lecture 3

image formation 3 - color

- RGB and HSV
- spectra
- trichromacy and photoreceptor sensitivity
- color displays
- CIE chromaticity diagram
- physical vs. perceived

Thursday Sept 17, 2015
Color and RGB

(0, 0, 1)
(1, 0, 1)
(1, 1, 1)
(0, 1, 1)
(1, 1, 0)
(0, 1, 0)

Red (1, 0, 0) -- hidden

Blue
Magenta
White
Cyan
Black
Yellow
Green
hue - which 'color'?
saturation - how pure?
luminance (value) - intensity
What is light? What is color?
Light consists of electromagnetic waves from 400-700 nm.
For a given light ray travelling through space, how is light energy (or power, i.e. energy per unit time) that is carried by that ray distributed over wavelength?
What determines a light spectrum?

- emitted light (sunlight, fire, tungsten bulb, LED, OLED, ....)

In this course, we don't care about these physics details.

- reflected light

\[ E(x, \lambda) = L(x, \lambda) \times R(x, \lambda) \]

illumination reflectance (fraction)
Retinal Images

Images are measured by light sensitive photoreceptor cells in the retina.
Two classes of photoreceptors: rods and cones

Rods are used at night (low light levels). They measure luminance only.

Cones are used during day (high light levels). They are involved in color vision, as we will discuss.
ASIDE: rod and cone density across the retina

![Diagram of retinal anatomy with rod and cone density graph]

**FIGURE 3-7** The distribution of rods and cones in the human retina. The left figure gives the locations on the retina of the “angle” relative to the optic axis on the right figure (based on Lindsay & Norman, 1977).

Plot shows photoreceptors per mm².
e.g. \[ 160,000 \text{ per } \text{mm}^2 \approx 400 \times 400 \text{ per } \text{mm}^2 \]

ASIDE: Rods have a high density in periphery. However, they are very noisy. They operate at low light levels, and so the signal is small compared to their (internal) noise.
\[
\frac{1 \text{ mm}}{20 \text{ mm}} \times \frac{57 \text{ degrees}}{\text{rad}} \approx 3 \text{ degrees}
\]

Peak density \(\sim \frac{140,100}{3 \times 3 \text{ (deg}^2)} \sim \frac{200 \times 200}{\text{deg}^2}\)
Approximate cone density across the retina ("eccentricity")

\[ \text{peak density} \approx \left( \frac{200}{\text{deg}} \right)^2 \]

The diagram shows the peak density at the center of the retinal image, decreasing as eccentricity increases. The density values are indicated at various eccentricities: 250, 150, and 100 at 3°, 5°, and 10° respectively.
Three types of cones
(defined by their light absorbing pigment)

L - sensitive to long wavelengths
M - sensitive to medium wavelengths
S - sensitive to short wavelengths

(You may assume for simplicity that these correspond roughly to RGB sensors in camera.)
Probability that a photon of wavelength $\lambda$ will be absorbed by each type of photoreceptor pigment.
(For illustration purposes, each curve is normalized to 1.)
(Optional) video for the mechanism of how a rod photoreceptor cell in the retina responds to light.

Khan Academy:
https://www.youtube.com/watch?v=CqN-XIPhMpo

in case you are interested in the basic details.....
A photoreceptor does not know the *distribution* of wavelengths of photons that it absorbs.

Rather it sums the energy of all absorbed photons.

- I will be loose with physical units here e.g. energy vs power, energy per photoreceptor vs per mm on retina, etc
$E(x, \lambda)$ - spectrum of light arriving at cone $x$

$C_{\text{LMS}}(\lambda)$ - spectral absorptance of a photoreceptor

More generally, $C$ is a 'color matching function'.
Cone absorptance $C_{RGB}$ may be easier to understand if we discretize the interval of visible light into $N$ bins.

$$I_{LMS}(x,y) = \sum_{\lambda} C_{LMS}(\lambda) E(x,y,\lambda)$$

This maps an $N$-D spectrum to a 3-D LMS image.
Metamers

It can easily happen that matrix C maps two different spectra $E_1(\lambda)$ and $E_2(\lambda)$ to the same cone absorption triple,

$$C \ E_1 = C \ E_2$$

Such spectra $E_1$ and $E_2$ are called 'metamers'. They are visually indistinguishable.
Color Blindness

