Optimizing Iterative Learning Control of Cyclic Production Processes with Application to Extruders

M. Pandit and K.-H. Buchheit

***Abstract*— Conventional and new methods for the control of cyclic processes are described and compared on the basis of their performance results achieved in an aluminum extruder plant. The thrust of the work lies in the area of iterative learning control systems. After a brief description of (linear) iterative learning control, the optimizing iterative learning control of cyclic processes is presented. In this method the control input is adjusted from cycle to cycle such that a prescribed quantitative performance index is made to take on an extremum. The results which the presented methods of cyclic control yield when applied to a simulation model of an aluminum extruder are compared with one another. Finally, results obtained in an actual industrial extruder plant are given. The new method yields an increase of production by 10% as compared to methods in current use.**

***Index Terms*—Learning control systems, metals industry, noncontact temperature measurement and control, optimal control, periodic control, process control.**

I. Introduction

CYCLIC processes occur in a variety of manufacturing plants. In such processes, one has often the task of applying suitable inputs such that one or more process variables follow prescribed target trajectories. In principle one could try to employ conventional closed loop control, e.g., with a proportional integral derivative (PID) type controller, for performing the task. In case of cyclic processes which have large time-lags and are nonlinear, one looks for simple alternative control schemes which exploit the cyclic nature of the process and yield better results.

One control scheme applicable for performing the task and investigated by various authors is linear iterative control [19], [4], [3]. For a basic and succinct exposition of linear iterative learning control the reader is referred to the survey paper of Moore *et al.* [13]. Examples of applications often cited by the authors are robot control [6], batch processes, extruders etc. A common feature of such processes is that each cycle starts with the same initial condition, has the same target trajectory and is of the same duration. In this paper, motivated by the concrete problem of the control of a cyclically operating extruder, the practical applicability of known iterative learning control schemes is examined. Then standard methods of calculus of variations, see, e.g., [12], are adapted for the control of cyclic processes to obtain an *optimizing iterative learning control* scheme. The effectiveness of noncausal filtering which can be employed in iterative learning control for dealing with low frequency disturbances without endangering system stability is established. Finally, results obtained in an actual industrial extruder are presented. The goal of the paper is to present a control scheme for cyclic processes with due consideration to implementation aspects and discuss the control performance obtained in an industrial extruder rather than prove convergence properties of controllers under various conditions. For the latter type of investigations the reader is referred to the literature on iterative learning control cited in this section and also to [2].

II. Control Approaches

A general formulation of the problem of control of a cyclic process with a single input and a single output is as follows: The system starting with the same initial state and driven successively in the $1, 2, \cdots, k$th cycle by a sequence of input trajectories is represented in terms of input $u_k(t)$, output $y_k(t)$ and disturbance $\underline{z}_k(t)$ in the cycle k by means of the nonlinear time varying operator[1] representing the plant

$$y_k(t) = \Phi_k\{u_k(t), z_k(t)\} \tag{1}$$

where $\Phi_k\{.,.\}$ denotes the operator which assigns $y_k(t)$ to $u_k(t)$ and $z_k(t)$. This is a two-dimensional dynamic process, i.e., a process in which the functions have two arguments, viz. continuous time t and the cycle number k. In the sequel it is assumed that the variations of the operator over two consecutive cycles are slow and can be neglected. Then the operator obtained by an identification performed in cycle k can be used for determining the input for the following cycle $k+1$. Furthermore, the disturbance is assumed to be identical over the cycles. Then we can write

$$y_{k+1}(t) = \Phi_k\{u_{k+1}(t)\}. \tag{2}$$

For each k, we denote by t the time in the interval $[0, T]$ of each cycle.

The objective is to determine and apply the input $u_{k+1}(t)$ such that the output $y_{k+1}(t)$ follows the target trajectory $y_d(t)$. Basically, the following control strategies can be employed.

1) Conventional closed-loop control. One can attempt to employ conventional feedback control, e.g., by employing a PID-controller, eventually with adaptation of the controller parameters from cycle to cycle. The scheme is unlikely to function satisfactorily if the plant possesses

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[1] Functions, operators and functionals are denoted by $(\cdot)$, $\{\cdot\}$, and $[\cdot]$, respectively.

a large time-lag and nonlinearities. In such cases one may still be able to obtain satisfactory performance by exploiting the cyclic nature of the process. This is indeed the situation in the case of temperature control in extruders described in a later section of the paper.

2) Iterative learning control. Iterative learning control schemes exploit the cyclic operation mode, and consequently yield better system performance. Two approaches to iterative learning control are:
   - Linear iterative learning control;
   - Optimizing iterative learning control in which an optimality criterion is successively improved.

In both case the input to the plant in the $(k+1)$th cycle is obtained as a function of time $u_{k+1}(t)$ which depends on the error and other functions in the preceding cycle(s) and up to the instant t in the current cycle. To avoid unnecessary generalization, we deal with the learning control law of first order of the form

$$u_{k+1}(t) = F_L\{u_k(t), y_k(t), y_d(t)\} \tag{3}$$

where $F_L\{\cdots\}$ represents a 'learning' operator, $y_d(t)$ is the target trajectory which $y(t)$ has to follow and the control action depends only on the input and output functions in the immediately preceding cycle k. Eventually the output up to the instant t in the current cycle $k+1$ can also be taken into account by superimposing direct feedback control on to the iterative learning control.

This method was first applied by Uchiyama [19] and developed by Arimoto and colleagues. The general *linear iterative learning controller of PID-type*

$$u_{k+1}(t) = u_k(t) + K_1\dot{e}_k(t) + K_2 e_k(t) + K_3\int_0^t e_k(\tau)\,d\tau. \tag{4}$$

where $e_k(t) = y_d(t) - y_k(t)$ is the error in the cycle K_1, K_2 and K_3 are constants was studied by Arimoto *et al.* [4]. It has been proved that if the transfer function of the plant, assumed to be rational, fulfills certain rather general conditions and the controller parameters are chosen appropriately, the error tends to zero as k tends to ∞ in the sense of a certain function norm (λ-norm). For sampled data systems Togai and Yamano [17] obtain a linear iterative learning controller which involve gradient type algorithms. For applying linear iterative learning control, however, the plant must fulfill the following conditions.

