that the scene consists only of the ground i.e. no objects.

2. To do this question, you need to do the previous one. Suppose an observer is moving forward through a long hollow cylindrical tunnel of radius $h$, such that the optical axis coincides with the axis of the cylinder, and the projection plane is $Z = f$:

   ![Diagram of a cylindrical tunnel](image)

   (a) What is the motion field as a function of $(x, y)$?
   (b) Sketch this motion field.

3. Suppose you are a passenger in a car and you wish to read a detailed road sign on the right side of the road as your car approaches the sign and moves past it. Describe how the three types of eye movements (saccades, smooth pursuit, VOR) are used for you to perform this reading task. In particular, what will be the directions of rotation of the eye relative to the head for the three types of eye movements?

4. When there is a depth gradient or depth discontinuity in the scene, the translation component of the image motion field will have a gradient or discontinuity as well, since the translation component of the motion field depends on (inverse) depth. Such local changes in the translation component of the motion field are called motion parallax. Note that motion parallax is only produced by the translation component. It is not produced by the rotation component of observer motion, since the rotation component of the motion field does not depend on the depth $Z$.

   (a) The image below depicts a scene viewed through a car window, i.e. the image boundary is the frame of the window. Two black dots are show on the right. The upper one marks a point on a road sign. The lower one marks a point on the ground. Sketch the 2D image velocity vectors at the two positions marked by black dots. Assume that you are looking in the direction that the car is driving i.e. the horizon point at the tip of the triangle.
   (b) Same question, but now suppose you are making a smooth pursuit eye movement to track the point marked by the upper dot. Draw the retinal image velocity vectors at these two marked points. Briefly explain.
Solutions

1. If the ground plane is at $Y = -h$, i.e. below the origin/eye, then from lecture 1, we have

$$y = -\frac{h}{Z}$$

where I am writing $y$ in radians here (by dividing by the $f$ which is the distance from lens to sensor) and making a small angle approximation.

In the lecture, we saw that the motion field for forward translation is

$$(v_x, v_y) = T\frac{Z}{Z}(x, y)$$

where ($x, y$) be horizontal and vertical angles away from the origin. Substituting for $\frac{1}{Z}$, we get

$$(v_x, v_y) = -T\frac{Z}{h}(xy, y^2)$$

This solution only holds for $y < 0$ since, above the horizon ($y = 0$), the sky is visible and the sky has depth $Z = \infty$ and so the velocity field is 0 there. Below is a sketch of the velocity field. Seem familiar?

![Velocity Field Sketch](image)

2. The trick to this question about the tunnel is to notice that the scene is radially symmetric about the $z$ axis. Thus, if the observer is moving forward, then the velocity field is radially symmetric too. So if we were to parameterize the image plane by polar coordinates ($r, \theta$) instead of ($x, y$), then the speed would depend on $r$ but not $\theta$.

Since the tunnel cylinder is of radius $h$, we can use the solution from the previous question. Take the plane $X = 0$. The velocity field on the image line $x = 0$ is

$$(v_x, v_y) = -T\frac{Z}{h}(0, y^2).$$

For a radially symmetric solution, the speed will depend on the distance $\sqrt{x^2 + y^2}$ from the origin. Since the speed for line $x = 0$ is $y^2$, the speed for general ($x, y$) must be $x^2 + y^2$. So,

$$(v_x, v_y) = -T\frac{Z}{h}(x^2, y^2).$$
A sketch of the field is shown here. This field is radially expanding, but it is different from the field one sees when moving toward a wall. The main difference is that the speed here goes like $x^2 + y^2$, whereas when one moves toward a wall, the speed varies with $\sqrt{x^2 + y^2}$.

3. • The sign is on the right side of the road and so the head will turn to the right as the car drives past.
• As the head turns to the right to keep the sign close to the center of gaze, the eyes automatically turn left to cancel the head motion.
• Pursuit eye movements are used to track the sign because the sign is moving relative to the car. If the head were fixed in place for a short time, the eyes would rotate to the right as the sign translates. When the eye is rotating, the VOR will cancel the head rotation, and the pursuit movement will need to be added on top.
• A saccade will be used initially to move the eye to the sign. Further saccades will be need to jump from word to word as you read the sign. The jumping is from left to right within a line, and then back to the left and down to start a new line.
4. For (a), the velocities point away from the direction of heading (the end of the road). The two vectors should be close to parallel. However, the speed varies with inverse depth, so the point on the sign has greater speed (longer vector) than the point on the ground.

For (b), the upper black point has zero retinal velocity since you are tracking it with a pursuit movement which cancels the velocity at that point in (a). For the lower one, you take the lower vector in (a) and subtract roughly the upper vector in (a), since this is the vector that comes from the smooth pursuit.