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$\mathcal{H}_2$ and $\mathcal{H}_\infty$-Optimal Gust Load Alleviation for a Flexible Aircraft

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Abstract

$\mathcal{H}_2$ (LQG), weighted-$\mathcal{H}_2$ and $\mathcal{H}_\infty$ techniques are used to establish a nominal performance baseline for the vertical acceleration control of a B-52 aircraft model with flexibilities. The aircraft is assumed to be subjected to severe wind gusts causing undesirable vertical motion. The purpose of the controllers is to reduce the transient peak loads on the airplane caused by the gust. Our designs account for the flexible modes of the aircraft model. We use the Dryden gust power spectral density model to guide the performance specifications and control designs, as well as for time-domain simulations. Motivation for the use of an $\mathcal{H}_\infty$ performance specification is given in terms of a new interpretation of the Dryden model. Both weighted-$\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal controllers are shown to reduce dramatically the effect of wind gust on the aircraft vertical acceleration when compared to the regular $\mathcal{H}_2$ controller.

1- INTRODUCTION

Gust load alleviation (GLA) systems use motion sensor feedback to drive the aircraft's control surfaces in order to attenuate aerodynamic loads induced by wind gusts. This paper compares three nominal baseline GLA controllers designed using $\mathcal{H}_2$ (*LQG*), weighted-$\mathcal{H}_2$ and $\mathcal{H}_\infty$ techniques for the longitudinal dynamics of a B-52 bomber. Motivation is given for each approach in terms of gust models and performance specifications. The dynamic model of the aircraft used for control design includes five flexible modes. Such a model is more realistic than a rigid-body model, but it can also make the GLA feedback control design problem more challenging [2]. Previous research on gust load alleviation reported in [4], [6] used an LQG approach without performance weights for flexible aircraft models. Recent works [1], [7] reported on applications of $\mathcal{H}_\infty$ techniques to flight control problems, but rigid-body models of the aircraft were used. The neglect of flexible modes may lead to robustness problems during and after system commissioning.

The gust signals are assumed to be generated by the Dryden power spectral density model. This model lends itself well to frequency-domain performance specifications in the form of weighting functions. Since the Dryden gust model is simply a white noise driving a stable real-rational linear time-invariant filter, it is natural to consider the design of an *LQG* controller, or its equivalent deterministic $\mathcal{H}_2$ counterpart. However, as a bumpy flight experience would suggest, wind gusts are acting only over brief periods of time. Air turbulence in a localized area may last for a time long enough to warrant the Dryden colored noise approximation. But the point here is that an aircraft momentarily passing through the turbulence will only experience its effect for a brief period of time. Thus, rather than using signals of finite average power for LQG GLA control design, we suggest that such behavior may be best captured by bounded-energy signals with spectral contents given by the magnitude of the Dryden filter. In the deterministic, worst-case $\mathcal{L}_2$ setting, this remark gives some motivation to design a baseline GLA controller with an $\mathcal{H}_\infty$ performance specification.

2- VERTICAL GUST MODELS

The classical Dryden representation of the power spectral density (PSD) function of atmospheric turbulence can be written as:

$$\Phi_w(\omega) = \frac{\sigma_w^2 L_w\left[1+3(L_w\omega/U_0)^2\right]}{\pi U_0\left[1+(L_w\omega/U_0)^2\right]^2} \qquad (1)$$

where σ_w is the RMS vertical gust velocity (*m/s*), L_w is the scale of turbulence (*m*), and U_o is the aircraft trim velocity (*m/s*). For thunderstorms, typical values used are L_w = 580 m (1750 ft), σ_w = 7 m/s (21 ft/s). Formally, the Dryden model generates gust signals through the relationship $w_g = G_w n$, where n is a zero-mean, unit-intensity gaussian white-noise process with PSD $\Phi_n(\omega) = 1$, and w_g is the random vertical gust process. An expression for the Dryden filter $G_w(s)$ can be found through spectral factorization of , which yields

$$G_w(s) = \sqrt{\frac{3U_0\sigma^2{}_w}{\pi L_w}\,\frac{\dfrac{U_0}{\sqrt{3}L_w}+s}{\left[\dfrac{U_0}{L_w}+s\right]^2}} \quad . \qquad (2)$$

