Shape from (regular) texture
Shape from (random) texture
What is the slope of this surface?
What is the slope of this surface?
What is the slope of this surface?
What is the slope of this surface?
What is the slope of this surface?
What is the slope of this surface?
Some Basic Theory of Shape from Texture

Special case: Horizontal plane (lecture 1)
View from side (YZ)

(-h, Z)

Ground plane

\[ Y = -\frac{hf}{Z} \]
Take one (unit) square on ground plane.

\[ \Delta X = 1 \quad , \quad \Delta Z = 1 \]

\[ \frac{\Delta x}{f} = \frac{\Delta X}{Z} = \frac{1}{Z} \]

\[ \Delta y \approx \frac{dy}{dZ} \Delta Z = \frac{hf}{Z^2} \Delta Z = \frac{hf}{Z^2} \]
Take a boundary edge of square on ground plane.

\[ \Delta X = 1 \quad \text{or} \quad \Delta Z = 1 \]

\[ \Delta x = \frac{f}{Z} \]

\[ \Delta y \approx \frac{hf}{Z^2} \]

\[ \frac{\Delta y}{\Delta x} \approx \frac{?}{Z} \]
Oblique 3D Plane

\[ Z = Z_0 + AX + BY \]

How to describe the orientation of this oblique plane?

Thumbtack on plane
Oblique 3D Plane

\[ Z = Z_0 + AX + BY \]

depth gradient:

\[ \nabla Z = \left( \frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y} \right) = (A, B) \]
'Slant' (σ)

Slant σ is the angle between frontoparallel plane and the oblique plane.

\[ Z = Z_0 + AX + BY \]

\[ \nabla Z = \sqrt{A^2 + B^2} \]

\[ \equiv \tan(\sigma) \]
‘Tilt’ ($\tau$)

$\tau$ is the direction of slope.

\[
\nabla Z = \left| \nabla Z \right| \ (\cos \tau , \sin \tau )
\]

\[
= \tan(\sigma) \ (\cos \tau , \sin \tau )
\]
Slant and Tilt

(Koenderink, van Doorn, Kappers 1992)
Slant and Tilt on a Curved Surface

\[ \text{tilt} = 180 \quad \text{tilt} = 0 \]

all slants

all tilts and all slants
Shape from Texture for smooth surfaces

Slant compresses the texture in the direction of the tilt. Thus, compression is a ‘cue’ to slant.
Surface normal of a smooth surface $Z(X,Y)$

$$Z(X_0 + \Delta X, Y_0 + \Delta Y) = Z_0 + \frac{\partial Z}{\partial X} \Delta X + \frac{\partial Z}{\partial Y} \Delta Y + H.O.T.$$  

$$\Delta Z \approx \frac{\partial Z}{\partial X} \Delta X + \frac{\partial Z}{\partial Y} \Delta Y$$

\[\left(\frac{\partial Z}{\partial x}, \frac{\partial Z}{\partial y}, -1\right) \cdot (\Delta X, \Delta Y, \Delta Z) \approx 0\]  

This vector is perpendicular to any step on the surface.
Unit Surface Normal

\[ N \equiv \frac{1}{\sqrt{\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 + 1}} \left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right) \]
ASIDE: Common Experimental Method is to estimate perceived shape using a gauge figure
(Koenderink, van Doorn, Kappers 1992)

[Image of Task → Sampling → Reconstruction process]

[Wijntjes et al. 2012]
Surface Curvature: another important perceptual shape property

Formal mathematical definitions (2\textsuperscript{nd} order derivatives) omitted.
We can say a lot about local surface orientation and curvature just from the bounding (occluding) contours.
Shape from shading

Drawings of Leonardo da Vinci
Shape from shading
(random shapes)
SFS on a Sunny Day

\[ I(x) = \vec{N}(x) \cdot \vec{L} \]
Shading on a sunny day

\[ I(x, y) = N(x, y) \cdot L \]
Shading on a sunny day

\[ I = N \cdot L \equiv \frac{1}{\sqrt{\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 + 1}} \left(\frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1\right) \cdot (L_x, L_y, L_z) \]
‘Shape from shading on a sunny day’ in computer vision

Given an image $I(x,y)$, estimate the direction $L$, and then use numerical methods to solve the partial differential equation.

Much research on this in the 1970’s and 1980’s...

Interesting stuff but...

• It only works under very restricted conditions
• There is no evidence that the brain does this.
**Linear Shading on a sunny day**

\[
I = N \cdot L \equiv \frac{1}{\sqrt{(\frac{\partial Z}{\partial X})^2 + (\frac{\partial Z}{\partial Y})^2 + 1}} \left( \frac{\partial Z}{\partial X}, \frac{\partial Z}{\partial Y}, -1 \right) \cdot (L_X, L_Y, L_Z)
\]

if surface has low relief (surface slope is small) then the nasty non-linearity can be ignored
Example: Crumpled paper
SFS on a Cloudy Day
[Langer and Zucker, 1993]
SFS on a Cloudy Day

\[ I(x) = \int \limits_{\theta(x)} \mathbf{N}(x) \cdot \mathbf{L} \, d\mathbf{L} \]

\[ \theta(x) = \text{angle of visible light source} \]
Shape from shading on a cloudy day in computer vision

Perception of Shape from Shading on a Cloudy Day

Notice the local brightness peaks within the valleys. The shading is more complicated than “dark means deep”, and we showed that the visual system knows this.

[Langer and Buelthoff, 2000]
Shape from specular reflections

matte  glossy  mirror

highlight
Shape from specular reflections

matte  glossy  mirror
Many open questions in shape from texture and shading

What properties of shape are perceived? (depth, orientation, curvature, ....)

What image cues does the visual system use to infer shape? (e.g. local oriented structure, contours)

How are cues combined? (texture, shading, contour)

What properties of lighting and material are perceived?