lecture 3
image formation 3 - color
- RGB and HSV
- spectra
- trichromacy and photoreceptor sensitivity
- color displays
- CIE chromaticity diagram
- physical vs. perceived

Thursday Sept 17, 2015

hue - which ‘color’?
saturation - how pure?
luminance (value) - intensity

Light Spectrum $E(\lambda)$
For a given light ray travelling through space, how is light energy (or power, i.e. energy per unit time) that is carried by that ray distributed over wavelength?

What determines a light spectrum?
- emitted light (sunlight, fire, tungsten bulb, LED, OLED, ...)
- reflected light

In this course, we don’t care about these physics details.

$E(z, \lambda) = L(z, \lambda) \times R(z, \lambda)$
illumination reflectance (fraction)

Retinal Images
Images are measured by light sensitive photoreceptor cells in the retina.
Two classes of photoreceptors: rods and cones

Rods are used at night (low light levels). They measure luminance only.

Cones are used during day (high light levels). They are involved in color vision, as we will discuss.

ASIDE: rod and cone density across the retina

\[
\text{e.g.: } 160,000 \text{ per mm}^2 \\
\approx 450 \times 400 \text{ per mm}^2
\]

ASIDE: Rods have a high density in periphery. However, they are very noisy. They operate at low light levels, and so the signal is small compared to their (internal) noise.

Approximate cone density across the retina ("eccentricity")

\[
\text{Prob. density} \approx \frac{140,000}{3 \times 3 (\text{deg}^2)} \approx \frac{200 \times 200}{\text{deg}^2}
\]

Three types of cones (defined by their light absorbing pigment)

- **L** - sensitive to long wavelengths
- **M** - sensitive to medium wavelengths
- **S** - sensitive to short wavelengths

(You may assume for simplicity that these correspond roughly to RGB sensors in camera.)

"Principle of Univariance"

A photoreceptor does not know the distribution of wavelengths of photons that it absorbs.

Rather it sums the energy of all absorbed photons.

- I will be loose with physical units here
- e.g. energy vs power
- energy per photoreceptor vs per mm on retina, etc.

Khan Academy:
https://www.youtube.com/watch?v=CqN-XIPhMpo
in case you are interested in the basic details.....
**E(x, λ)** - spectrum of light arriving at cone \(x\)

**C_{LMS}(λ)** - spectral absorptance of a photoreceptor

More generally, \(C\) is a 'color matching function'.

\[
\int_{LMS} (x, y) = \int_{\lambda} C_{LMS}(\lambda) E(x, y, \lambda) \, d\lambda
\]

Cone absorptance \(C_{RGB}\) may be easier to understand if we discretize the interval of visible light into \(N\) bins.

\[
\int_{LMS} (x, y) = \int_{\lambda} \sum_{\lambda=1}^{N} C_{LMS}(\lambda) E(x, y, \lambda)
\]

This maps an N-D spectrum to a 3-D LMS image.

**Metamers**

It can easily happen that matrix \(C\) maps two different spectra \(E_1(\lambda)\) and \(E_2(\lambda)\) to the same cone absorption triple,

\[
C \cdot E_1 = C \cdot E_2
\]

Such spectra \(E_1\) and \(E_2\) are called 'metamers'. They are visually indistinguishable.

**Color Blindness**

Many people (~8% of males and ~0.5% of females) are missing a gene for one of the three cone pigments. This leads to three types of "color blindness", depending on which type is missing. "Color blind" doesn't mean the person can't see any colors. Rather, it means that they cannot distinguish some spectra that color normal people can distinguish. (Such spectra are metamers for the color blind person.)

**Color Displays (e.g. monitors, projectors)**

Color displays have three primary lights (RGB). Their emittance spectra can be represented by an \(N\times3\) matrix \(P\) ("phosphor emission spectrum") of basis vectors, such that the net emitted light spectra from a pixel is:

\[
E(\lambda) = P \cdot \text{matrix of capture sensitivity} \cdot \text{pixel display}
\]

For a given image to be displayed, two different monitors (\(P_1\) and \(P_2\) matrices) may produce different spectra and hence different captured RGB values. (See Exercises for display matching problem.)

**Anaglyph 3D Displays**

Anaglyph (definition): a stereoscopic photograph with the two images superimposed and printed in different colors, producing a stereo effect when the photograph is viewed through correspondingly colored filters.

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How does it work? See Exercises.
Monochromatic spectra (laser)

\[
\begin{bmatrix}
I_L \\
I_I \\
I_S
\end{bmatrix}
= \begin{bmatrix}
3 \times 1 \\
I_L \\
3 \times 1
\end{bmatrix} \times \begin{bmatrix}
C
\end{bmatrix}
\]

maximum saturation

Any spectrum \( E(\lambda) \) is a linear combination of monochromatic spectra (with positive coefficients).

\[
\begin{bmatrix}
E(\lambda) \\
\vdots \\
E(\lambda_k)
\end{bmatrix} = \begin{bmatrix}
E(\lambda_1) \\
\vdots \\
E(\lambda_N)
\end{bmatrix} + \begin{bmatrix}
\vdots \\
\vdots \\
\vdots
\end{bmatrix}
\]

The thick black curve below shows RGB points that are the columns of the matrix \( C \) (see two slides back). These are the points \( C E_k \) where \( E_k \) is the \( k \)th monochromatic spectrum. The rays from the origin through each RGB point \( C E_k \) are defined by varying the strength of each monochromatic spectrum by multiplying it by a constant (as on previous slide). The main idea here is that any spectrum \( E(\lambda) \) is mapped to a linear combination of the locus of points shown below.

CIE Chromaticity Diagram

The 3D surface on the previous slide is difficult for novices to visualize, so it is common to display a planar slice through it. The interior below is defined by convex combinations of the boundary points. This is another way to show a color palette.

A particular display has three spectra that it can produce, namely the columns of matrix \( P \) from earlier. The measured LMS values must lie within convex combinations of these three spectra. It must be convex because you cannot have a negative intensity value at a pixel.

"Luminance" (value, intensity)

- weighted average light power over visible wavelengths (independent of hue or saturation)

- physical quantity

"Brightness"

- perceived luminance (not physically measurable, only measurable behaviorally, i.e. based on people's responses in some task)

Example

Paper on the left is in shadow. It is darker (lower physical intensity) and it appears darker (lower perceived intensity)
Image is processed so that the right paper is given same image intensities as left paper. Now, right paper appears darker. Why?

Physically...
\[ \text{surface luminance (x,y)} = \text{surface reflectance (x,y)} \times \text{illumination (x,y)} \]

Perceptually... ?

The brightness of a surface is often more determined by the perceived reflectance than the perceived luminance.

Indeed, when we talk about color of things we see, we are typically talking about material properties rather than properties of light.

Many perception studies have used simple images to explore relationships between perceived and physical quantities.

Small gray squares have equal luminance but the square on the left appears brighter. (The left half does not appear to be a shadow, however.)

The light and dark small grey bars in fact have the same luminance, but the ones on the left are much brighter. This is a bigger effect than on the previous slide. See Exercise on why this effect occurs.

The same questions arise in color vision.

The small squares have the same RGB image values but the one on the left appears more yellowish. Why?