A proposed alternative use of the Dryden model is to consider the noise n to be any deterministic finite-energy signal in $\mathcal{N} := \left\{n \in \mathcal{L}_2[0,\infty) : \|n\|_2 \le 1\right\}$. The gust signal then lives in $\mathcal{W} := \left\{G_w n : n \in \mathcal{N}\right\} \subset \mathcal{L}_2[0,\infty)$, and its energy is bounded by

$\|w_g\|_2 \le \|G_w\|_\infty = 0.9\sqrt{2}\sigma_w\sqrt{L_w/\pi U_0}$. Furthermore, such signals taper off at infinity in the time domain. Hence, they may be more representative of real wind gusts acting on an aircraft passing through a turbulence. Although the stochastic nature of the signal is lost, the resulting set of bounded-energy gust signals can be used for a worst-case $\mathcal{H}_\infty$ design, which may be desirable in a safety critical application such as GLA.

3- FLEXIBLE AIRCRAFT MODEL DESCRIPTION

The short-period approximation for the rigid-body motion of the B-52 aircraft is considered. The rigid-body dynamics are augmented by a set of modal coordinates associated with the normal bending modes of the B-52. The i^{th} flexible mode is represented by the following equation in terms of its modal coordinate η_j:

$$\ddot{\eta}_i + 2\varsigma_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \rho_i\phi_i, \quad (3)$$

where ς_i, ω_i, ρ_i are the damping ratio, frequency and gain of the i^{th} flexible mode, and ϕ_i is its corresponding generalized force. Thus, the rigid-body dynamics may be augmented with pairs of first-order equations corresponding to each flexible mode considered. Five structural flexible modes were considered significant and were kept in the B-52 aircraft longitudinal dynamic model taken from [6]. The control inputs for the longitudinal motion are the deflection angles (in radians) of the elevator δ_{el} and the horizontal canard δ_{hc}. The longitudinal dynamics of the flexible aircraft are given by:

$$\begin{aligned}\dot{x} &= Ax + Bu + B_g w_g \\ y &= Cx + Du\end{aligned} \quad (4)$$

where $x(t) \in \mathbb{R}^{12}$ is the state vector given by:

$$x^T = [\alpha\ q\ \eta_1\ \dot{\eta}_1\ \eta_5\ \dot{\eta}_5\ \eta_7\ \dot{\eta}_7\ \eta_8\ \dot{\eta}_8\ \eta_{12}\ \dot{\eta}_{12}]$$

$u(t) = [\delta_{el}\quad \delta_{hc}]^T \in \mathbb{R}^2$ is the control vector (rad) $y(t) \in \mathbb{R}$ is the vertical acceleration (g), $w_g(t) = [w_{g1}\quad w_{g2}\quad w_{g3}]^T \in \mathbb{R}^3$ is the vertical gust velocity at three stations along the airplane (m/s), $\alpha(t) \in \mathbb{R}$ is the angle of attack (rad), $q(t) \in \mathbb{R}$ is the pitch rate (rad/s). There are couplings between the flexible modes and the rigid-body mode. The eigenvalues of A corresponding to the rigid-body mode are $\lambda_{1,2} = -1.803 \pm j2.617$. The five flexible modes are listed in Table 1 below.

Table 1: flexible modes

	1	2	3	4	5
ω_i (rd/s)	7.60	15.22	19.73	20.24	38.29
ς_i	0.393	0.056	0.011	0.067	0.023

Note that the second and third gust signals w_{g2} and w_{g3} are actually delayed versions of w_{g1}. The second gust is delayed by $\tau_1 = U_0/x_1 = 0.06$ s, where x_1 is the distance from the first body station to the second. The third gust input is delayed by $\tau_2 = U_0/x_2 = 0.145$ s. First-order lag approximations of the time delays are used, e.g., $w_{g2} = \dfrac{1}{0.06s+1}w_{g1}$.