1) The target trajectory $y_d(t)$ which $y(t)$ has to follow is identical for every cycle.
2) The system parameters are fixed or are very slowly time-varying.
3) The disturbance $z(t)$ is identical in every cycle.
4) The initial state $\underline{x}_k(0) = \underline{x}_0$ of the system is identical for every cycle.
5) Each cycle has the fixed duration T.
6) The control input and control error are not subjected to any further constraints.

Often, as in the case of the problem of temperature control in an extrusion plant, Conditions 1 to 5 are approximately fulfilled, but Condition 6 is not fulfilled. The λ-norm of the error is not suitable for practical applications and restrictions on the control variable and input trajectories have to be taken into account. Also it should be possible to allow for changes in the duration of the cycle time by specifying a constraint equation. Therefore we look for a more general control law of the form

$$u_{k+1}(t) = F_L\{u_k(t), y_k(t), y_d(t)\} \tag{5}$$

where the specifications on the control input and output, such as a smoothness and boundedness of the input $u(t)$ are taken into account.

These goals are achieved with an *optimizing iterative learning control law*, which is derived on the basis of the gradient of the functional Q which denotes the prescribed quantitative performance index associated with the input function $u(t)$.

III. Design of Optimizing Iterative Learning Control and Signal Processing Schemes for a Cyclic Process

To retain the advantage of the iterative reduction of the control error and at the same time to take the nonlinearity into account we apply the control law of the form

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t). \tag{6}$$

The trajectory $\Delta u_{k+1}(t)$, which represents the necessary modification of the control input of the kth cycle is calculated after completion of cycle k and before commencement of cycle $k+1$ with the objective that the improvement of the control performance from cycle k to cycle $k+1$ is maximal. Thereby the performance index of the control in the cycle $(k+1)$ is taken to be a functional of the form

$$Q[u_{k+1}(t), y_{k+1}(t)] = \int_0^T L(u_{k+1}(t), \dot{u}_{k+1}(t), \cdots, y_{k+1}(t)) \\ \dot{y}_{k+1}(t), \cdots, y_d(t))\,dt. \tag{7}$$

The functions $u_{k+1}(t)$ and $y_{k+1}(t)$ in the performance index are connected by the constraint equation corresponding to the plant model given by (2), viz. by

$$y_{k+1}(t) = \Phi_k\{u_{k+1}(t)\} \tag{8}$$

where $\Phi_k\{\cdot\}$ denotes a general operator. The function $L(\cdot)$ is so chosen that stipulations both on the behavior of control error and input function are taken into account. Then the optimal trajectory $u_{k+1}(t)$ is to be determined such that $Q[u_{k+1}(t), y_{k+1}(t)]$ takes on the required extremum. In the sequel we assume that the desired extremum is a minimum and that this is the only extremum which exists.

In keeping with the principle of iterative learning control, we consider the increments

$$\begin{aligned}\Delta y_{k+1}(t) &= y_{k+1}(t) - y_k(t)\\ e_{k+1}(t) &= y_d(t) - y_{k+1}(t) = e_k(t) - \Delta y_{k+1}(t)\end{aligned} \tag{9}$$

of the functions from cycle k to cycle $k+1$. The performance index $Q_\Delta[\Delta u_{k+1}(t), \Delta y_{k+1}(t)]$ with the *increments* as arguments is

$$\begin{aligned} Q_\Delta[\Delta u_{k+1}(t), \Delta y_{k+1}(t)] \\ = Q[u_k(t) + \Delta u_{k+1}(t), y_k(t) + \Delta y_{k+1}(t)] \end{aligned} \quad (10)$$

and the relationship between (i.e., the constraint equation) $\Delta u_{k+1}(t)$ and $\Delta y_{k+1}(t)$ is given by

$$\begin{aligned} \Delta y_{k+1}(t) &= \Phi_{k+1}\{u_k(t) + \Delta u_{k+1}(t)\} - \Phi_k\{u_k(t)\} \\ &= \psi_{k+1}\{u_k(t), \Delta u_{k+1}(t)\} \end{aligned} \quad (11)$$

where $\psi_{k+1}\{u_k(t), \Delta u_{k+1}(t)\}$ represents the plant operator for the incremental function $\Delta u_{k+1}(t)$ in the neighborhood of $u_k(t)$. Under the assumption that the system changes slowly with time, the local system operator for cycle $(k+1)$ is replaced by the operator for the cycle k. Furthermore it is approximated by a linear convolution operator, whose parameters, however, may depend on the trajectory $u_k(t)$. With $h_k(t) =$ step response of the linearized plant valid for the cycle k and the corresponding input, one can then write (11) using the convolution symbol $\otimes$ as

$$\Delta y_{k+1}(t) = \dot{h}_k(t) \otimes \Delta u_{k+1}(t). \quad (12)$$

For the optimizing iterative learning control, the trajectory $\Delta u_{k+1}(t)$ is calculated at the beginning of the cycle $(k+1)$ and superimposed on the function $u_k(t)$. If the system were exactly given by the linear operator then it would achieve the target trajectory after one cycle (if the disturbance is constant). However, if the system is only approximately described by the operator, it achieves the optimal performance only after a few cycles. The fact that the target trajectory is indeed achieved after some cycles is intrinsic to iterative learning which is realized because the error trajectory of a completed cycle is taken into account while determining the input at the commencement of the succeeding cycle. As is known, one would obtain a linear control algorithm in case of a linear plant model and a quadratic performance index. In this case the optimizing iterative learning control is identical with the linear iterative learning control whose convergence properties have been well studied [2].

