Questions

1. (a) Consider two Gaussian functions \( G(x, \sigma_1) \) and \( G(x, \sigma_2) \). Assume \( \sigma_1 < \sigma_2 \). Is the following DOG ON-center/OFF-surround or OFF-center/ON-surround?

\[
\text{DOG}(x, \sigma_1, \sigma_2) = G(x, \sigma_1) - G(x, \sigma_2)
\]

(b) Consider an array of retinal ganglion cells, each modelled as identical ON-center OFF-surround DOG function. Sketch the array of responses of these cells to the image of a bright disk on a black surround. Assume the bright disk is much smaller than the DOG. The DOG and disk are circularly symmetric, so you can sketch your response in 1D.

(c) Same as (b), but now suppose the bright disk is much larger than the DOG.

(d) Suppose you have an image \( I(x, y) \) which is a vertical white line at \( x = x_0 \) and otherwise is black. Sketch the result of the 2D cross-correlation \( \text{DOG}(x, y, \sigma_1, \sigma_2) \otimes I(x, y) \), that is, the responses of DOG cells. Be sure to label where \( x = x_0 \).

(e) How would your answer change if the image were a dark line on a white background?

2. In the lecture, the response of a single DOG cell centered at \( (x_0, y_0) \) was defined to be:

\[
r(x_0, y_0) = \sum_{x,y} \text{DOG}(x - x_0, y - y_0) I(x, y).
\]

and the response of a family of DOG cells was defined to be:

\[
\text{DOG} \otimes I(x, y) = \sum_{x', y'} \text{DOG}(x', y') I(x + x', y + y').
\]

The formulas are slightly different. How and why?

3. Nerve cells cannot spike at negative rates, but the linear models that we’ve looked at (such as the inner product of a DOG and an image) can define negative responses. How can ”negative responses” be encoded by spiking cells? (This is a review from the lecture.)
Solutions

1. (a) ON-center, OFF-surround
(b) If the disk is very small, then the image can be approximated by a delta function. But a delta function convolved with any function (in particular, a DOG) is just that function itself. So, answer is a response profile that is identical to a DOG.
(c) The response is a 1D profile that looks like this:

\[
000000000000000000+++0000000000000+++000000000
\]

where 0 is zero response, - is negative response, and + is positive response.
(d) \( x_0 \) is here

\[
\begin{array}{l}
000000000000000000+++0000000000000+++000000000
\\
| \\
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
ETC
\end{array}
\]

Note that the image is constant in the \( y \) direction. So the result of the convolution should be constant in the \( y \) direction too.
(e) It would flip sign:

\[
\begin{array}{l}
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
000000000000000000+++0000000000000+++000000000
\\
ETC
\end{array}
\]

2. The response of a single DOG cell centered at \((x_0, y_0)\) is

\[
 r(x_0, y_0) = \sum_{x,y} DOG(x-x_0, y-y_0) \cdot I(x, y) .
\]

Here \((x_0, y_0)\) is a specific location of one DOG cell, and the \( x,y \) variables refer to all other positions in the image. The formula could have be written equivalently as:

\[
 r(x_0, y_0) = \sum_{x,y} DOG(x, y) \cdot I(x_0 + x, y_0 + y)
\]

where now the \((x, y)\) are positions in the coordinate system whose origin \((x, y) = (0, 0)\) is the center of the DOG’s receptive field. Note that this equivalent way of writing the response is the same as

\[
 DOG \otimes I(x_0, y_0) = \sum_{x', y'} DOG(x', y') \cdot I(x_0 + x', y_0 + y') .
\]

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which is the formula used the question, with renamed variables.

[ASIDE: There is a difference between talking about the response of a particular DOG at some fixed location versus the responses of a family of DOGs at many locations. But the mathematical notation sometimes doesn’t differentiate the two. This should be familiar to you: “f(x)” can mean “the function f at a particular value of x”, or it can mean a function of that maps each x to some value.]  

3. We considered two ways. One is to have two copies of each cell, whose weights are sign flipped. For example, for every ON center / OFF surround cell, we have an OFF center / ON surround cell. We then ”half wave” rectify each cell’s response and encode that output. So if an image yielded a negative response from some cell, then its polar opposite cell would yield a positive response (and vice-versa).

The second way is to use a sigmoidal response where the ”zero” response would correspond to some positive firing rate (say 10 spikes per second – which is the ”resting rate” of the cell) and a negative response would be some firing rate between 0 and the resting rate. A positive response would be greater than the resting rate.

Both of the above are generally believed to be used in the retina and V1.