4- PROBLEM SETUP

A block diagram of the closed-loop GLA design problem with weighting functions is shown in Figure 1 below:

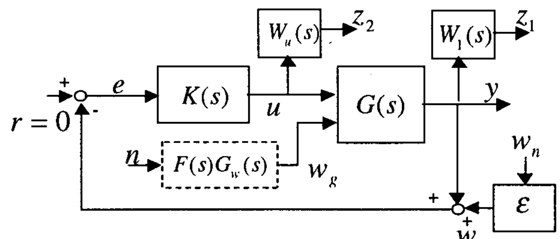

Fig. 1: Problem setup

where $w_{n1}(t) \in \mathbb{R}$ is an acceleration measurement noise used to regularize the problem, with amplitude specified by $\varepsilon = 10^{-4}$ as w_n is normalized, $r(t) \in \mathbb{R}$ is the vertical acceleration setpoint ($r(t) = 0$), $e(t) \in \mathbb{R}$ is the measured error, $z_1(t) \in \mathbb{R}$ is the weighted measured acceleration, $z_2(t) \in \mathbb{R}^2$ is the weighted controller output, and $F(s)$ contains the lags approximating the delays. The plant transfer matrix $G(s)$ mapping $[u\ w_g{}^T]^T$ to y is given by

$$G(s) = \left[\begin{array}{c|c} A & [B\ B_g] \\ \hline C & [D\ 0] \end{array}\right]. \quad (5)$$

The GLA problem of Figure 1 can be recast into the standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal control problem of Figure 2.

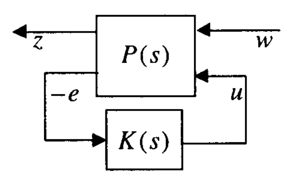

Fig. 2: Standard control problem setup

The vector of exogenous inputs in Figure 2 is $w := [n\ w_n]^T$ for the $\mathcal{H}_2$ controller designs, or $w := [w_g{}^T\ w_n]^T$ for the $\mathcal{H}_\infty$ controller design. The signals to be minimized are collected in $z := \begin{bmatrix} W_1 y \\ W_u u \end{bmatrix}$. The nominal generalized plant

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \quad (6)$$

has a minimal state-space realization

$$\begin{aligned} \dot{x} &= A_P x + B_{P1} w + B_{P2} u \\ z &= C_{P1} x + D_{P11} w + D_{P12} u \\ y &= C_{P2} x + D_{P21} w + D_{P22} u \end{aligned} \quad (7)$$

which combines the aircraft model, the Dryden filter (for the $\mathcal{H}_2$ controller) and the weighting functions. For convenience, we will use the notation $T_{xy} := x \mapsto y$ for closed-loop transfer matrices.

5- $\mathcal{H}_2$ CONTROL

Assuming that the origin of the gust is a zero-mean gaussian white noise of unit intensity, we first design an unweighted $\mathcal{H}_2$ controller. This control design is based on the generalized plant with exogenous inputs $w = [n \;\; w_n]^T$ and outputs $z = \begin{bmatrix} z_1 & z_2^T \end{bmatrix}^T$. Thus, the gust vector generator FG_w is embedded in the generalized plant P. Note that all our control designs were carried out following [5] and using the Matlab μ-Synthesis and Analysis Toolbox [3]. For regular $\mathcal{H}_2$ control design, we used the weighting functions $W_1 = 1$ and $W_u = 1$. Closed-loop performance is indirectly specified by G_w. For instance, choosing $\sigma_w = 7 m/s$ and $L_w = 580 m$ in G_w for simulation purposes, we obtain a gust that has most of its power concentrated in the frequency band $[0.1, 6]\,\mathrm{Hz}$. The objective of the $\mathcal{H}_2$ controller design is to minimize

$$\|T_{wz}\|_2 = \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Tr}\{T_{wz}^*(j\omega)T_{wz}(j\omega)\}d\omega\right]^{\frac{1}{2}}, \quad (8)$$

the $\mathcal{H}_2$ norm of T_{wz}, over all finite-dimensional, linear time-invariant stabilizing controllers $K(s)$. Assuming that $z(t)$ is ergodic, its average power is then minimized as $\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\|z(t)\|^2 dt = E\left\{\|z(t)\|^2\right\} = \|T_{wz}\|_2^2$. This has the effect of decreasing the $\mathcal{H}_2$ norm of submatrices of T_{wz} by virtue of the fact that $\|[G_1 \;\; G_2]\|_2^2 = \|G_1\|_2^2 + \|G_2\|_2^2$. For example, the closed-loop vertical acceleration of the aircraft can be written in terms of the uncorrelated white noises n and w_n as follows:

$$y = T_{wy} w = T_{ny} n + T_{w_n y} w_n. \quad (9)$$

The $\mathcal{H}_2$ norms of the transfer matrices T_{ny} and $T_{w_n y}$ satisfy $\|T_{ny}\|_2^2 + \|T_{w_n y}\|_2^2 \leq \|T_{wz}\|_2^2$. The $\mathcal{H}_2$ controller obtained had 20 states and achieved $\|T_{wz}\|_2 = 0.2$. Figure 3(a) shows the spectral norms of $T_{ny}(j\omega)$ and $T_{w_{n1}y}(j\omega)$ with the $\mathcal{H}_2$ controller for comparison with the other controllers.

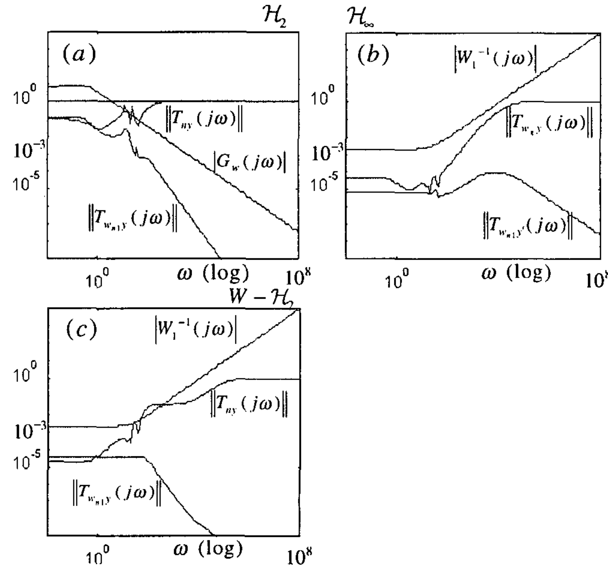

Fig. 3: Norms of closed-loop transfer matrices

6- WEIGHTED $\mathcal{H}_2$ AND $\mathcal{H}_\infty$-OPTIMAL CONTROL

6.1- Weighted $\mathcal{H}_2$ Controller

In order to improve the performance obtained with the regular $\mathcal{H}_2$ controller, we introduce a performance weighting function

$$W_1(s) = \frac{k_1}{a_0 s + 1} \quad (10)$$

on the acceleration y, with $k_1 = 500$, $a_0 = 0.05$. This weighting function is the same as the one used for the $\mathcal{H}_\infty$ design in order to compare the results obtained for both controllers. We obtained $\|T_{wz}\|_2 = 1.5$ with the weighted-$\mathcal{H}_2$ controller, which means that $\|W_1 T_{ny}\|_2^2 + \|W_1 T_{w_n y}\|_2^2 \leq 2.25$. Figure 3(c) shows that the weighting function led to much better GLA performance at low frequencies by trading off sensor noise rejection, which deteriorated slightly, but remained acceptable.

6.2- $\mathcal{H}_\infty$-Optimal Design

As gusts act over a relatively short period of time, they can be considered as signals with finite energy. Such signals can have spectral contents similar to the *PSD* of stochastic Dryden gust signals by using $G_w(s)$ as a filter. This remark provides motivation for $\mathcal{H}_\infty$ GLA control design as

$$\min_{K\in S}\max_{w\in\mathcal{W}}\|T_{wz}w\|_2 = \min_{K\in S}\max_{n\in\mathcal{N}}\|T_{wz}G_w n\|_2 = \min_{K\in S}\|T_{wz}G_w\|_\infty$$

where S is the set of all stabilizing controllers.