A. Determination of the Optimal Trajectory $\Delta u_{k+1}(t)$

The optimal trajectory $\Delta u_{k+1}(t)$ which minimizes the performance index in (10) is determined using the method of calculus of variations described in textbooks such as [12] and [18]. Some relevant formulas are summarized in the Appendix. The conditions for minimum which $\Delta u_{k+1}(t)$ fulfills are

$$\begin{aligned} \nabla_{\Delta u} Q_\Delta[\Delta u_{k+1}(t), \Delta y_k(t)] \equiv 0 \\ \Delta y_{k+1}(t) = \dot{h}_k(t) \otimes \Delta u_{k+1}(t) \\ 0 \preceq t \preceq T \end{aligned} \quad (13)$$

where $\nabla_{\Delta u} Q_\Delta$ is the gradient of the functional Q_Δ with regard to $\Delta u_{k+1}(t)$. As the direct analytical solution of (13) is seldom possible, they are solved numerically. To do this, (13) are reduced to their discrete form: By sampling the gradient, i.e., by setting $t = i \cdot T_A$, $i = 0, 1, 2, \cdots, n-1$

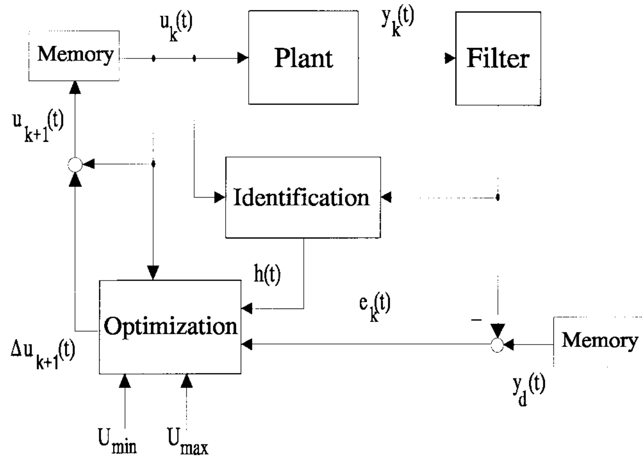


Fig. 1. Structure of the optimal iterative learning control system.

in the expression for the gradient and constraint equations, approximating integrals by sums and differentiation by differences and using the notation $\Delta u_{k+1}(i) = \Delta u_{k+1}(i \cdot T_A)$ and $\Delta y_{k+1}(i) = \Delta y_{k+1}(i \cdot T_A)$, one obtains a set of equations (generally nonlinear) of the form

$$\begin{aligned} &g_i(\Delta u_{k+1}(0), \cdots, \Delta u_{k+1}(n-1), \Delta y_{k+1}(0), \cdots \\ &\quad \Delta y_{k+1}(n-1), u_k(0), \cdots, u_k(n-1), y_k(0), \cdots \\ &\quad y_k(n-1), y_d(0), \cdots, y_d(n-1)) = 0 \\ &\qquad\qquad i = 0, 1, 2, \cdots, n-1 \end{aligned} \quad (14)$$

$$\Delta y_{k+1}(i) = \dot{h}_k(i) \otimes \Delta u_{k+1}(i), \quad i = 0, 1, 2, \cdots, n-1. \quad (15)$$

Note that $u_k(i)$ and $y_k(i)$ as well as $h_k[\cdot]$ in (14) and (15) are known from the previous cycle. These equations are solved numerically using steepest descent, Newton or other iterative algorithms to obtain $\Delta u_{k+1}(i)$.

Fig. 1 shows the principle structure of the optimal iterative learning control system.

B. Example for the Determination of the Control Input Trajectory

To illustrate the application of the optimizing iterative learning control in industrial practice, we consider the task of determining the control input of the extruder described in the next Section IV which yields a constant temperature of the extruded aluminum. From considerations of extrusion technology, the performance index for the $(k+1)$th cycle was chosen to be (see Section IV)

$$\begin{aligned} Q[\dot{u}_{k+1}(t), y_{k+1}(t)] = \frac{\lambda}{T} \int_0^T (\dot{u}_{k+1}(t))^2\, dt \\ + \frac{1}{T} \int_0^T (y_d(t) - y_{k+1}(t))^2\, dt. \end{aligned} \quad (16)$$

For small deviations of the input the equation

$$\Delta y_{k+1}(t) = \int_0^t \Delta \dot{u}_{k+1}(\tau) h_k(t-\tau)\, d\tau \quad (17)$$

is taken to hold, where $h_k(t)$ represents the step response identified after the kth cycle using $\Delta \dot{u}_k(t)$ and $\Delta y_k(t)$ by deconvolution. For the initial cycle, $k = 0$, the input $u(t)$ is chosen to be a step function of suitable amplitude. In this case identification of the step-response reduces to scaling the output. For increasing k, as the error decreases, the richness of excitation may be lost. One has to employ a supervisory loop to "freeze" the identification and continue the optimization with the last set of reliable parameters. Using (17), first the optimal trajectory $\Delta \dot{u}_{k+1}(t)$ is calculated and then by integration the trajectory $\Delta u_{k+1}(t)$.

For the functional in the $(k+1)$th cycle we thus have

$$\begin{aligned} Q_\Delta[\Delta \dot{u}_{k+1}(t), \Delta y_{k+1}(t)] &= \frac{\lambda}{2T}\int_0^T (\dot{u}_k(t) + \Delta \dot{u}_{k+1}(t))^2\, dt \\ &\quad + \frac{1}{2T}\int_0^T (e_k(t) - \Delta y_{k+1}(t))^2\, dt, \end{aligned} \tag{18}$$

$$e_k(t) = y_d(t) - y_k(t).$$

The gradient of the functional in (18) is obtained by inspection of (34) and (39) in the Appendix as

$$\begin{aligned} \nabla_{\Delta u} Q_\Delta[\Delta \dot{u}_{k+1}(t)] &= \frac{\lambda}{T}(\dot{u}_k(t) + \Delta \dot{u}_{k+1}(t)) \\ &\quad - \frac{1}{T}\int_t^T \left(e_k(\tau) - \int_0^t \Delta \dot{u}_{k+1}(\xi) \right. \\ &\quad \left. \times\, h_k(t-\xi)\, d\xi \right) h_k(\tau - t)\, d\tau. \end{aligned} \tag{19}$$

For the computation of $\Delta \dot{u}_{k+1}(t)$ one has to equate the right hand side of the above equation to zero and solve it under consideration of the constraint. In practice this is performed numerically. An advantage of first calculating the continuous gradient and subsequently sampling it is that the sampling can be performed at the appropriate—not necessarily equidistant—instants. In the following example, however, the gradient is sampled at equidistant instants. In this case one could also start with discrete models as in [17].

