lecture 1

- two's complement
- floating point numbers
- hexadecimal

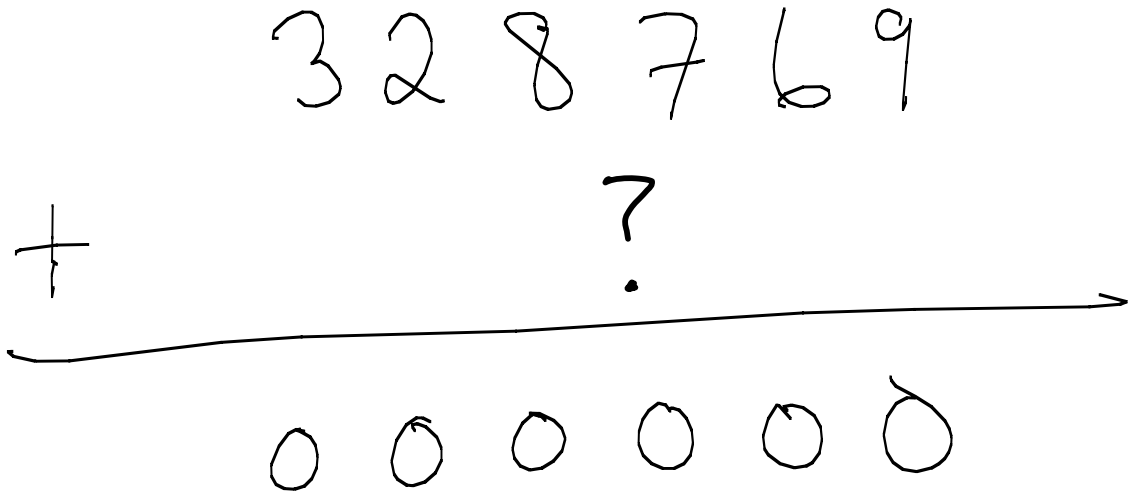
Mon. January 11, 2016

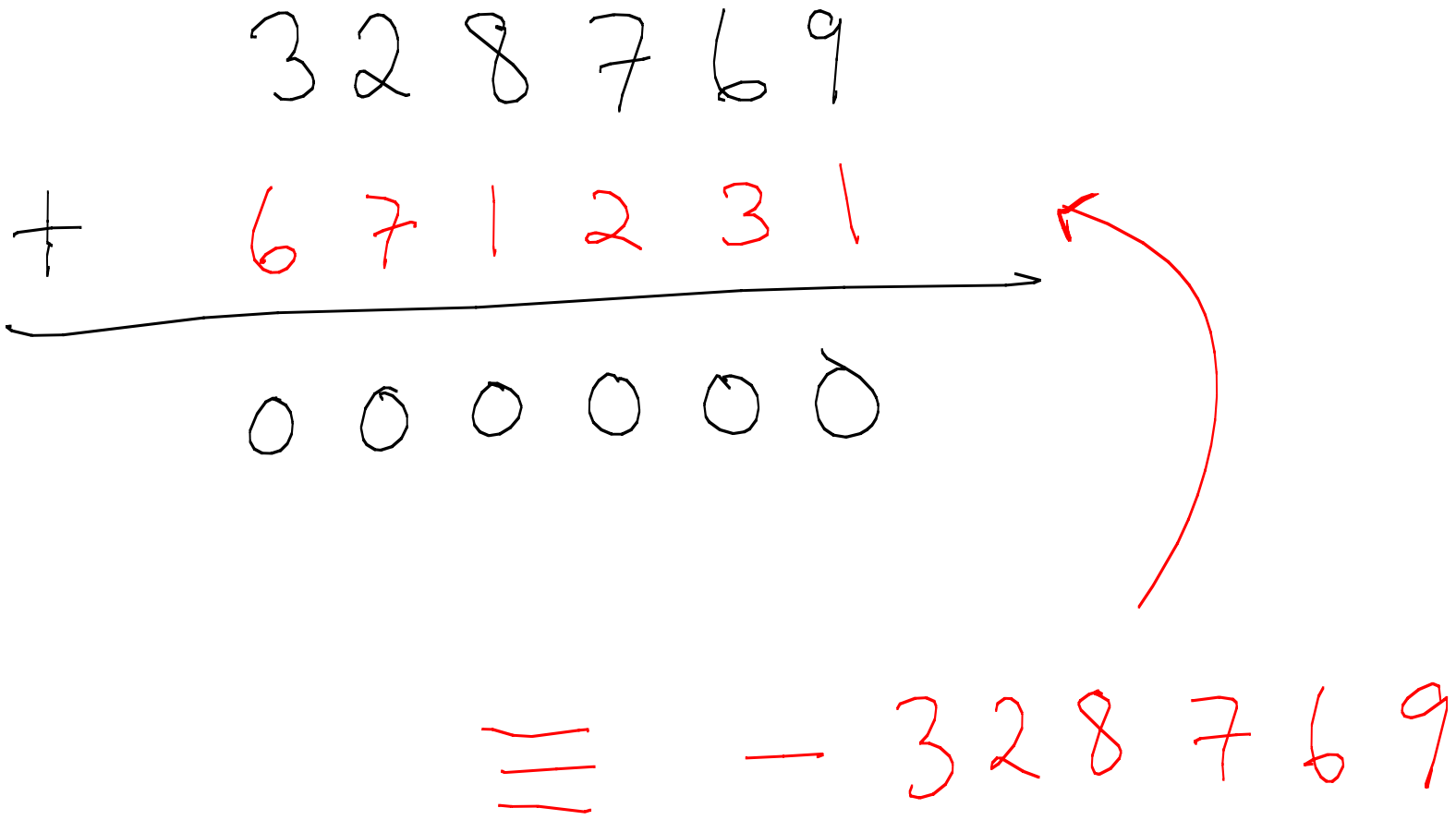
Car odometer (fixed number of digits)


```
  0 0 0 0 9 9 9
+ 0 0 0 0 0 0 1
-----------------
  0 0 0 1 0 0 0
```

