lecture 2

binary numbers

- decimal to binary conversion
- fractional numbers

$$238$$
$$= 2 \times 10^2 + 3 \times 10^1 + 8 \times 10^0$$

$$(10011)_{two}$$
$$= 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 16 + 2 + 1$$
$$= 19$$

$$238 \times 10$$
$$= (2 \times 10^2 + 3 \times 10^1 + 8 \times 10^0) \times 10$$
$$= 2 \times 10^3 + 3 \times 10^2 + 8 \times 10^1 + 0 \cdot 10^0$$
$$= 2380$$ i.e. shift left

What is the corresponding property for binary numbers?

$$m = (10011)_{two}$$

$$m \times 2$$
$$= (1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) \times 2$$
$$= 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$
$$= (100110)_{two}$$

- Multiplying by 2 $\Rightarrow$ shift left

Similar idea with division:

$$238 / 10 = 23$$

i.e. ignore remainder 8

(Shift right and drop last digit)

$$\Rightarrow 238 = 23 \times 10 + 8$$

$$m = \underbrace{(m / 10)}_{\text{integer division}} \times 10 + \underbrace{m \% 10}_{\text{"mod"}}$$

What is the corresponding property in binary?

$$m = (m/2) \times 2 + m \% 2$$

Example:

$m = (10011)_{two}$

$m/2 = 1001$

$(m/2) \times 2 = 10010$

$m \% 2 = 1$

$$m = \sum_{i=0}^{n-1} b_i \, 2^i$$

$$= (b_{n-1} b_{n-2} \cdots b_2 b_1 b_0)_{two}$$

$$m/2 = \sum_{i=1}^{n-1} b_i \, 2^{i-1}$$

$$= (b_{n-1} b_{n-2} \cdots b_2 b_1)$$

$$m \% 2 = b_0$$

Algorithm: given m in decimal, convert it to binary

```
i ← 0
while  m > 0 {
    b[i] ← m % 2
    m    ← m / 2
    i    ← i + 1
}
```

Example $m = 241 = (11110001)_{two}$

m	b[i]
241	
$= 120 \times 2 +$	1
60	0
30	0
15	0
7	1
3	1
1	1
0	1

$= \_\_\_ \times 2 + \_\_\_$

Fractional Numbers

"decimal point"

$$26.375 = 2 \times 10^1 + 6 \times 10^0 + 3 \times 10^{-1} + 7 \times 10^{-2} + 5 \times 10^{-3}$$

"binary point"

$$(11010.011)_{two} = 2^4 + 2^3 + 2^1 + 2^{-2} + 2^{-3}$$

$$= 16 + 8 + 2 + 0.25 + 0.125$$

$$= 26.375$$

Multiplying by 2 means

Shifting bits to left

i.e. shifting binary point to right.

$$(11010.011)_{two} \times 2$$

$$= (110100.11)_{two}$$

Dividing by 2 and <u>not</u> ignoring remainder means shifting bits to right, i.e. shifting binary point to left.

$$(11010.011)_{two} / 2$$
$$= (1101.0011)_{two}$$

Converting fractional numbers from binary to decimal is straightforward.

But how do we convert from decimal to binary?

$$19.243 = ?$$

m	$b[i]$
19	
9	1
4	1
2	0
1	0
0	1

$\therefore\ 19 = (10011)_{two}$

$$19.243 = (10011.\_\_\_\_)_{two}$$

$$\begin{aligned} .243 &= .243 \times 2 \times 2^{-1} \\ &= .486 \times 2^{-1} \\ &= .972 \times 2^{-2} \\ &= 1.944 \times 2^{-3} \end{aligned}$$

But

$$1 \times 2^{-3} < 1.944 \times 2^{-3} < 2^{-2}$$

Thus, $.243 = .001\_\_\_\_$

$$19.243 = (10011.001\_\_\_\_)_{two}$$

<u>Note</u>: We will not get an exact representation using a finite number of bits for this example.

Another Example

$$26.375 = (11010.011)_{two}$$

$$\begin{aligned} &.375 \\ &= .75 \times 2^{-1} \\ &= 1.5 \times 2^{-2} \\ &= 3.0 \times 2^{-3} \\ &= (11)_{two} \times 2^{-3} \\ &= (.011)_{two} \end{aligned}$$