C. Implementation Aspects of the Control

For simplifying the calculations and at the same time to attain smooth control, practical tests show that it is expedient to apply the control input in the form shown in Fig. 2. Here the control variable is sampled at the rate of $1/T_a$ and the control input changed only at the instants $m \cdot T_a$ where m is fixed at a value of, say, 5. Then we have

$$u_k(t) = \sum_{j=0}^{n-1} du_k(j) \cdot \sigma(t - jmT_a) \tag{20a}$$

$$\dot{u}_k(t) = \frac{1}{T_A}\sum_{j=0}^{n-1} du_k(j) \cdot \delta(t - jmT_a)$$
$$i = 0, 1, \cdots, n \cdot m - 1. \tag{20b}$$

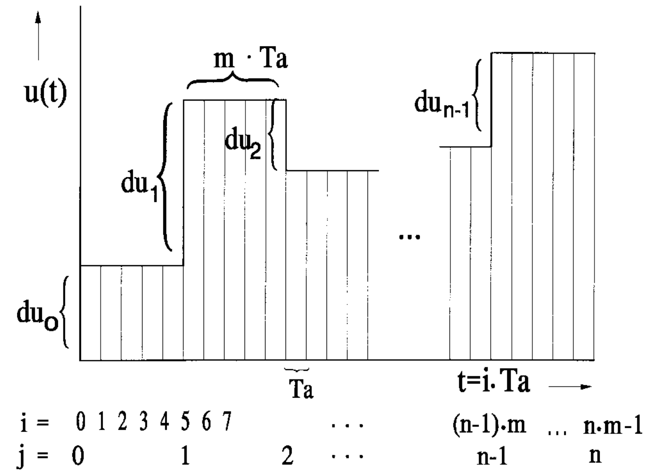


Fig. 2. Schematic run of input trajectory.

The plant operator is written as

$$y_k(t) = \sum_{j=0}^{n-1} du_k(j) \cdot h(t - jm). \tag{21}$$

The gradient of the functional is obtained by substituting (20) in (19)

$$\begin{aligned} &\nabla_{\Delta u} Q_\Delta[\Delta \dot{u}_{k+1}(t)] \\ &= \frac{\lambda}{T}\left[\frac{1}{T_a}\sum_{j=0}^{n-1}(du_k(j) + \Delta du_{k+1}(j))\delta(t - jmT_a)\right] \\ &\quad - \frac{1}{T}\int_t^T \left(e_k(\tau) - \sum_{j=0}^{n-1}(du_k(j) + \Delta du_{k+1}(j)) \right. \\ &\quad \left. \times\, h(\tau - jmT_a) \right) h_k(\tau - t)\, d\tau. \end{aligned} \tag{22}$$

By approximating integral by a sum and putting

$$t = i \cdot T_A, \quad i = 0, 1, 2, \cdots, m \cdot n - 1 \quad \text{and}$$
$$\tau = my \cdot T_A; \quad T = n \cdot m \cdot T_A; \quad d\tau = T_A$$

we get the equations fulfilled by the *iscrete gradient* at the instants $t = 0, mT_a, 2mT_a, \cdots (n-1)mT_a$ to be

$$\begin{aligned} &\frac{\lambda}{nmT_A^2}(du_k(i) + \Delta\, du_{k+1}(i)) \\ &\quad - \frac{1}{nm}\sum_{my=im}^{nm-1}\left(e_k(my) - \sum_{j=0}^{n-1}\Delta\, du_{k+1}(j) h(my - j) \right) \\ &\quad \times h(my - i \cdot m) = 0 \quad i = 0, 1, 2, \cdots n-1. \end{aligned} \tag{23}$$

The set of equations (23) is solved for $\Delta du_{k+1}(i)$ with the help of a numerical iterative algorithm (e.g., Newton or quasi-Newton algorithm). To take the hitting of the limits of the control action into account, the numerical algorithm is supplemented by the Kuhn–Tucker–technique [15]. Effects of disturbances in the form of changes in the initial condition

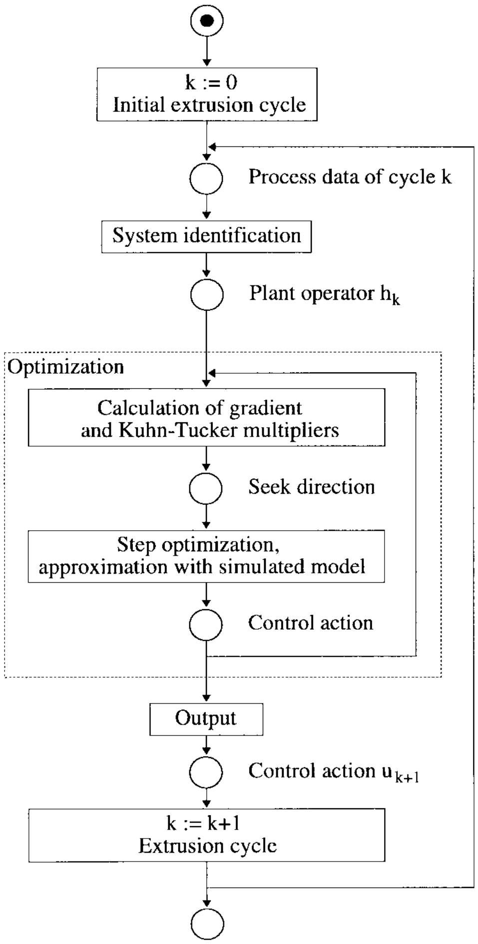


Fig. 3. Flow diagram for computing input trajectory.

are suppressed by calculating and superimposing a correction factor $\Delta y_{d,k+1}(i)$ onto the target trajectory y_d.

The actual calculation of $u_{k+1}(i)$ is carried out as follows.

