COMP 546

Lecture 6

orientation 2: complex cells
binocular cells

Tues. Jan. 30, 2018
Recall last lecture: simple Cell

Linear response → half wave rectification
Recall last lecture: simple Cell

Linear response → half wave rectification

[Diagram showing linear response and half wave rectification]
“Complex Cell” (Hubel and Wiesel)

Responds to preferred orientation of line *anywhere* in receptive field.
How to construct a complex cell? (1)

Use several simple cells with common orientation and neighboring receptive field locations. If we sum up their rectified responses then we get a response to image structure of that orientation anywhere in the overlapping receptive fields.
Now suppose these even cell and odd cells have *the same receptive field locations* (perfect overlap). Again sum up their rectified responses and the result is a response anywhere in the receptive field.
How to construct a complex cell? (3)

This is the same as the last model but now we square the positive values. This model is more commonly used than model (2) and so we’ll use this one.
Unit circle

\[
\cos^2 \varphi + \sin^2 \varphi = 1
\]
The response to an image $I(x, y)$ is modelled as the Euclidean length of the vector, i.e. L2 norm

$$\| (< \cos Gabor(x, y), I(x, y) >, < \sin Gabor(x, y), I(x, y) >) \|_2$$
Complex cell response modelled as the length of 2D vector:

\[
\langle \cos\text{Gabor}(x, y), I(x, y) \rangle, \\
\langle \sin\text{Gabor}(x, y), I(x, y) \rangle
\]
We can model complex cells of any orientation.

Cosine Gabor

Sine Gabor
Example: image cross correlated with four complex cells
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Left halves of retina map to V1 in left hemisphere

Right halves of retina map to V1 in right hemisphere

$\phi = 90$

$\phi = -90$

$\phi = 90$

$\phi = -90$

$\phi = \text{eccentricity}$

$\phi = \text{eccentricity}$
Superimposed left and right eye images

Negative disparity

Positive disparity

Zero disparity (eyes verged at this depth)
How to estimate binocular disparity?

*Computer vision-ish* approach:

For each \((x_0, y_0)\), find disparity value \(d\) that minimizes:

\[
\sum_{x,y} \left( I_{left}(x - d, y) - I_{right}(x, y) \right)^2
\]

where sum is over a neighborhood of \((x_0, y_0)\).
How to build ‘disparity tuned’ binocular cells?

We use vertically oriented cells only.
Left eye
\( Gabor(x - x_0 - d, y - y_0) \)

\( (x_0 + d, y_0) \)

Right eye
\( Gabor(x - x_0, y - y_0) \)

\( (x_0, y_0) \)
Idea 1: (analogous to computer vision)

To compute disparity at \((x_0, y_0)\), find the \(d\) that minimizes:

\[
\begin{align*}
( & < \cos \text{Gabor}(x - x_0 - d, y - y_0), I_{left}(x, y) > \\
- & < \cos \text{Gabor}(x - x_0, y - y_0), I_{right}(x, y) > )^2 \\
+ & < \sin \text{Gabor}(x - x_0 - d, y - y_0), I_{left}(x, y) > \\
- & < \sin \text{Gabor}(x - x_0, y - y_0), I_{right}(x, y) > )^2
\end{align*}
\]
Idea 1: (analogous to computer vision)

To compute disparity at \((x_0, y_0)\), find the \(d\) that 
minimizes:

\[
\begin{align*}
&\left( < \cos \text{Gabor}(x - x_0 - d, y - y_0), I_{\text{left}}(x, y) > 
\right) - \\
&\left( < \cos \text{Gabor}(x - x_0, y - y_0), I_{\text{right}}(x, y) > 
\right)^2 \\
&+ \\
&\left( < \sin \text{Gabor}(x - x_0 - d, y - y_0), I_{\text{left}}(x, y) > 
\right) - \\
&\left( < \sin \text{Gabor}(x - x_0, y - y_0), I_{\text{right}}(x, y) > 
\right)^2
\end{align*}
\]

If \(I_{\text{left}}(x + d, y) = I_{\text{right}}(x, y)\) for all \((x, y)\) in 
receptive fields, then the minimum should be 0.
Let $y$ position of these cells be the same and $x$ positions of left and right be separated by $d$. This binocular cell is tuned to disparity $d$. 
Idea 1 (computer vision):

find the disparity $d$ that minimizes the squared differences:

$$(c_l - c_r)^2 + (s_l - s_r)^2$$

$$= c_l^2 + c_r^2 + s_l^2 + s_r^2 - 2 (c_l c_r + s_l s_r)$$

where $c_l$ and $s_l$ depend on $d$. 
Idea 2 (biological vision):

find the shift $d$ that maximizes the squared sums:

$$\left( c_l + c_r \right)^2 + \left( s_l + s_r \right)^2$$

$$= c_l^2 + c_r^2 + s_l^2 + s_r^2 + 2 \left( c_l c_r + s_l s_r \right)$$

where $c_l$ and $s_l$ depend on $d$. 
Q: What happens if you close an eye?

A: The cell behaves like a monocular complex cell.
Recall (monocular) complex cell response to white line

Response of complex cell
Response of binocular complex cell tuned to $d = 0$ when disparity of white line is 2 pixels.
Response of binocular complex cell tuned to $d = 0$ when disparity of white line is 10 pixels.
Response of binocular complex cell tuned to $d = 0$ when disparity of white line is 18 pixels.
I will finish this next lecture.
Each binocular cell has receptive field location centered at \((x_l, y_l)\) and \((x_r, y_r)\) in the two eyes.

Q: What disparity is each cell tuned for?

Q: How to visualize the set ("population") of cells?
Disparity Space

\[ d < 0 \quad d = 0 \quad d > 0 \]

\[ x_{\text{right}} \]

\[ x_{\text{left}} \]
Disparity Tuned Cells

Previous examples of vertical lines

$d < 0$

$d = 0$

$d > 0$
Disparity Tuned Cells

\[ x_{\text{right}} \]

\[ d < 0 \quad d = 0 \quad d > 0 \]
To estimating the disparity at an image point, the visual system must select which of these binocular complex cells gives the largest response?