COMP 546

Lecture 1

Image Formation: Geometry

Thurs. Jan. 11, 2018
Origins of spatial vision
(500 million years ago?)

photoreceptor array (eye)

“brain”

legs
Origins of spatial vision
Origins of spatial vision
Origins of spatial vision

Predator arrives, but no change in light level received by this cell.
Origins of spatial vision

predator

Some change in light level received by this cell.
Origins of spatial vision

If right cell measures decrease in light, then move right.
Evolution of eyes

As pit becomes more concave, angular resolution improves (but amount of light decreases)
large aperture

poor angular resolution
	small aperture

good angular resolution
Radians

\[ \theta \text{ radians} = \frac{\text{arclength on circle}}{\text{radius of circle}} \]
Radians vs. degrees

\[ \theta \text{ radians} \times \frac{180 \text{ degrees}}{\pi \text{ radians}} = \theta \times \frac{180}{\pi} \text{ degrees} \]

1 radian \approx 57 \text{ deg}
Small angle approximation

\[ \theta \approx 2 \tan \frac{\theta}{2} \]
Aperture angle from a few slides ago....
“F number” (photography)

\[ F \text{ number} \equiv \frac{f}{A} \approx \frac{1}{\theta} \]
ASIDE: camera

5 mm aperture $A$

“focal length” $f$
50 mm

$F\ number \equiv \frac{f}{A} = \frac{50}{5} = 10$
eye  (ignore lens)

\[ F \text{ number} \equiv \frac{f}{A} = \frac{25}{5} = 5 \]
Visual Angle

\[ \alpha \approx \frac{\text{object height}}{\text{distance}} \]
Visual Angle

\[ \alpha \approx \frac{\text{image size of object}}{\text{diameter of eyeball}} \]
Two different concepts

Aperture angle

Visual angle
Visual Angle Example 1

\[ \alpha \approx \frac{\text{object height}}{\text{distance}} = \frac{1 \text{ cm}}{\frac{180}{\pi} \text{ cm}} = 1 \text{ degree} \]
Visual Angle Example 2

\[ \alpha \approx \frac{\text{object height}}{\text{distance}} \]

\[ = \frac{\pi}{10} \frac{m}{18 \ m} = \frac{\pi}{180} \text{ radians} = 1 \text{ degree} \]
Example 3: moon

Visual angle of moon is about \( \frac{1}{2} \text{ deg} \).
Units of visual angle

1 radian = \( \frac{180}{\pi} \) deg

1 deg = 60 minutes (or “arcmin”)

1 minute = 60 seconds (or “arcsec”)

Image position

(X, Y, Z)

pinhole camera mode

(x, y)
Pinhole camera

\((X, Y, Z)\)

\((x, y)\)

\((0, 0)\)

image plane behind pinhole
View from side (YZ)

\[ \frac{y}{f} = \frac{Y}{Z} \]

Y, Z

pinhole position (0, 0, 0)

image plane
\[ Z = -f \]
View from above (XZ)

\[ \frac{x}{f} = \frac{X}{Z} \]

pinhole position \((0, 0, 0)\)

image plane \(Z = -f\)
Image position \textit{in radians}*

\[
\begin{pmatrix}
\frac{x}{f'} \\
\frac{y}{f'} \\
\end{pmatrix}
= \begin{pmatrix}
\frac{X}{Z'} \\
\frac{Y}{Z'} \\
\end{pmatrix}
\]

*assuming small angle approximation
Visual direction *in radians*

\[
\begin{pmatrix}
x \\
y \\
\frac{1}{f} \\
\frac{1}{f'} \\
\end{pmatrix}
= \begin{pmatrix}
\frac{X}{Z'} \\
\frac{Y}{Z} \\
\end{pmatrix}
\]
Example  (ground and horizon)
Image projection
(upside down and backwards)
Visual direction
(image plane in front of pinhole)

Image projection
(image plane behind pinhole)
The mapping $Z(x, y)$ from image positions $(x, y)$ to depth $Z$ values on a 3D surface is called a “depth map”.
What is the depth map of a ground plane?

Ground plane

\[ Y = -h \]
What is the depth map of a ground plane?

Ground plane \( Y = -h \)

Thus, \( Z = \frac{-hf}{y} \)

\[
\frac{y}{f} = \frac{Y}{Z}
\]
Visual direction

(image plane in front of pinhole)

\[ Z = -h \left( \frac{f}{y} \right) \]
Binocular Vision

Assume eyes are separated by $T_X$ in the X direction. $T_X$ is the interocular distance.
What is the *difference* in or visual direction (or image position) of each 3D object in the left and right images?

How does this difference depend on depth?
View from above (XZ)
Binocular disparity \( \equiv \frac{x_l}{f} - \frac{x_r}{f} \)

is the difference in visual direction of a 3D point as seen by two eye.
Binocular disparity \( \equiv \frac{x_l}{f} - \frac{x_r}{f} \)

\[
\frac{x_l}{f} = \frac{X_0}{Z_0}
\]

\[
\frac{x_r}{f} = \frac{X_0 - T_x}{Z_0}
\]

Thus, binocular disparity \( = \frac{T_x}{Z_0} \)
Superimposing left and right eye images

Zero disparity

\[ \text{binocular disparity} = \frac{T_x}{Z_0} = \frac{T_x}{-h} \left( \frac{y}{f} \right) \]
Vergence (rotating the eyes)

Here we assume horizontal rotation only ("pan").
Vergence

Negative disparity

Positive disparity

Zero disparity

Example: verge on far person
Let $\theta_l$ and $\theta_r$ be the rotations of the left and right eyes due to vergence.

The rotations can be *approximated* by a shift in image position.

Binocular disparity $\equiv \left( \frac{x_l}{f} - \theta_l \right) - \left( \frac{x_r}{f} - \theta_r \right)$

$$= \left( \frac{x_l}{f} - \frac{x_r}{f} \right) - (\theta_l - \theta_r)$$