1) The step response $h_k(i)$ is identified using $\Delta u_k(i)$ and $\Delta y_k(i)$ with the aid of a deconvolution algorithm. If the control error fall below a certain threshold, the identified parameters are rejected and the parameters used in the previous cycle are retained.
2) The correction factor $\Delta y_{d,k+1}(i)$ for changes in the initial condition is calculated.
3) The optimal function $\Delta du_{k+1}(i)$, and from this by summation the function $\Delta u_{k+1}(i)$, are calculated in an iteration loop using a numerical algorithm.
5) The function $\Delta u_{k+1}(i)$ calculated is superimposed on to the input function $u_k(i)$ to obtain $u_{k+1}(i)$.

Fig. 3 shows the procedure schematically.

D. Signal Processing in Iterative Learning Control Systems

An important advantage offered by iterative learning schemes is that they can cope with cyclic processes in which the measurement of the controlled variable is strongly corrupted by sensor noise. For controlling such processes, conventional direct feedback, as used e.g., in conjunction with a PID controller, allows only causal filtering. This inevitably leads to large time delays and sluggishness of the controlled process. Stability and sufficiently short response times are difficult to achieve.

The iterative learning control offers however the possibility of noncausal filtering with its inherent potential for smoothing signals without lag. Basically, noncausal filtering is performed after a cycle has been completed by off-line processing. It is expedient to start off the signal processing by passing the measured signal through a short length median filter to remove impulsive noise.

Various methods can be employed for realizing noncausal filters, e.g., using allpass filters or Hilbert transformators [9]. Of these, we adopt the method of synthesis using causal filters and design procedures for causal IIR-filters. A noncausal digital filter with transfer function $H(z)$ can be obtained by connecting two filters with transfer functions $\Gamma(z)$ and $\Gamma'(z) = \Gamma(1/z)$ in cascade: $H(z) = \Gamma(z) \cdot \Gamma'(z) \cdot w$ with $\Gamma(z) =$ transfer function of a causal IIR filter. The frequency response of the cascade is

$$H(e^{j\Omega}) = \Gamma(e^{j\Omega}) \cdot \Gamma(e^{-j\Omega}) = |\Gamma(e^{j\Omega})|^2. \qquad (24)$$

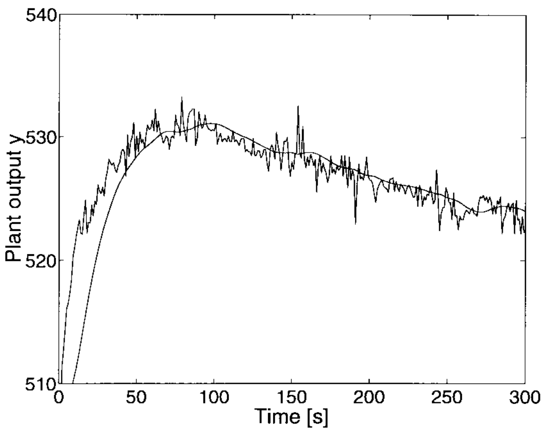
The task of determining the appropriate $\Gamma(z)$ is accomplished using design methods of causal IIR-Filters and bearing in mind, that at the chosen cutoff frequency the filter has a damping of 6 dB instead of 3 dB as the filter order is doubled. For suppressing edge effects, filter initialization or signal extension must be applied. Off-line filtering allows signal extension (the signal is extended by a reflected version of the input sequence).

Fig. 4(a) and (b) shows typical results of filtering pyrometer output signals measured in the extrusion plant with causal and noncausal second-order Butterworth filters.

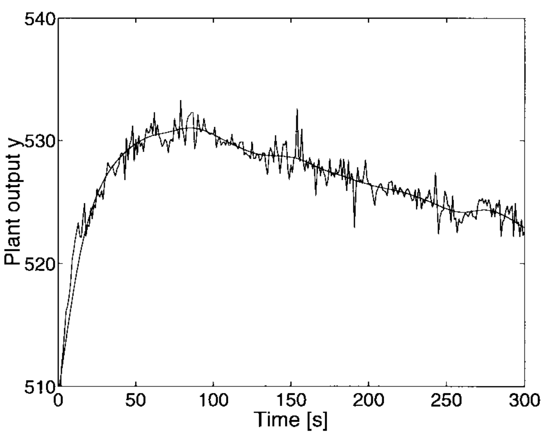
IV. Results Obtained with a Temperature Control System for Extruders

The optimizing iterative learning control was primarily developed for temperature control in an aluminum extruder for manufacturing bars. In an extruder, Fig. 5, a billet *Bi* of the metal is heated to a temperature of about 500°C in a furnace *Fu* and loaded into the receptacle *Re*. A hydraulic ram *Ra* squeezes the metal through an orifice with the appropriate geometry in a die D and forms the bars *Ba*. Subsequently, the next billet (heated in the meantime in the furnace) is loaded into the extruder and the process repeated successively. The heat generated by the deformation and the friction causes a rise of the temperature of the aluminum in the die. Metallurgical and extruder technology experience indicates that the quality of the product is best and the productivity high if the exit temperature ϑ_P of the aluminum bar as it leaves the die is maintained at a constant value of 540°C and the ram velocity has a smooth run [1]. This exit temperature ϑ_P can be influenced by varying either the temperature to which the billets are preheated and/or the velocity with which the ram squeezes the metal out of the die.

The goal of *isothermal extrusion* is to operate the extruder in such a way that the exit temperature ϑ_P is constant. Simulated isothermal extrusion, which is based on calculating the control



(a)



(b)

Fig. 4. (a) Causal and (b) noncausal filtering of pyrometer output.