6.2.1- Weighting Functions for Nominal Performance

The specification is that our controller has to be able to regulate the vertical acceleration in the gust bandwidth with an amplitude attenuation of at least -54 dB (500). The closed-loop vertical acceleration of the aircraft can be written in terms of the gust vector w_g and the noise w_n:

$$y = T_{w_g y}w_g + T_{w_n y}w_n. \quad (11)$$

The gust alleviation performance specification on $\|T_{w_g y}(j\omega)\|$ can be enforced through the use of a weighting function W_1 of magnitude at least 500 over $[0.1, 6]$ Hz, as long as we get $\|W_1 T_{w_g y}\|_\infty < 1$ with the controller K, which implies

$$\|T_{w_g y}(j\omega)\| < |W_1(j\omega)|^{-1}. \quad (12)$$

The weighting function is as given in (10). A plot of $|W_1(j\omega)|^{-1}$ is shown in Figure 3(b)(c). The controller outputs consist of deflection angles (in radians) of the aircraft's elevators and horizontal canards. In order to keep these angles within acceptable limits, we used a suitable weighting function W_u on u such that $\|W_u T_{wu}\|_\infty < 1$. The weighting function W_u is a constant diagonal matrix $W_u = \mathrm{diag}\{k_{u_1}, k_{u_2}\}$ so that the above $\mathcal{H}_\infty$-norm condition implies

$$\|T_{wu1}(j\omega)\| < k_{u_1}{}^{-1} \quad (13)$$

and

$$\|T_{wu2}(j\omega)\| < k_{u_2}{}^{-1}. \quad (14)$$

where $T_{wu} = \begin{bmatrix} T_{wu1} \\ T_{wu2} \end{bmatrix}$. Here $k_{u_1}{}^{-1} = 0.2$, $k_{u_2}{}^{-1} = 0.5$ are the maximum allowed control gains in closed loop. It is preferable to use a constant weighting matrix for this application, since it does not increase the order of $P(s)$. Hence, the resulting $\mathcal{H}_\infty$ controller has a lower order.

6.2.2- $\mathcal{H}_\infty$-Optimal Controller

The overall objective in this $\mathcal{H}_\infty$ controller design was to minimize $\|T_{wz}\|_\infty$ over all finite-dimensional, linear time-invariant stabilizing controllers $K(s)$, in order to get $\|T_{wz}\|_\infty < 1$. This would guarantee that the performance specification is satisfied. A norm of $\|T_{wz}\|_\infty = 0.68$ was achieved. Figure 3 shows that our $\mathcal{H}_\infty$ controller meets the GLA performance specification given above. We can see that the maximum singular value of $T_{w_g y}$ is well below 10^{-5} over $2\pi[0.1, 6]$ rd/s. The spectral norms of the frequency responses of T_{wu1} and T_{wu2} satisfy the constraints of (13) and (14) respectively.

7- SIMULATION RESULTS

We used the Dryden model with parameters $\sigma_w = 7\,m/s$, $L_w = 580m$ to generate severe wind gust signals for simulation purposes. Figure 4(a),(b),(c) shows the resulting gust signals. The open-loop vertical acceleration response of the B-52 to these gust signals is shown in Figure 4(d).

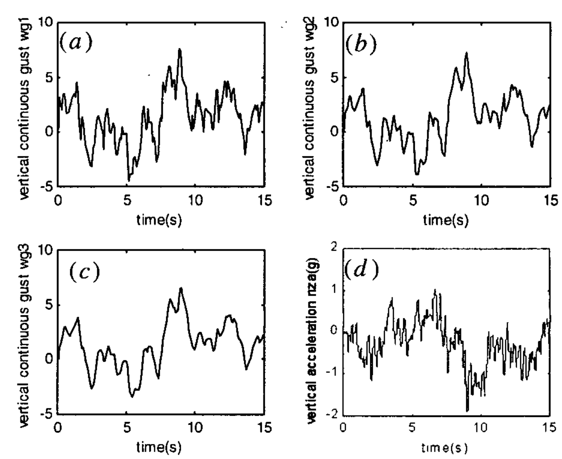

Fig. 4: Gust signals and open-loop response

Figure 3(a) shows a magnitude plot of the frequency response of the Dryden filter for the simulation. We see that the performance specification enforced by the weighting function W_1 used for the weighted-$\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers (Figure 3(b)(c)) should result in efficient gust alleviation. Time-domain simulations were conducted and results are presented below. The results shown in Figure 5(a)(b) confirm that the $\mathcal{H}_\infty$ controller can dramatically reduce the effect of wind gusts on the vertical acceleration of the aircraft for the nominal model. This controller also seems suitable from the point of view of control effort. From

Figure 5(a), it can be seen that the elevator angle remained within ±0.25 radians (±14°) in the simulation. The $\mathcal{H}_\infty$ controller gave the best GLA performance without exciting the flexible modes of the model or generating large control angles that could saturate the control surfaces. A comparison of peak vertical accelerations with the $\mathcal{H}_\infty$ controller (Figure 5(b)) and without any feedback control (Figure 4(d)) indicates that the $\mathcal{H}_\infty$ controller reduced the acceleration by a factor of 10^5. This could translate into dramatic improvements in flight comfort and reduced airframe loads.