$$
\begin{array}{r}
9\ 9\ 9\ 9\ 9\ 9 \\
+\ 0\ 0\ 0\ 0\ 0\ 1 \\
\hline
0\ 0\ 0\ 0\ 0\ 0
\end{array}
$$

$$
\Rightarrow \quad 999999 \equiv -1
$$

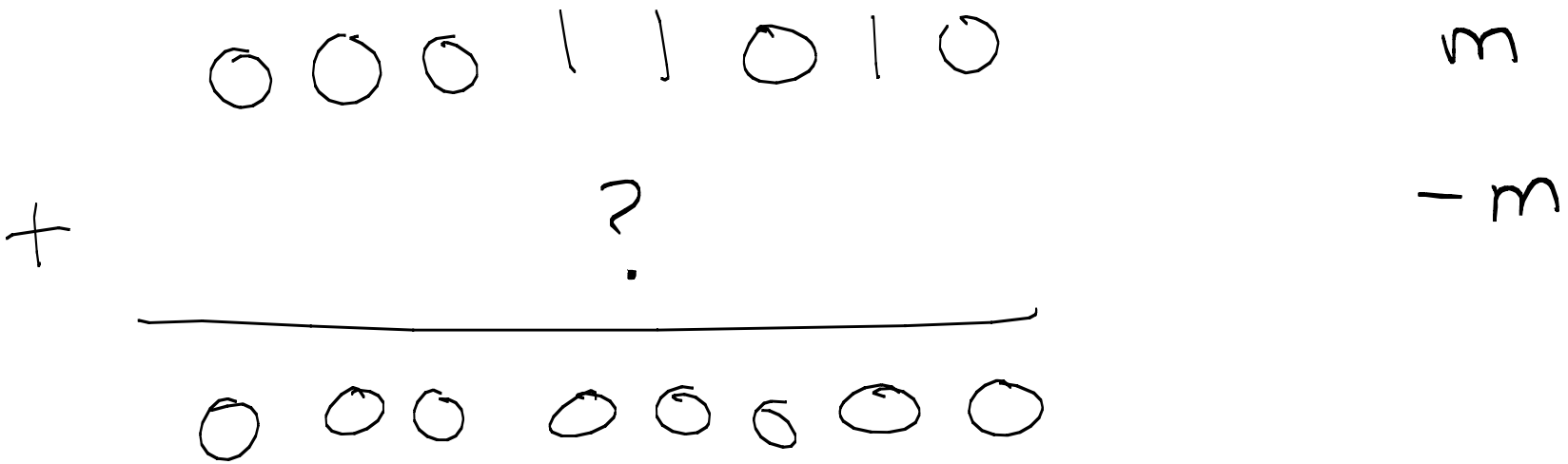

32769
+ ?
000000



If you know what "modular arithmetic" is (MATH 240), then you recognize this: addition of integers mod 10^6.

Q: How to represent negative numbers in binary ?

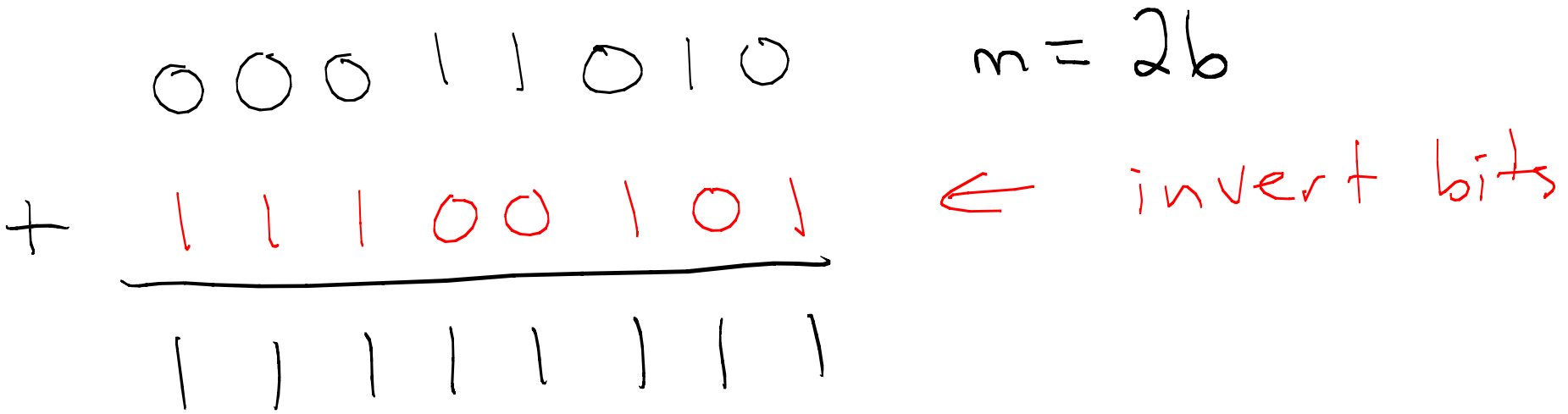
A: Given an 8 bit binary number m, define -m so that m + (-m) = 0.

