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Robust Control of Large Flexible Space Structures Using a Coprime Factor Plant Description

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Dynamics of large flexible space structures (LFSS) are characterized by their high order and their significant number of closely-spaced, lightly-damped, clustered low-frequency modes. Finite-element (FE) models of LFSS are known to be accurate only for the first few modes of the structure. Moreover, these models do not provide the modal damping ratios. Model identification of LFSS is often impractical because such structures are assembled in space. Thus it would be desirable to have a design procedure that would directly use an uncertain FE model to produce a controller that could be implemented on real LFSS with good confidence. However, most robust design techniques (e.g. [1]) require accurate knowledge of the damping ratios of the most significant modes to put bounds on the uncertainty sets. Unfortunately, those bounds are very sensitive to the amount of damping in the model. Overly large uncertainty subsets of $\mathcal{H}_\infty$ are required to cover the lightly damped error dynamics of LFSS when additive or multiplicative uncertainty models are used. This results in loss of nominal performance to maintain robustness [4]. This note presents a simple description of uncertainties in LFSS as stable perturbations in the factors of a nominal left-coprime factorization (LCF) of LFSS dynamics. This leads to a better, less conservative description of the uncertainty set and hence improves achievable closed-loop performance. Because of space limitations, many details which can be found in the full report [2] are omitted here.

1. An LCF of LFSS Dynamics

We start with LFSS dynamics in modal coordinates including three rigid body modes to account for the attitude (3 DOF) of the rigid part. Assume only displacements and rotations are measured. Then the dynamic equations in modal coordinates can be written as

$$\ddot{\eta} + D\dot{\eta} + \Lambda\eta = B_1 u\ , \quad y = C_1\eta \qquad (1)$$

where y is the vector of measured outputs, u is the vector of torque and force inputs, $D = \text{diag}\{0,0,0,2\zeta_4\omega_4,\ldots,2\zeta_n\omega_n\}$ and $\Lambda = \text{diag}\{0,0,0,\omega_4^2,\ldots,\omega_n^2\}$; ζ_i is the nominal damping ratio and ω_i is the nominal frequency of the i^{th} mode. Taking the Laplace transform in (1) yields

$$\hat{y}(s) = C_1\left[s^2I + sD + \Lambda\right]^{-1} B_1\,\hat{u}(s) \qquad (2)$$

The assumptions here are: (A1) No pole-zero cancellation occurs when the product $C_1\left[s^2I+sD+\Lambda\right]^{-1}B_1$ is formed, (A2) The uncertainty in the output matrix C_1 can be lumped in with the input uncertainty. Let the polynomial $s^2 + as + b$ be Hurwitz with real zeros. The stable, proper matrices $\tilde{M}(s) := [s^2I+sD+\Lambda]/(s^2+as+b)$ and $\tilde{N}(s) := B_1/(s^2+as+b)$ form a left-coprime factorization of the transfer matrix from $\hat{u}$ to $\hat{\eta}$. Note that scalings would be performed on the factors to reduce conservativeness in an actual controller design. The uncertainty modeling process proposed here uses the a priori knowledge of the bounds between which lie the values of $\{\zeta_i\}_{i=4}^n$, $\{\omega_i\}_{i=4}^n$, of each mode and the entries of B_1. This information is used to derive a bound on the norm of the coprime factor perturbations at each frequency. The perturbations $\Delta\boldsymbol{N}_{rp}$ and $\Delta\boldsymbol{M}_{rp}$ of the coprime factors resulting solely from the perturbed real parameters are easily computed. Let $\boldsymbol{\Delta}_{rp} = [\Delta\boldsymbol{N}_{rp}\ -\Delta\boldsymbol{M}_{rp}]$. Given the above parametric uncertainty, a relatively tight first-order weight r which covers $\|\boldsymbol{\Delta}_{rp}(j\omega)\|$ is derived. Note that $\boldsymbol{R} = rI$ and $\boldsymbol{W}_1 = w_1 I$ in Figure 1.

