COMP 546

Lecture 5

Orientation Selection in V1

Tues. Jan. 24, 2017
Brief review of a few ideas from last lecture which are relevant to A1.
Last lecture: DOGs, cross correlation

DOG

image
Responses of a *family* of DOGs

Cross correlation operator

\[ L(x, y) \equiv DOG(x, y, \sigma_1, \sigma_2) \otimes I(x, y) \]

\[ \equiv \sum_{x', y'} DOG(x', y') \cdot I(x' + x, y' + y) \]
How do such cells respond to a "step edge"?

\[ I(x) \]
\( \text{DOG}(x, y) \)

\[
\text{DOG} \otimes I(x, y)
\]
Mach Bands

Are they the result of center-surround coding in the retina?
Mach bands are well known problem for interpreting x-ray images, since very subtle changes in dark-bright must be detected.
Visual pathway (retina to V1)
Left (right) visual field of both eyes is imaged on right (left) half of retina and processed by right (left) brain hemisphere.
LGN

V1

retina

feedback
Primary Visual Cortex  
(Hubel & Wiesel 1959)

The moving slide (see 35 sec and on...)  
http://www.youtube.com/watch?v=IOHayh06LJ4

3 minutes of exploration:  
https://www.youtube.com/watch?v=Cw5PKV9Rj3o
“Simple Cell”

Temporal effect (discussed later)
V1 Orientation Tuning Curve

Hubel & Wiesel, 1968
Hubel and Wiesel suggested this mechanism for receptive field profile of V1 simple cell.
Towards a model of orientation selectivity in V1

Line Detector (even)

- + -
- + -
- + -
- + -

Edge Detector (odd)

- +
- +
- +
- +

Such cells have also been found.
Recall: half-wave rectification

Response (spike rate)

How to encode the negative responses?
Line Detector (even)

Edge Detector (odd)
“Gabor”: classical model of simple cell

Line (even)  

Edge (odd)
1D Cosine Gabor

\[ \text{Gaussian} \ast \text{cosine} = \text{cosine Gabor} \]
1D Sine Gabor

\[ \text{sine} \ast \text{Gaussian} = \text{sine Gabor} \]
(Sampled) Cosine

\[
\cos\left(\frac{2\pi}{N} k_x x \right)
\]

e.g. \( k_x = 8 \)

\( N = 256 \)
1D Cosine Gabor

\[ \cos \left( \frac{2\pi}{N} k_x x \right) \]

\[ \frac{1}{\sqrt{2\pi \sigma}} e^{-\frac{x^2}{2\sigma^2}} \]
1D Sine Gabor

\[ \sin\left(\frac{2\pi}{N} k_x x\right) \]

\[ \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \]
2D cosine

$$\cos \left( \frac{2\pi}{N} (k_x x + k_y y) \right)$$

e.g. \( k_x = 4 \)

\( k_y = 0 \)

\( N = 256 \)
2D sine

\[
\sin \left( \frac{2\pi}{N} (k_x x + k_y y) \right)
\]

- e.g. \( k_x = 8 \)
- \( k_y = 2 \)
- \( N = 256 \)
model of simple cell: 2D Gabor

\[ G(x, y, \sigma) \cos \left( \frac{2\pi}{N} (k_x x + k_y y) \right) \]

\[ G(x, y, \sigma) \sin \left( \frac{2\pi}{N} (k_x x + k_y y) \right) \]
• What is the response of a single Gabor cell to a parameterized family of images? 
  
  e.g. thin white line at different positions in receptive field

• What is the response of a family of Gabor cells to a single image? 
  
  e.g. shift the Gabor cells across an image
\[
< \cosGabor(x, y, \sigma_1, \sigma_2), I(x, y) > 
\equiv \sum_{x', y'} \cosGabor(x', y') I(x', y')
\]
\[ < \text{cosGabor}(x, y, \sigma_1, \sigma_2), I(x, y) > \]

\[ \equiv \sum_{x', y'} \text{cosGabor}(x', y') I(x', y') \]

Non-zero only at \( x \) position of vertical line

\[ G(x, y, \sigma) \cos \left( \frac{2\pi}{N} (k_x x) \right) \]
\[
< \text{sineGabor}(x, y, \sigma_1, \sigma_2), \quad I(x, y) >
\]

\[
\equiv \sum_{x', y'} \text{sineGabor}(x', y') \ I(x', y')
\]

Non-zero only at \(x\) position of vertical line

\[
G(x, y, \sigma) \ \sin \left( \frac{2\pi}{N} (k_x x) \right)
\]
Gaussian envelope
What is the response of a family of Gabor cells to a single image?
cross correlation with four cosine Gabors
cross correlation with four sine Gabors
“Complex Cell”

Responds to preferred orientation of line *anywhere* in receptive field
Unit circle

\[ \cos^2 \varphi + \sin^2 \varphi = 1 \]
Model of a Complex Cell

The response of a complex cell to an image $I(x,y)$ is modelled as the lengths of the above vector. The $<>$ notation is inner product.
Complex cell response
Model of Family of Complex Cells

The complex cell responses to an image $I(x, y)$ are a function of $(x, y)$. 

$$\text{Model: } (\cos \text{Gabor}(x, y) \otimes I(x, y), \sin \text{Gabor}(x, y) \otimes I(x, y))$$
Obtained from cross correlation with four cosine & sine Gabor pairs
Upcoming lectures

• How are orientation and positions laid out in V1?

• How are images from left and right eyes combined?

• What are V1 cell’s temporal properties? (motion?)

• What happens when the eyes move? (rotation or translation?)