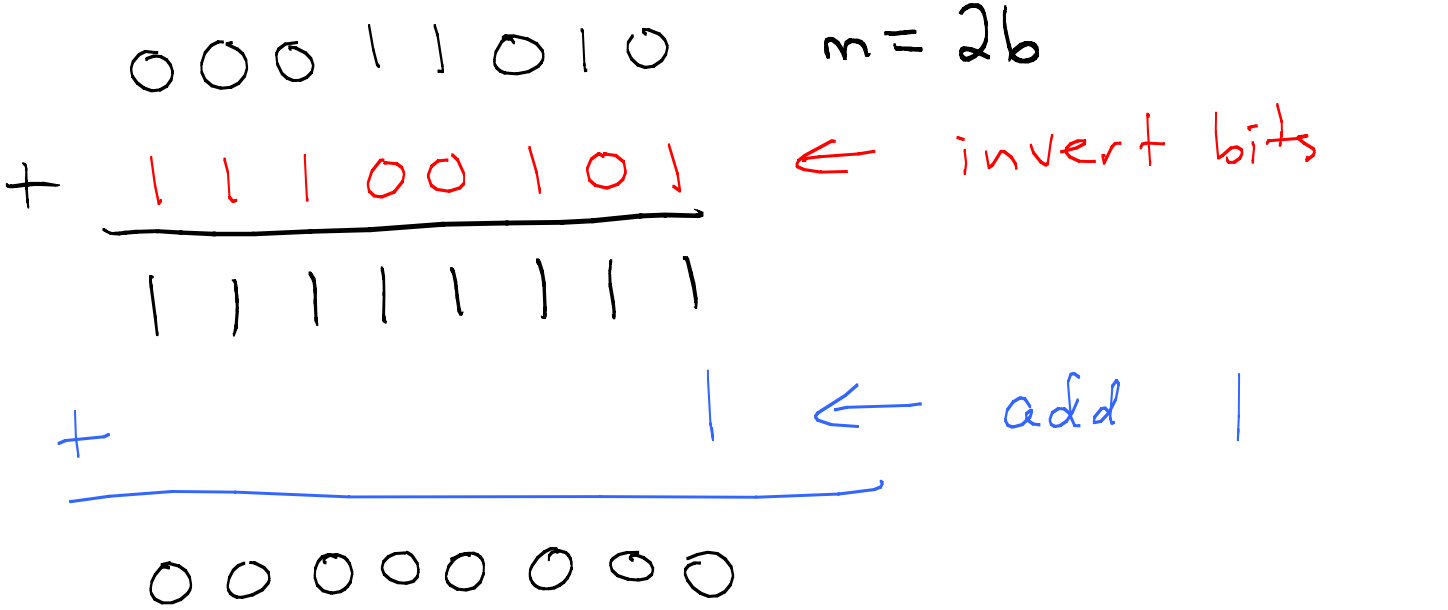


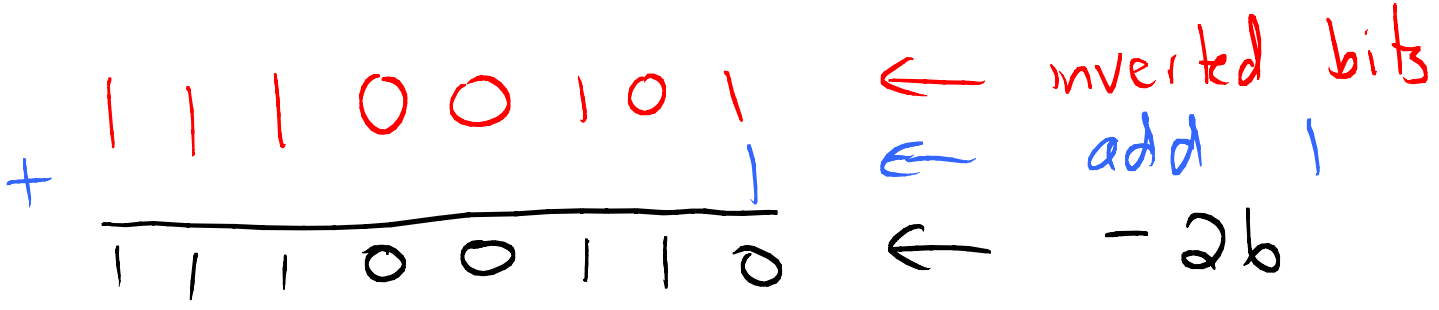
Two's complement representation of integers

Example: How to represent -26 ?

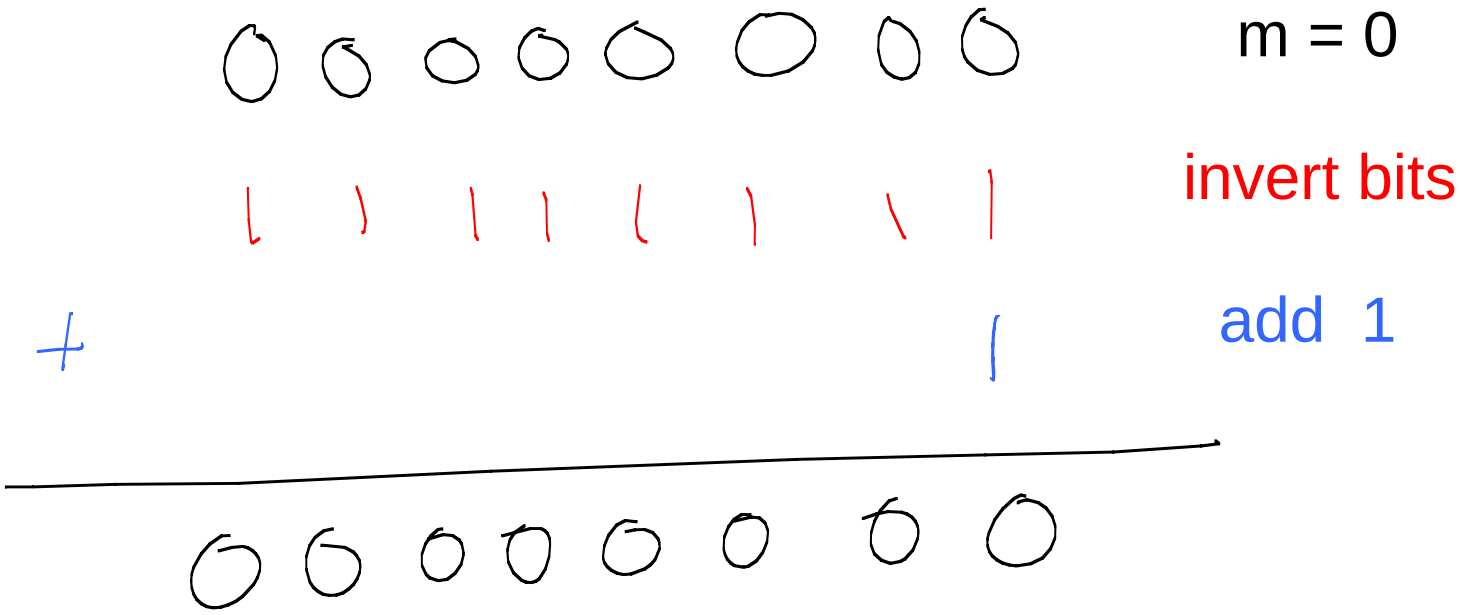
Use a trick!