2. A Robust $\mathcal{H}_\infty$ Design Example

Figure 1 shows the scaled closed-loop system with all the weights for designing a $\boldsymbol{K}$ providing robust

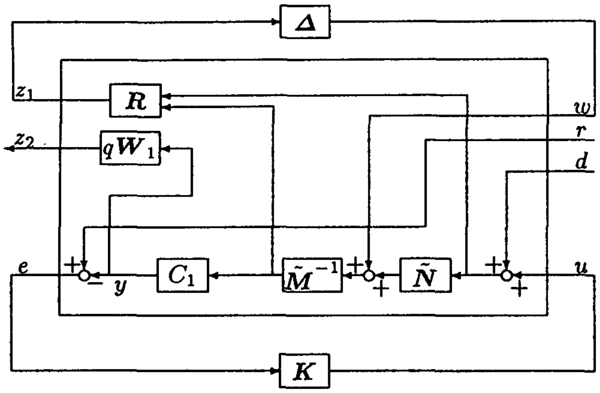

Figure 1: Generalized Plant with Scaled Perturbation and Controller

stability and nominal performance. Suppose the requirements for nominal performance are good tracking and good torque/force disturbance rejection at low frequencies. These requirements can be translated into desired shapes for the norms of the sensitivity matrices $S_r := r \mapsto e$ and $S_d := d \mapsto y$. For example, $\|S_r(j\omega)\| \leq |w_1^{-1}(j\omega)|$ and $\|S_d(j\omega)\| \leq |w_1^{-1}(j\omega)|$, $\forall \omega$. The basic goal of the proposed $\mathcal{H}_\infty$ design is to achieve $\left\| w \mapsto \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\|_\infty \leq 1$ for $\Delta \equiv 0$. The method described so far is illustrated by designing a robust controller for a reduced model of the Daisy experimental setup [3] using the $\mathcal{H}_\infty$ design method. The reduced 10^{th}-order model is comprised of two of Daisy's flexible modes together with its three rigid-body modes. The modal parameters are listed in Table 1 with their uncertainties. It is assumed that Daisy's three torque wheels actuate the hub and that two of Daisy's jet thrusters actuate the flexible part of the structure. There is up to 20% uncertainty in the entries of the input matrix B_1. Also, the three hub Euler angles and two rib angles are measured. The

i	1(r)	2(r)	3(r)	4(f)	5(f)
ω_i	0	$.37 \pm .03$	$.37 \pm .05$	$.70 \pm .10$	$.70 \pm .05$
ζ_i	0	$.11 \pm .05$	$.09 \pm .05$	$.02 \pm .01$	$.04 \pm .02$

Table 1: Modal parameters of reduced model

generalized plant for the robust $\mathcal{H}_\infty$ design is built according to Figure 1, and the weighting functions are $w_1 = \frac{100}{s^2/(.01)^2+2\times.7s/.01+1}$, $r = \frac{.01s+\sqrt{2}}{2.9s+1}$ (q is a scalar). The controller obtained is of the 40^{th} order and stable. The less-damped pair of complex closed-loop poles has a damping ratio of 0.37. Robust stability was achieved since $\|w \mapsto z_1\|_\infty = .92$ and Figure 2 shows $\|S_r(j\omega)\|$ and $\|S_d(j\omega)\|$. Closed-loop output responses of the nominal system to hub torque step disturbances (0, -5.4, 5.4 Nm steps around the x, y, and z axes respectively) are plotted in Figure 3. The maximum values of the simulated control torques and forces are 6.8 Nm and 0.31 N respectively, below the saturation limits. As can be seen, a good tradeoff between nominal performance and robust stability was achieved despite significant modal parameter uncertainties.

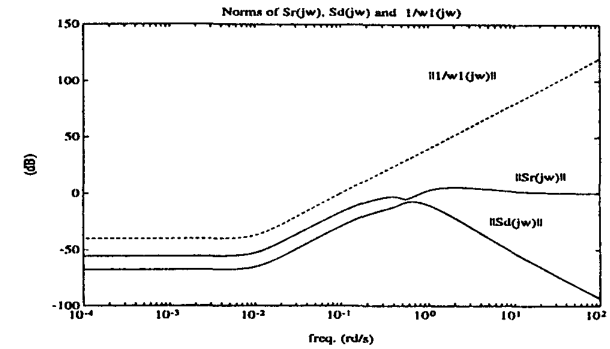

Figure 2: Norms of $S_d(j\omega)$ and $S_r(j\omega)$

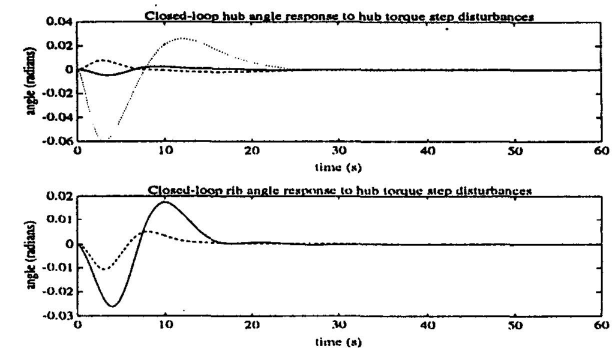

Figure 3: Closed-loop responses to hub torque step disturbances

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