Many people (~8% of males and ~0.5 % of females) are missing a gene for one of the three cone pigments. This leads to three types of "color blindness", depending on which type is missing. "Color blind" doesn't mean the person can't see any colors. Rather, it means that they cannot distinguish some spectra that color normal people can distinguish. (Such spectra are metamers for the color blind person.)
Color Displays (e.g. monitors, projectors)

Color displays have three primary lights (RGB). Their emittance spectra can be represented by an $N \times 3$ matrix $P$ ("phosphor emission spectrum") of basis vectors, such that the net emitted light spectra from a pixel is:

$$E(\lambda) = P \cdot 1_s$$
For a given image to be displayed, two different monitors (P1 and P2 matrices) may produce different spectra and hence different captured RGB values. (See Exercises for display matching problem.)
Anaglyph (definition): a stereoscopic photograph with the two images superimposed and printed in different colors, producing a stereo effect when the photograph is viewed through correspondingly colored filters.
How does it work? See Exercises.
Monochromatic spectra (laser)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} \text{Maximum saturation} \\ \text{Red} \\ \text{Green} \end{bmatrix}_{3 \times N} \cdot \begin{bmatrix} C \end{bmatrix}_{N \times 1}.$$
Any spectrum $E(\lambda)$ is a linear combination of monochromatic spectra (with positive coefficients).

\[
\begin{bmatrix}
E(\lambda_1) \\
\vdots \\
E(\lambda_N)
\end{bmatrix} = E(\lambda_1) \begin{bmatrix} 0 \end{bmatrix} + E(\lambda_4) \begin{bmatrix} 0 \end{bmatrix} + \ldots + E(\lambda_N) \begin{bmatrix} 0 \end{bmatrix}
\]
The thick black curve below shows RGB points that are the **columns** of the matrix $C$ (see two slides back). These are the points $C_{Ek}$ where $Ek$ is the $k$th monochromatic spectrum. The rays from the origin through each RGB point $C_{Ek}$ are defined by varying the strength of each monochromatic spectrum by multiplying it by a constant (as on previous slide). The main idea here is that any spectrum $E(\lambda)$ is mapped to a linear combination of the locus of points shown below.

$$I_{\text{LMS}} = \sum_{\lambda} C(\cdot\lambda) E(\lambda)$$
The 3D surface on the previous slide is difficult for novices to visualize, so it is common to display a planar slice through it. The interior below is defined by convex combinations of the boundary points. This is another way to show a color palette.

Saturation increases *radially* from 0 at the 'white' point near the middle to a max at the boundary.
A particular display has three spectra that it can produce, namely the columns of matrix P from earlier. The measured LMS values must lie within convex combinations of these three spectra. *It must be convex because you cannot have a negative intensity value at a pixel.*
Exercise: what is relationship between previous slide and the figure below (from the beginning of this lecture)?
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color

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"Luminance" (value, intensity)

- weighted average light power over visible wavelengths (independent of hue or saturation)
- physical quantity

"Brightness"

- perceived luminance
  (not physically measurable, only measurable behaviorally i.e. based on people's responses in some task)
Example

Paper on the left is in shadow. It is darker (lower physical intensity) and it appears darker (lower perceived intensity)
Image is processed so that the right paper is given same image intensities as left paper. Now, right paper appears darker. Why?
Physically...

\[
\text{surface luminance (x,y)} = \text{surface reflectance (x,y)} \times \text{illumination (x,y)}
\]

Perceptually...  

The brightness of a surface is often more determined by the perceived reflectance than the perceived luminance.

Indeed, when we talk about color of things we see, we are typically talking about material properties rather than properties of light.
Many perception studies have used simple images to explore relationships between perceived and physical quantities.

Small gray squares have equal luminance but the square on the left appears *brighter*. (The left half does not appear to be a shadow, however.)
The light and dark small grey bars in fact have the same luminance, but the ones on the left are much *brighter*.

This is a bigger effect than on the previous slide.

See Exercise on *why* this effect occurs.
The same questions arise in color vision.

The small squares have the same RGB image values but the one on the left appears more yellowish. Why?