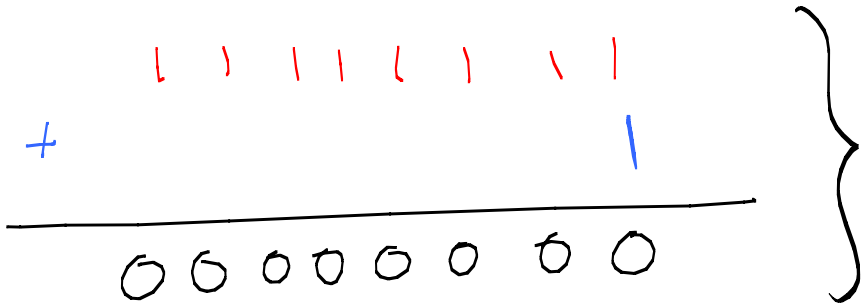



00011010
m = 26
+ 11100101
← invert bits
11111111
+ 1
← add 1
00000000


11100101
← inverted bits
+ 1
← add 1
11100110
← -26

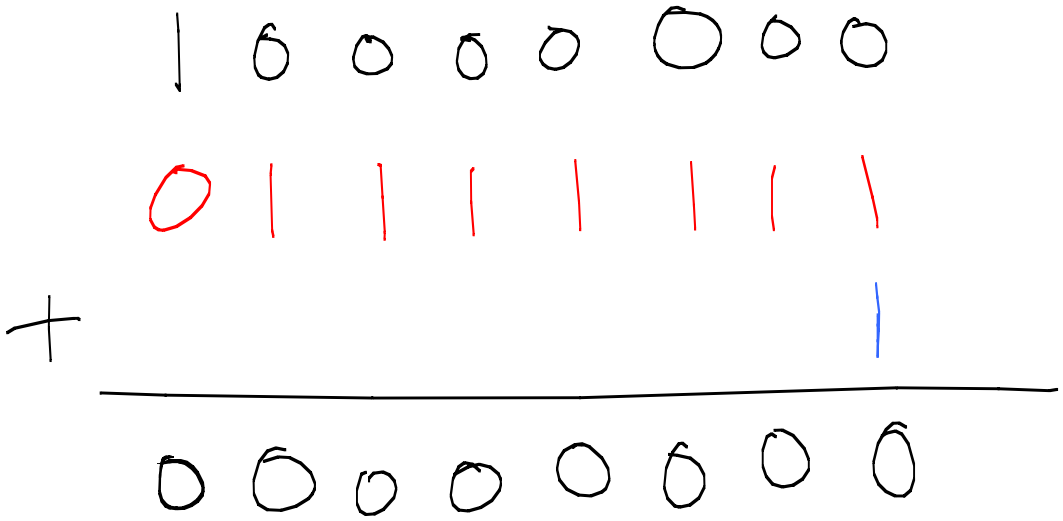
Another example: What is -0 ?