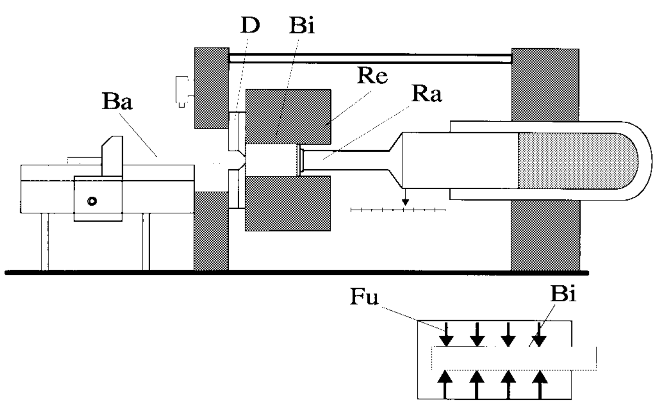


Fig. 5. Schematic of aluminum extruder.

input using a parallel plant model, has been proposed [7], [16]. Another method proposed [10] recommends the collection of a repertoire of suitable velocity profiles corresponding to different alloys and ambient conditions in a data bank and selecting and applying the appropriate one in actual operation. Whether the exit temperature ϑ_P is really maintained at a constant value or not with these methods in an industrial application is a matter of speculation as the *temperature* ϑ_P is *not measured*. Attempts to control the temperature ϑ_P by measuring it and employing a feedback control configuration had no prospects of success till of late, primarily due to the nonavailability of suitable contactless temperature sensors with the required accuracy (of about $\pm 5^\circ$ K).

A. Process Description and Simulation Model

The extrusion process is a very complicated one, we give here only a rudimentary mathematical description of it. The exit temperature of the bar ϑ_P(t) depends on the initial temperature in the deformation zone $\vartheta_I(t)$ and the temperature rise $\Delta\vartheta_D(t)$. The temporal temperature variation is given by

$$\dot{\vartheta}_P(t) = -\frac{1}{T_2}\vartheta_P(t) + \frac{K_2}{T_2}(\vartheta_I(t) + \Delta\vartheta_D(t)) \qquad (25)$$

where the constants K_2 and T_2 depend on the temperature of the die and the velocity of ram $v_{\mathrm{Ra}}(t)$, which in turn depends on the hydraulic pressure $p(t)$. The initial temperature $\vartheta_I(t)$ is calculated from the billet temperature $\vartheta_B(t)$, the receptacle temperature ϑ_R, which is assumed to be constant within a cycle and the velocity of ram $v_{\mathrm{Ra}}(t)$ from the equation

$$\vartheta_I(t) = \vartheta_B(t) + c_1\left[\int_0^t v_{\mathrm{Ra}}(\tau)\,d\tau - c_2(\vartheta_B(t) - \vartheta_R)\sqrt{t}\right] \qquad (26)$$

by applying a model of [11].

The temperature change $\Delta\vartheta_D(t)$ in the deformation zone is determined by a nonlinear equation in which the velocity of the ram $v_{\mathrm{Ra}}(t)$ and the initial temperature $\vartheta_I(t)$ appear as follows:

$$\Delta\dot{\vartheta}_D(t) = -\frac{1}{T_1}\Delta\vartheta_D(t) + \frac{K_1}{T_1}\left(c_3 \cdot v_{\mathrm{Ra}}^{\kappa}(t) \cdot e^{(c_4\vartheta_I(t))}\right). \qquad (27)$$

The constants K_1, T_1, c_1, c_2, c_3, c_4 and κ depend on the aluminum alloy, material properties of the plant and geometry of the profile and die. Variations of the alloy composition of the aluminum, aging of the die, the duration of operation of the plant etc. influence the relation considerably. Apparently one has to deal with a nonlinear, time variant, distributed parameter model with a large time-lag time for the plant. The simplified mathematical model—which turns out to be accurate enough for simulation studies and for determining initial settings of the new control scheme—is shown in Fig. 6.

B. Setup of Plant and Control System

Technological progress has made pyrometers employing sophisticated algorithms involving the measured radiation at several wavelengths available, which in principle offer the accuracy required for noncontact aluminum temperature measurement and control [8]. As also mentioned in [7], the pyrometers however have two drawbacks which make control difficult: 1) they exhibit an inherent time-lag and 2) their output is corrupted by nonstationary noise. Causal filtering (which would have to be inevitably employed for a PID-type control) would introduce additional lag. These disadvantages can be surmounted, thanks to the characteristic features of the iterative learning control. By employing the instruments in conjunction with optimizing iterative learning control and

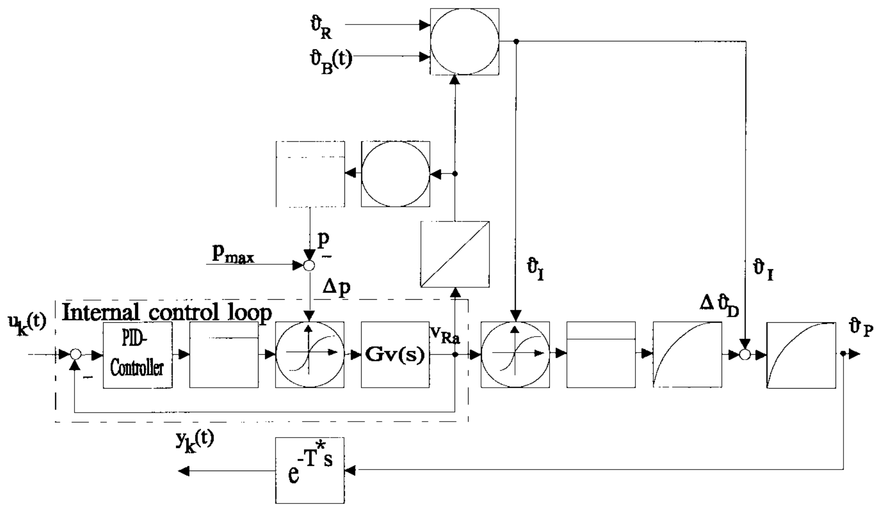


Fig. 6. Block diagram of aluminum extruder.

noncausal filtering, it is indeed possible to achieve satisfactory performance in a closed-loop control.

The performance index Q is so chosen that deviations in the output temperature from the desired temperature are minimized and at the same time the control action $u(t)$ is as smooth as possible. Trials with various expressions for the performance index led to the choice

$$Q[\dot{u}(t), y(t)] = \frac{1}{T}\int_0^T [\lambda(\dot{u}(t))^2 + (y_d(t) - y(t))^2]\, dt \quad (28)$$

where $y_d(t)$ and $y(t)$ correspond to the desired temperature $\vartheta_{\mathrm{PD}}(t)$ and the measured temperature $\vartheta_P(t)$ in a cycle respectively. The factor λ is a weighting factor which lies between zero and one and T is the cycle period.

