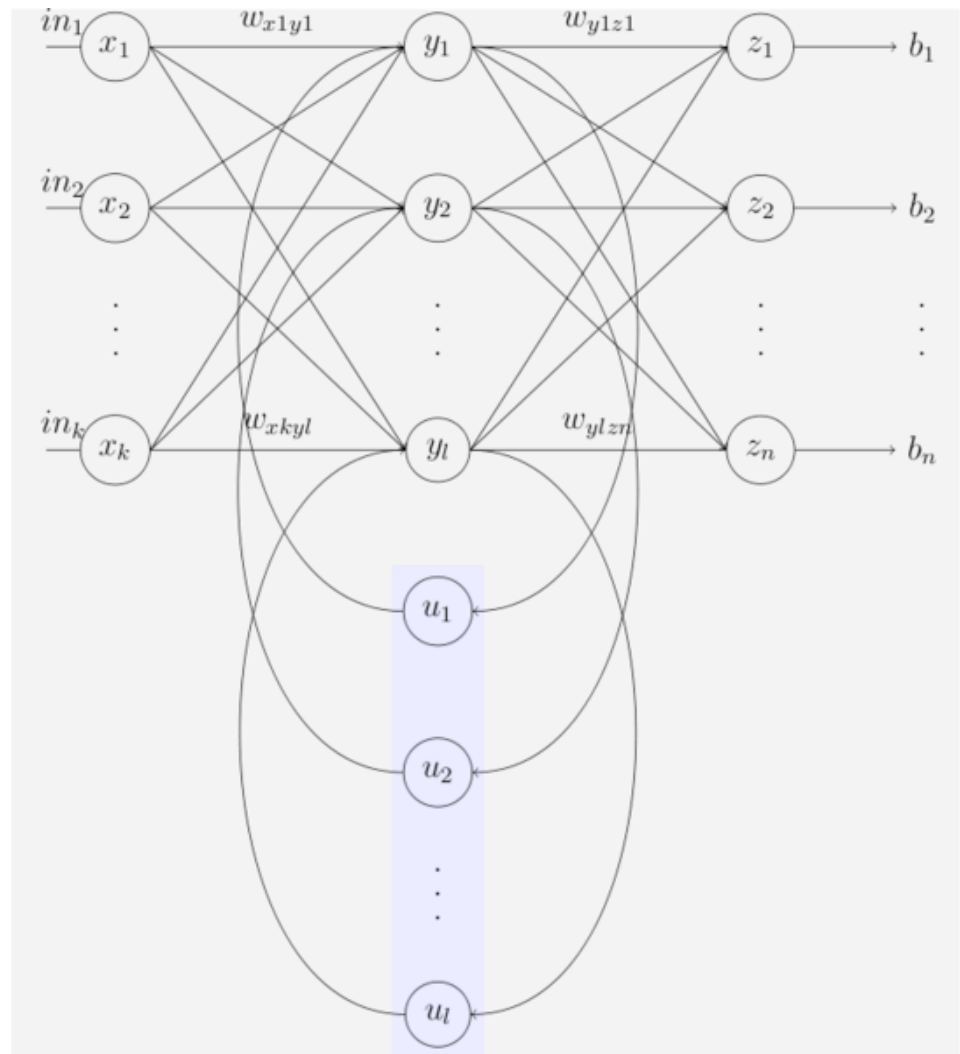


# **Recurrent (or Auto-Associative) Neural Networks**

# Elman Network

- includes a layer of “context units” connected from hidden layer with weight of 1
- at each iteration, hidden unit outputs copied into context units



# Jordan Network

- similar to Elman network but:
  - context units connected to output layer
  - values in context units ( $c_i$ ) are updated, not replaced by values of output units ( $o_i$ ), i.e.,  
$$c_i = o_i + \gamma c_i$$

# Associative Memory

- store a set of  $p$  patterns
- when presented with a new pattern, network responds by producing closest stored pattern “in memory”
- uses relaxation learning

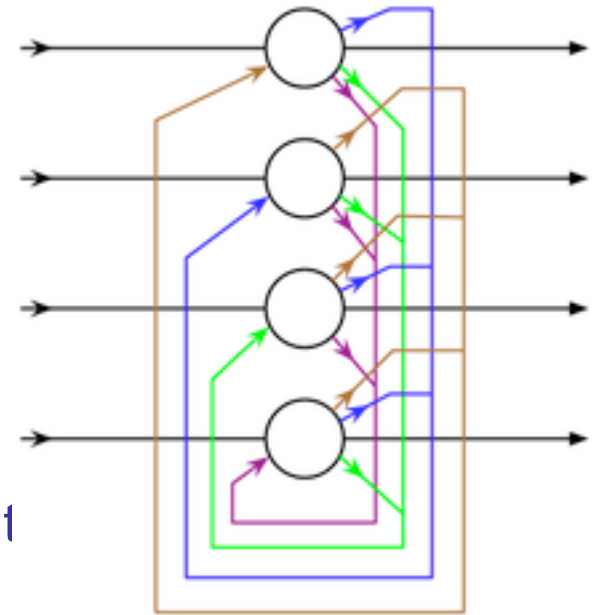
# Relaxation Learning

- Important questions:
  - will the network settle?
  - how long does it take to settle?
  - how are the weights acquired?
  - how much information does each unit need to convey to neighbours?

# Hopfield Networks

- N interconnected neurons, potentially completely recurrent
- all neurons are both input & output; all weights are symmetric (but  $w_{ii}=0$ )
- activation values are binary (typically -1/+1)

- activation rule:  $a_j = \begin{cases} +1 & \text{if } \sum_i w_{ij} a_i > \theta_j \\ -1 & \text{otherwise} \end{cases}$   
where  $\theta_j$  is the threshold



# Learning and Recall

- set weights to ensure learned patterns are stable
- pattern  $x^p$  stable if, when clamped (activation fixed for the given neurons), all neurons stable
- recalling a pattern is achieved by activating (possibly a subset of) neurons with the corresponding inputs and then letting the network settle

# Hebbian learning

- $w_{ij} \leftarrow$  correlation of activations (+1)
- for  $n$  patterns  $x^p$

$$w_{ij} = \sum_p x_i^p x_j^p \text{ if } i \neq j, \text{ else } 0$$

- limits of learning: for a network of  $N$  neurons, recall errors become severe above  $0.15N$  “memories”
  - stored patterns become unstable
  - spurious stable states appear

# Why simple Hebbian Learning Fails

- don't know how to set weights to or from hidden units  
← can't generalize beyond simple Hopfield Networks