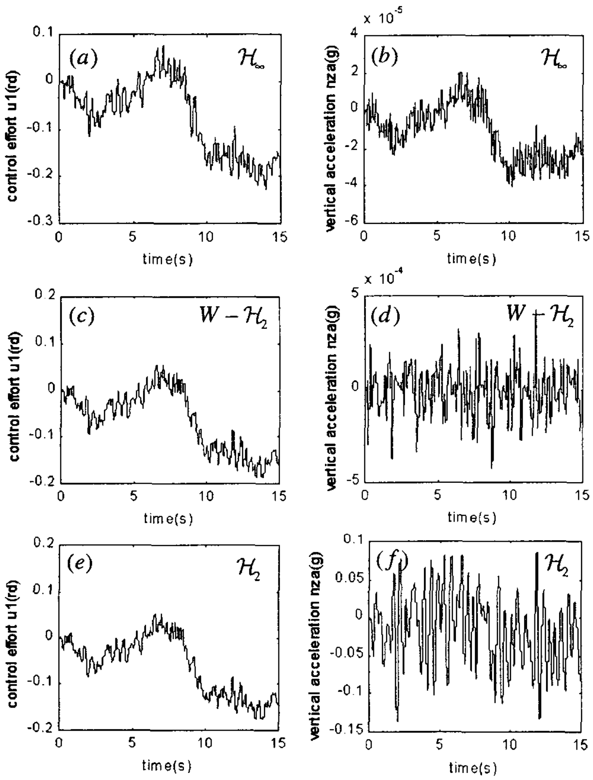

Fig. 5: Simulation results

In contrast to the $\mathcal{H}_\infty$ controller, the $\mathcal{H}_2$ controller (similar to the one proposed in [4]) only reduced the peak acceleration by a factor of 14 as seen in Figure 5(f). This controller is also of higher order than the $\mathcal{H}_\infty$ controller, which can be a disadvantage for implementation.

A significant performance improvement was obtained over regular $\mathcal{H}_2$ by using a weighted-$\mathcal{H}_2$ control approach, as evidenced by Figure 5(d). The performance obtained with the weighted-$\mathcal{H}_2$ controller is more comparable to the performance of the $\mathcal{H}_\infty$ controller, although still about 20 times worse in terms of peak acceleration. The $\mathcal{H}_\infty$ controller appeared to control the flexible modes better, which is not surprising in view of its low closed-loop $\left\|T_{w_g y}(j\omega)\right\|$ in the frequency range of the flexible modes (see Figure 3(b)).

8- CONCLUSION

We presented a comparison of $\mathcal{H}_2$, weighted-$\mathcal{H}_2$ and $\mathcal{H}_\infty$ GLA controller designs for the nominal model of a B-52 aircraft with flexible modes. The gust was modeled by a Dryden filter whose input was assumed to be white noise for the two $\mathcal{H}_2$ designs. We argued that gust signals may be better represented by a set of filtered deterministic, bounded-energy signals, with the Dryden transfer function used as the filter. This motivated the use of $\mathcal{H}_\infty$ control for GLA. This kind of model lends itself well to frequency-domain performance specifications in the form of weighting functions. The $\mathcal{H}_\infty$ controller was shown to meet the desired performance specification with reasonably small control surface deflection angles. It outperformed both the $\mathcal{H}_2$ and weighted-$\mathcal{H}_2$ controllers in the simulation. It was also shown that it may prove beneficial to weight the outputs in an $\mathcal{H}_2$ GLA control design. Future investigations will address the robustness issues related to uncertain modal parameters and different flight envelopes.

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