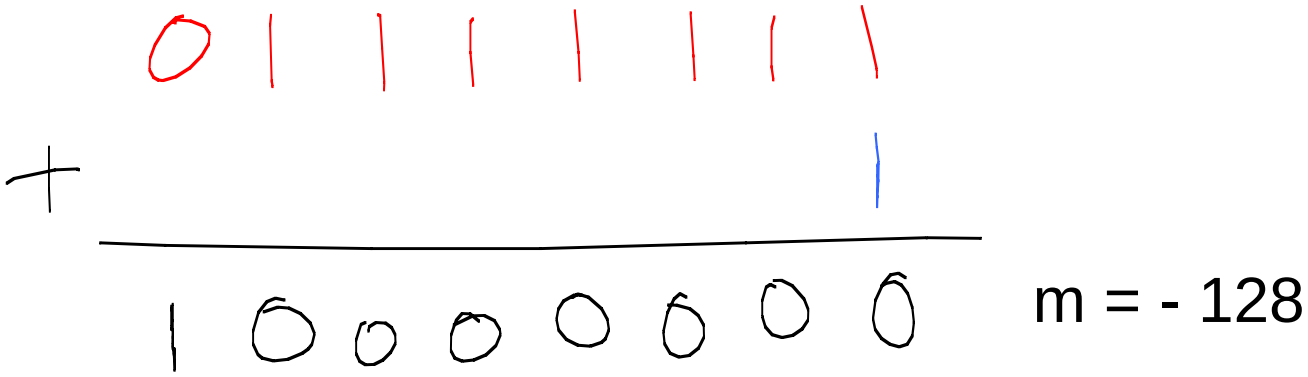




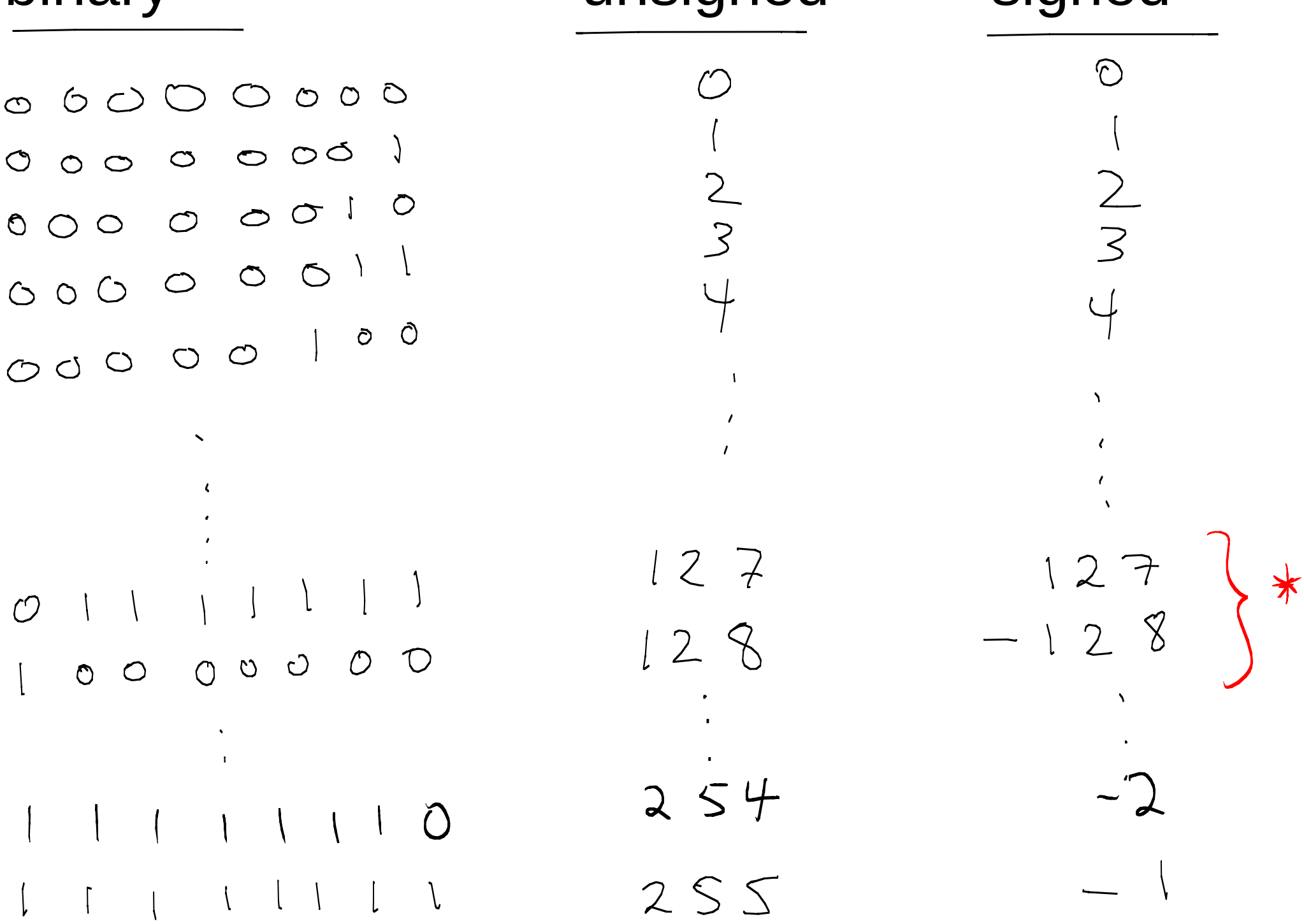
We have verified that -0 = 0.

What about m = 128 ? What is -128 ?