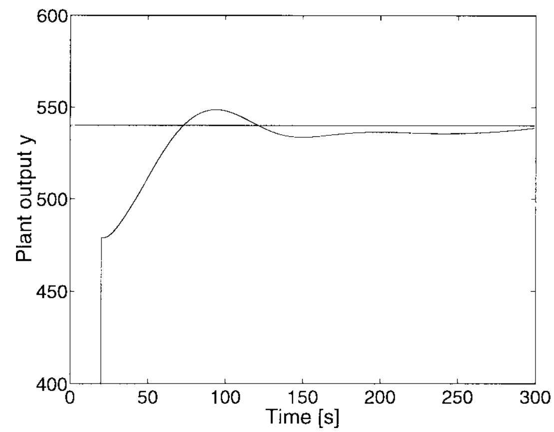
C. Performance Results

The control schemes described above were tested both with a simulated extruder model and an industrial extruder. Due to economics of the extruder plant, only the optimizing iterative learning control scheme could be tested with the industrial extruder.

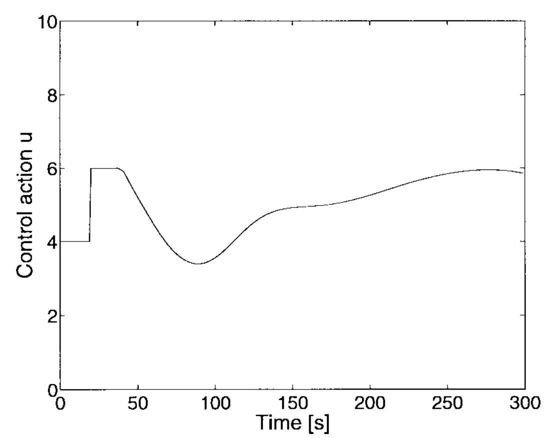
Figs. 7–9 depict the results obtained in simulation tests. Fig. 7 shows the control variable and the control input runs obtained with a PID controller with controller settings which yield a good compromise between overshoot and fast settling time. Linear iterative learning control applied to the model yields the results in Fig. 8. Noncausal filtering was employed. The optimizing iterative learning control described in Section III in conjunction with a noncausal filter tested with the model yields the runs in Fig. 9.

Tests with an industrial extruder with optimal iterative learning control yield the results shown in Fig. 10. These results are indeed impressive as they show a decrease in the extrusion time of about 10%.

Iterative control of an extruder opens the possibility of reducing the initial billet temperature. Successive lowering of the initial billet temperature automatically leads to an increase of the extrusion rate, while the control tends to maintain the exit temperature at a constant value by doing this. Tests with the industrial extruder led to a 25% reduction of the extrusion time per billet. Whether such a reduction of billet temperature can be allowed has to be decided on the basis metallurgical considerations. At this stage it is up to the metallurgists to specify the ideal conditions of extrusion—achieving them with



(a)



(b)

Fig. 7. (a) Control variable and (b) control input of control system PID-controller.

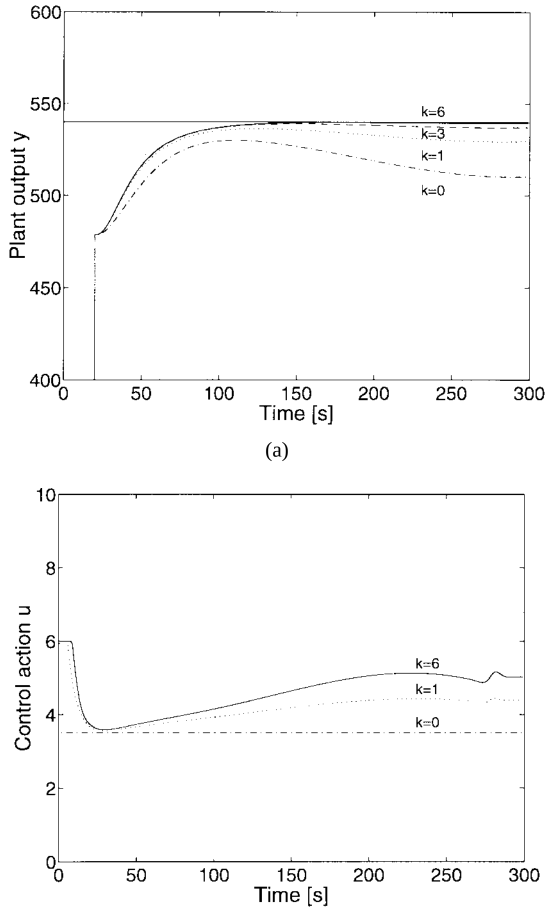


Fig. 8. (a) Control variable and (b) control input with linear iterative learning control obtained in cycles 0, 1, 3, and 6 with simulated system.

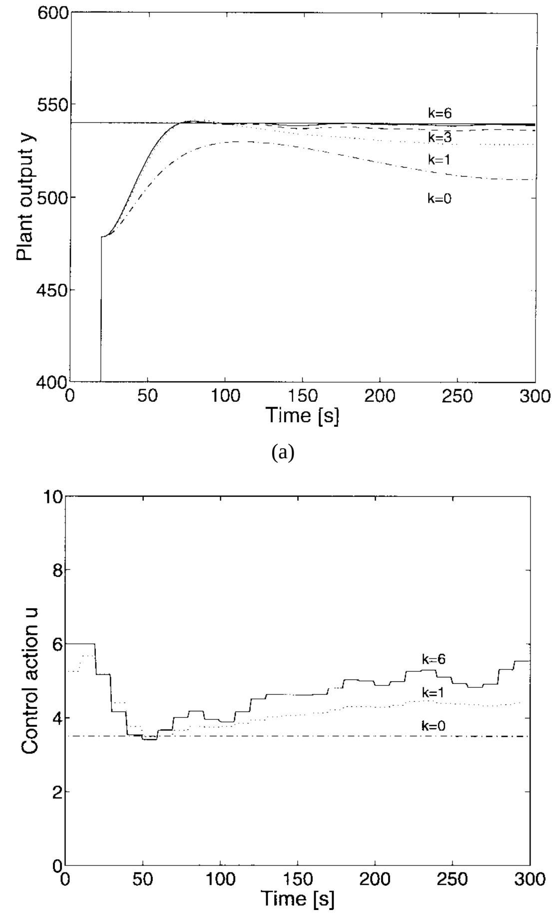


Fig. 9. (a) Control variable and (b) control input with optimal iterative learning control obtained in cycles 0, 1, 3, and 6 with simulated system.

present day hardware and software seems to have moved into the domain of the viable [14].

