Mid-Term Exam 2

COMP 251 Algorithms and Data Structures
Tues. March 11, 2014
Prof. Michael Langer

LASTNAME: ____________________ FIRSTNAME: _______________________ ID:________________

Instructions:

- You may use a small number of pages of notes.
- No electronic devices are allowed.
- If your answer does not fit on a page, then use the reverse side.

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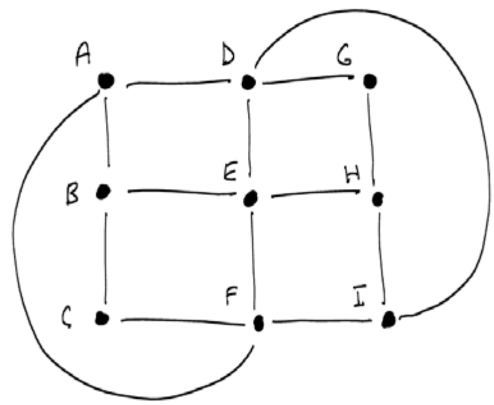
question	points	score
1	7	
2	7	
3	6	
4	10	
TOTAL	30	

1. (7 points)

a) Is the following a bipartite graph? If yes, give a partition of the vertices. If no, why not?



b) Consider the preferences shown below for the stable marriage problem.

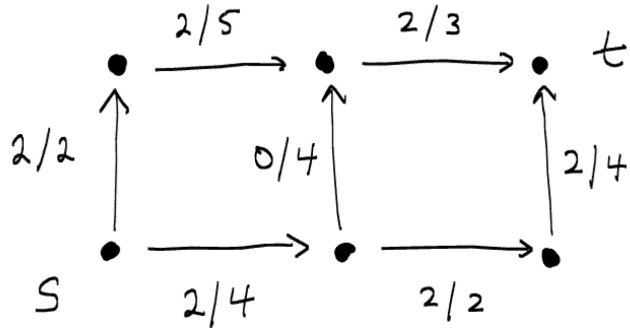
A	A's preferences		B	B's preferences
$\alpha1$	$\beta2$ $\beta1$ $\beta3$		$\beta1$	$\alpha1$ $\alpha2$ $\alpha3$
$\alpha2$	$\beta3$ $\beta2$ $\beta1$		$\beta2$	$\alpha2$ $\alpha3$ $\alpha1$
$\alpha3$	$\beta2$ $\beta3$ $\beta1$		$\beta3$	$\alpha2$ $\alpha3$ $\alpha1$

Use the Gale-Shapley algorithm to find a stable matching.

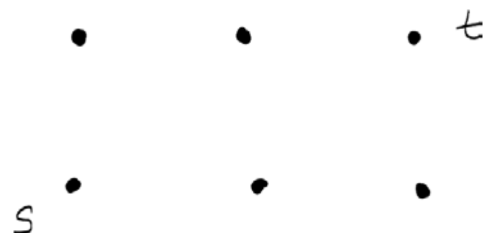
c) The matching $(\alpha1,\beta1)$, $(\alpha2,\beta2)$, $(\alpha3,\beta3\ )\}$ is unstable. Why?

2. (7 points)

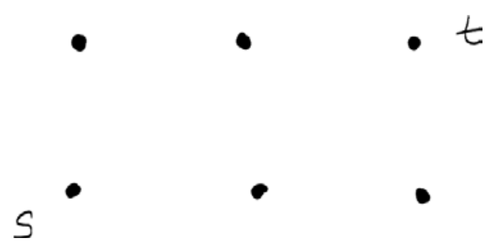
For the flow network below, each edge (u,v) has a label of the form f(u,v) / c(u,v) where f(u,v) is the flow and c(u,v) is the capacity.



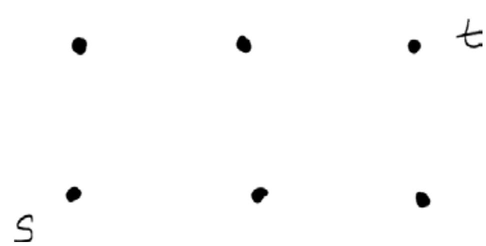
a) Draw the residual graph. Indicate the capacity and direction of each edge. (You do not need to label each edge as backwards or forward.)



b) Draw an augmenting path for the residual graph in (a).



c) Draw the minimum s-t cut of the original graph which is defined by edge capacities c(u,v). You do not need to draw the edges again.



3. (6 points)

Suppose we have m types of coins with values **v1 > v2 > … > vm**. For the example of quarters, dimes, nickels, and pennies, we have m=4 and v1=25, v2=10, v3=5, v4 = 1.

Let f(n) be the minimum number of coins whose values add up to exactly **n**.

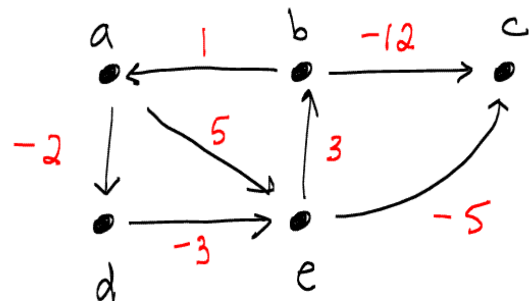
a) Give a greedy algorithm for solving for f(n). This algorithm should provide a good solution but not necessarily an optimal solution. You may describe the algorithm in words rather than pseudocode.

b) Using trial and error, find the optimal solution to this coin change problem for **n= 43**, and for the following values: **v1= 16, v2 = 12, v3 = 3, v4 = 1**.

c) For the general coin change problem described at the top of the page, write a recurrence for f(n) in terms of the values of the m types of coins. Assume for simplicity that at least one solution exists.

4. **(10 points)**

a) Run the Bellman-Ford shortest path algorithm on the following graph, starting from vertex **a**. Specifically, fill in the tables below.



#edges	a	b	c	d	e
0	0	infinity	infinity	infinity	Infinity
1					
2					
3					
4					

vertex	a	b	c	d	e
prev[vertex]					

b) Suppose you are given a directed graph that has negative values on some of the edges. The Bellman Ford algorithm will give the correct solution for shortest paths from some starting vertex s, *if there are no negative cycles*. But suppose you don't know if the given graph has negative cycles. One way to check for negative cycles is to run Bellman Ford for n iterations, instead of n-1. Explain.