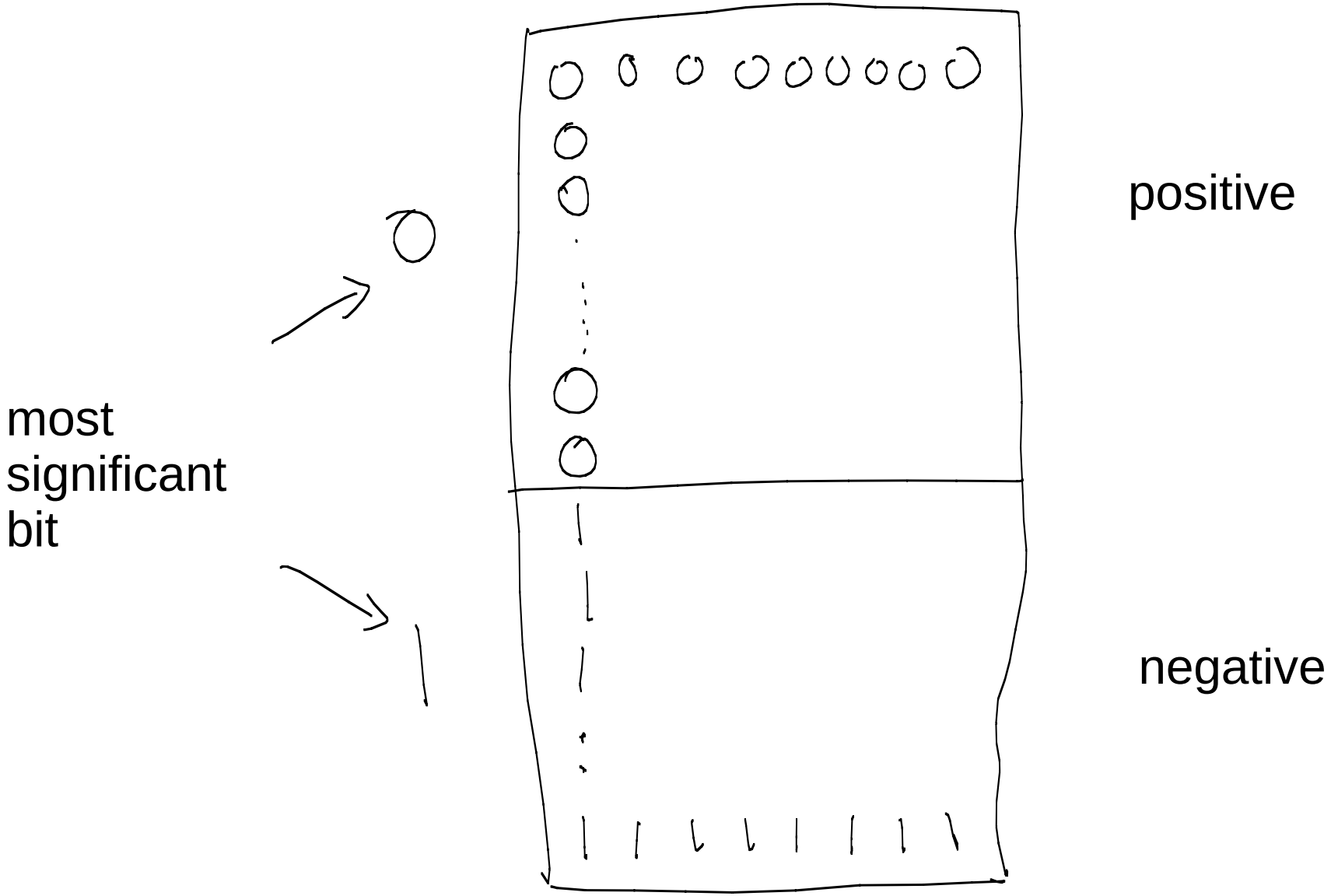




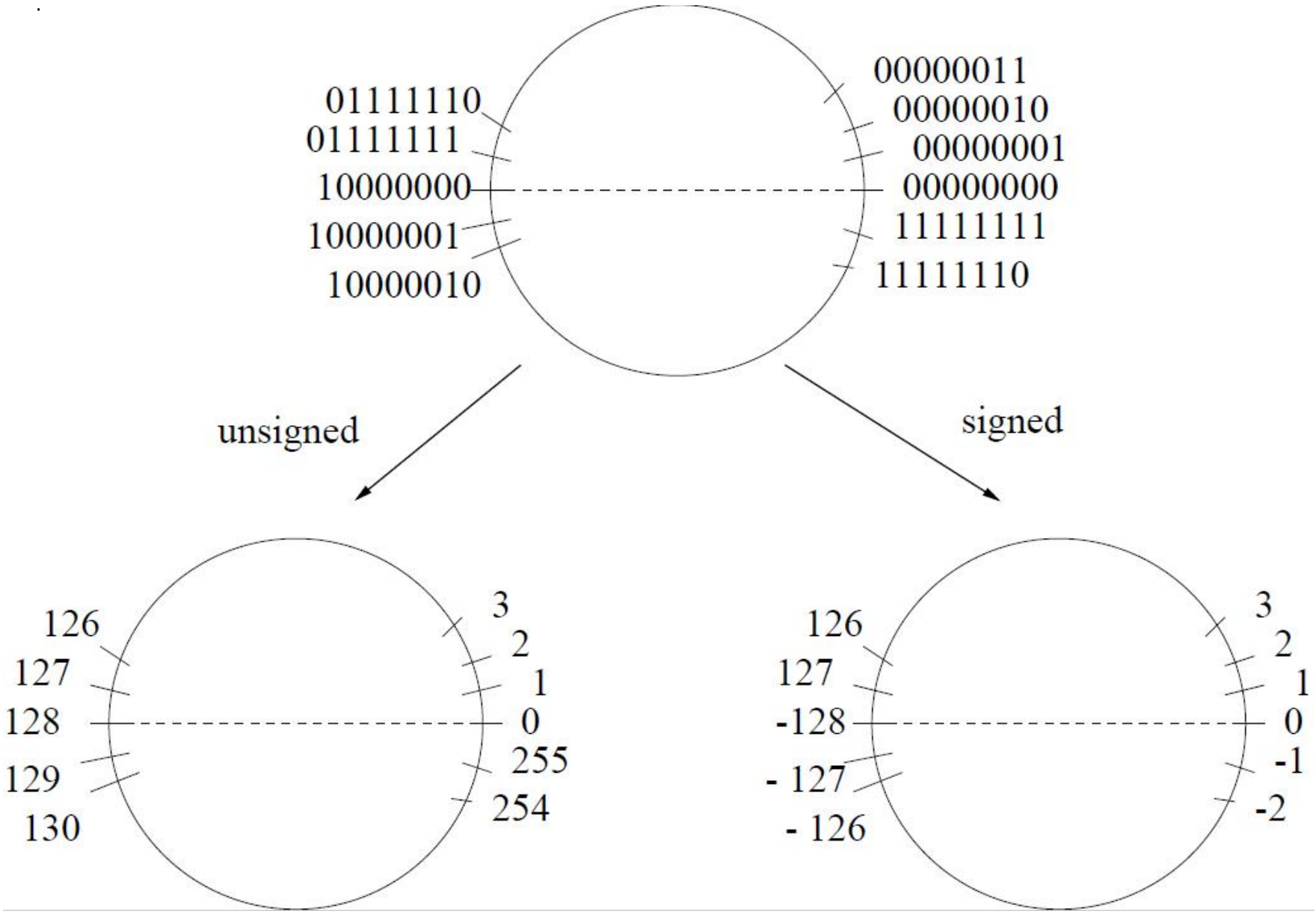
Thus, 128 is equivalent to -128.