V. Conclusions and Perspectives

Known and new control schemes have been presented for the control of cyclic processes. The proposed optimal iterative learning control has been applied for nonlinear systems as an adaptive control system with a linearised model. By defining an appropriate performance index, it is possible to take account of specifications on the control variable and the input. The implementation of the control is illustrated with reference to the isothermal operation of an extruder. Results of tests with simulated models and an actual extruder have been cited. The results indicate that the optimal iterative learning control scheme developed in the paper in conjunction with noncausal filtering seems to be ideally suited for automatizing cyclic production processes such as the extrusion process. Future work will be directed toward implementation in other industrial processes.

Appendix

1) Extremum of a Functional: Let it be required to determine the condition under which the functional $Q[u(t)]$ takes on a minimum. First, it is known that the condition for a minimum, i.e.,

$$u(t) : Q[u(t)] = \min_{u(t)} Q[u(t)] \tag{29}$$

is

$$\delta Q[u(t), \xi(t)] \equiv 0 \quad \text{for a differential } \xi(t). \tag{30}$$

The expression $\delta Q[u(t), \xi(t)]$ denotes the Gateaux differential

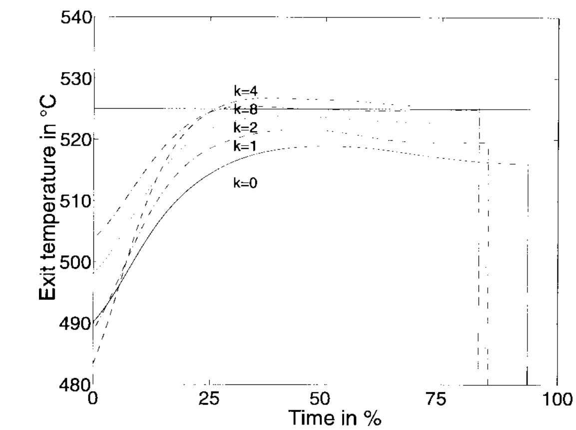
$$\delta Q[(u(t), \xi(t)] = \lim_{\gamma \to 0} \frac{Q[u(t) + \gamma \xi(t)]}{\gamma}. \tag{31}$$

The differential $\delta Q[u(t), \xi(t)]$ is rewritten using the gradient $\nabla Q[u(t)]$ in the form

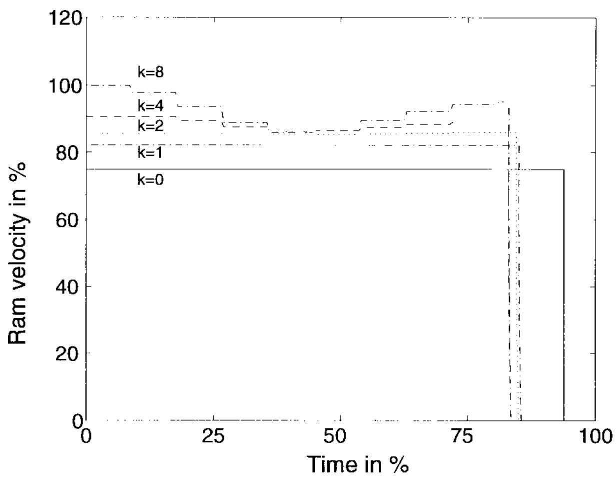
$$\delta Q[u(t), \xi(t)] = \int_0^T \xi(t) \cdot \nabla Q[u(t)]\, dt. \tag{32}$$

As (32) holds for any arbitrary $\xi(t)$, the function sought $u(t)$ in (29) fulfills the equation

$$\nabla Q[u(t)] \equiv 0. \tag{33}$$



(a)



(b)

Fig. 10. (a) Control variable and (b) control input measured in an industrial extruder with optimizing iterative learning control in cycles 0, 1, 2, 4, and 8.

2) Example for Calculation of the Gradient: We consider the functional described in Section III, viz.

$$Q[\dot{u}(t)] = \int_0^T \frac{\lambda}{T}\dot{u}^2(t) + \frac{1}{T}(y_d(t) - y(t))^2\, dt. \quad (34)$$

with the constraint equation

$$\phi\{\dot{u}(t)\} = y(t) = u(0^+)h(t) + \int_0^t \dot{u}(\tau)h(t-\tau)\, d\tau. \quad (35)$$

The Gateaux differential of the operator is

$$d\phi\{\dot{u}(t), \xi(t)\} = \int_0^t \xi(\tau)h(t-\tau)\, d\tau. \quad (36)$$

From (31), (34), and (36), the Gateaux differential $d\delta Q[u(t), \xi(t)]$ is

$$\delta Q[\dot{u}(t), \xi(t)] = \int_0^T \xi(t)\left[\frac{2\lambda}{t}\dot{u}(t) - \frac{2}{T}\int_0^T (y_d(\tau) - y(\tau))h(\tau - t)\, d\tau\right] dt. \quad (37)$$

A comparison of (32) and (37) yields the gradient

$$\nabla Q[\dot{u}(t)] = \frac{2\lambda}{T}\dot{u}(t) - \frac{2}{T}\int_t^T (y_d(\tau) - y(\tau))h(\tau - t)\, d\tau. \quad (38)$$

Finally, substituting for $y(t)$ from (35), we obtain the gradient

$$\nabla Q[\dot{u}(t)] = \frac{2\lambda}{T}\dot{u}(t) - \frac{2}{T}\int_t^T \Bigg(y_d(\tau) - u(0^+)h(\tau) - \int_0^\tau \dot{u}(\zeta)h(t-\zeta)\, d\zeta\Bigg)h(\tau - t)\, d\tau. \quad (39)$$

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