binary
"unsigned"
"signed"
0
1
2
3
4
127
128
254
255
0
1
2
3
4
127
-128
-2
-1
*

signed integers



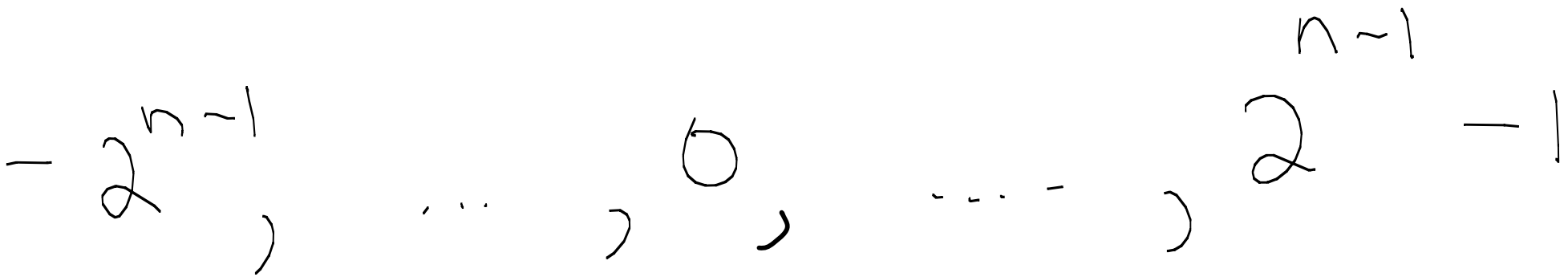
8 bit integers (unsigned vs. signed)



n bits defines 2^n integers

unsigned

signed



Take n = 32.

The largest signed integer is 2^31 - 1.

2 ^ 10 = 1024 ~ 10 ^ 3 = one thousand.

2 ^ 20 ~ 10 ^ 6 = one million

2 ^ 30 ~ 10 ^ 9 = one billion

2 ^ 31 ~ 2,000,000,000 = two billion

Java Example

```
int  j = 4000000000;   // 4 billion  >  2^31
```

This gives a compiler error. "The literal of type int is out of range."

```
int  j = 2000000000;  //   2 billion <  2^31

System.out.println( 2 * j );

//  This prints out  -294967296.
//   To understand why these particular digits are printed,  you
//   would need to convert 4000000000 to binary, which I don't
//    recommend.)
```

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Mon. January 11, 2016

Floating Point

"decimal point"

$$26.375 = 2\times10^{1} + 6\times10^{0} + 3\times10^{-1} + 7\times10^{-2} + 5\times10^{-3}$$

"binary point"

$$(11010.011)_2 = 2^4 + 2^3 + 2^1 + 2^{-2} + 2^{-3}$$

$$= 16 + 8 + 2 + 0.25 + 0.125$$

$$= 26.375$$

Convert from binary to decimal

We must use both positive and negative powers of 2.

n	2^n
-1	.5
-2	.25
-3	.125
-4	.0625
-5	.03125
-6	.015625
-7	.0078125
⋮	etc.

Sum up the contributing 1 bits as on previous slide.

How to convert from decimal to binary ?

$$26.375 = (\underline{\quad ? \quad} . \underline{\quad ? \quad})_2$$

To find the bits for the positive powers of 2, use the algorithm from last lecture ("repeated division") .

m	b_i
26	
13	0
6	1
3	0
1	1
0	1

$\Rightarrow$ 26

$= (11010)_2$

What about negative powers of 2 ?

In general, note that multiplying by 2 shifts bits to the left (or shifts binary point to the right)

Example:

$$(11010.011)_2 \times 2$$

$$= (11010\ 0.11)_2$$

Similarly....dividing by 2 and not ignoring remainder shifts bits to the right (or shifts binary point to the left)

$$(1101.011)_2 / 2$$

$$= (1101.0011)_2$$

For the negative powers of 2, use "repeated multiplication"

$$
\begin{aligned}
& .375 \\
= \; & .375 \times 2^{1} \times 2^{-1} \\
= \; & .75 \times 2^{-1} \\
= \; & 1.5 \times 2^{-2} \\
= \; & 3.0 \times 2^{-3}
\end{aligned}
$$

convert decimal to binary

$$
\begin{aligned}
= \; & (11)_{2} \times 2^{-3} \\
= \; & (.011)_{2}
\end{aligned}
$$

A more subtle example:

$$19.243 = (\ ?\ )_2$$

First, find the bits for the positive powers of 2 using "repeated division" (last lecture).

m	b_i
19	
9	1
4	1
2	0
1	0
0	1

$\therefore 19 = (10011)_2$

Then find the bits for the negative powers of 2 using repeated multiplication.

$$= .243$$

$$= .243 \times 2^{1} \times 2^{-1}$$

$$= (0).486 \times 2^{-1}$$

$$= ?$$

Then find the bits for the negative powers of 2 using repeated multiplication.

$$.243$$

$$= (0)_2 \; .486 \times 2^{-1}$$

$$= (00)_2 \; .972 \times 2^{-2}$$

$$= (001)_2 \; .944 \times 2^{-3}$$

$$= (0011)_2 \; .888 \times 2^{-4}$$

Thus $(.243)_{10} = (.0011)_2 + \sum_{i=-5}^{-\infty} b_i 2^i$

Note the summation is over bits bi from -5, -6, ..., -infinity.

$19.243 = (10011.0011\overline{})_2$

We cannot get an exact representation using a finite number of bits for this example.

Can we say anything more general about what happens ?

$$.(05)_{10}$$

$$= (0)_2 .1 \times 2^{-1}$$

$$= (00)_2 .2 \times 2^{-2}$$

$$= (000)_2 .4 \times 2^{-3}$$

$$= (0000)_2 .8 \times 2^{-4}$$

$$= (00001)_2 .6 \times 2^{-5}$$

$$= (000011)_2 .2 \times 2^{-6}$$

$$= .00\ \underline{0011}\ \underline{0011}\ \underline{0011}\ \text{etc.}$$

6 6 4

This will repeat over and over again.

When we convert a floating point decimal number with a finite number of digits into binary, we get:

- a finite number of non-zero bits to left of binary point

- an infinitely repeating sequence of bits to the right of the binary point

Why ?

[Note: sometimes the infinite number of repeating bits are all 0's, as in the case of 0.375 a few slides back.]

Recall previous example...

$$.243$$

$$= (0)_2 \; .486 \times 2^{-1}$$

$$= (00)_2 \; .972 \times 2^{-2}$$

$$= (001)_2 \; .944 \times 2^{-3}$$

$$= (0011)_2 \; .888 \times 2^{-4}$$

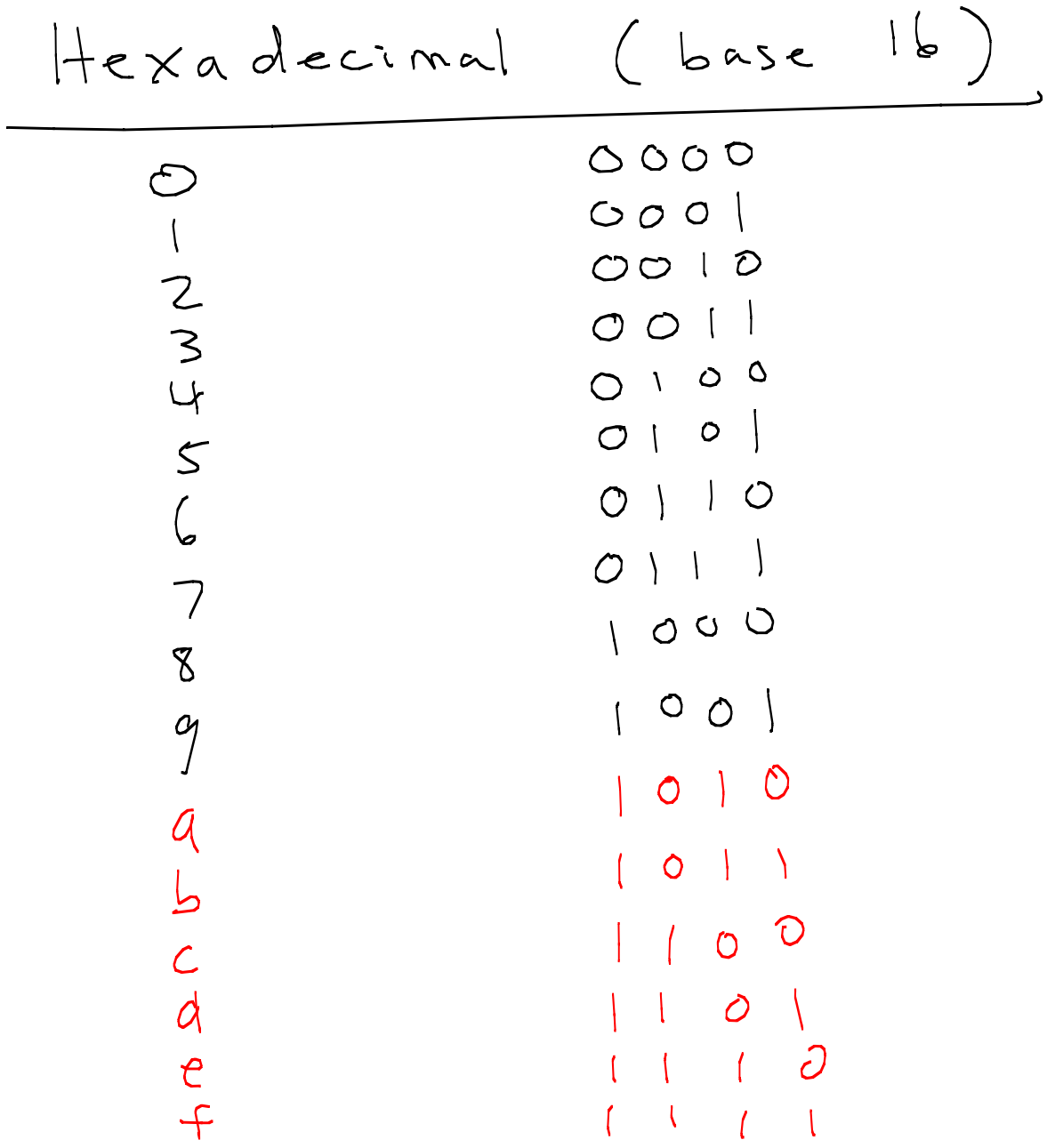
$$= \text{etc}$$

Eventually, the three digits to the right of the decimal point will enter a cycle that repeats forever. This will produce a bit string that repeats forever.

Hexadecimal

Writing down long strings of bits is awkward and error prone.

Hexadecimal simplifies the representation.



Examples of hexadecimal

1) 0010 1111 1010 0011

2 f a 3

We write 0x2fa3 or 0X2FA3.

2) 101100

We write 0x2c (10 1100), not 0xb0 (1011 00)

See Exercises 1 Questions 1-7.

Quiz 1 will be on Monday Jan 18.
(15 minutes at start of class)

Waiting list issues. (I will speak with admin.)

OSD issues. They don't handle 15 min quizzes.