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A Gain Scheduling Approach for a Flexible Aircraft

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Abstract

A technique based on interpolation of the solutions of Riccati equations for frozen $\mathcal{H}_\infty$ designs is used for gain scheduling over the full flight envelope of a flexible aircraft. The aircraft model consists of a linear parameter-varying model of a flexible B1 bomber that varies with the Mach number from 0.2 to 0.8Mach. The performance objectives are to track the pitch demand and to alleviate the loads due to wind gusts. The flexibility of the aircraft is taken into account through four flexible modes. The method presented is shown to provide good results over the full flight envelope, while allowing for a simple implementation structure.

1-Introduction

Gain scheduling research in aerospace control has been very active over this last decade. To achieve performance over a large flight envelope, it is necessary to schedule the multivariable controllers designed for frozen operating points. The multivariable scheduling problem is more difficult to deal with than the single-variable case, for which the zeros and poles of the controllers can easily be interpolated. Switching techniques [1], based on bumpless switching, interpolation techniques [1], and on linear interpolation of the parameters of the observer-form controller, have been presented. Methods based on the recent results on LPV designs [2,3] are computationally complicated and the solution to get an LPV controller is not guaranteed.

Some criticism regarding the lack of stability guarantees have been directed at methods based on the interpolating techniques, even though a judicious choice of the frozen operating design points with respect to a tolerated level of parameter variation should preserve the stability for the closed-loop system. However, the ease of real-time implementation of those techniques is their major advantage. Flexibility of the aircraft is taken into account by including the important flexible modes. Modelling the structural flexibility existing in large aircraft is sensible, and makes the control system design challenging for gust load alleviation objectives. In fact, the turbulence (gusts) acting on the aircraft is more influent through the bending modes existing in the aircraft.

At frozen linear models covering the flight envelope, controllers based on the mixed-sensitivity $\mathcal{H}_\infty$ technique were designed. The same weighting functions for performance were used for the different controller designs. This allows us to automate the design and to unify the performance objectives along the Mach number variation. The scheduling technique used to cover the full flight envelope while preserving performance is based on linear interpolation of the solutions of the two Riccati equations used in the $\mathcal{H}_\infty$ controller design. A version of this method was first introduced and successfully implemented for a small missile model in [4]. Our approach is different from the version presented in that paper in the sense that the controller at the intermediate design points is a function of only the interpolated Riccati solutions variables. In [4], the controller depends on the interpolated solution of the Riccati equation and other parameters viewed as exogenous inputs upon which the nonlinear dynamics are dependent. Another major advantage to be noticed, in this paper, is the use of a linear parameter-varying model. In fact, in [4], a nonlinear model is linearized at each point of the flight envelope to be used in the construction of the controller. This will certainly complicate the implementation of the controller over the flight envelope. We believe that the mixture of the LPV models and the interpolating techniques solving the control design problem offers easier implementation and good results. We propose this approach since the classical LPV control design is time-consuming and suffers from the difficulty of solving an LMI problem for high-order systems.

2-Problem setup

2.1-Aircraft model

A B-1 bomber is the aircraft under study, and a model of it [5,6] taking into account the structural flexibility is used. The aircraft's dynamics are assumed typical for a high-speed transport aircraft, and a ride quality control system (RQCS) was installed to diminish the unacceptable vibration due to the turbulence that was apparent in the previous version of that aircraft.

The longitudinal, linear parameter varying, B-1 model can be represented as:

$$\begin{cases} \dot{x} = A(\rho)x + B(\rho)u \\ y = C(\rho)x + D(\rho)u \end{cases} \tag{1}$$

where: $x = [\alpha, q, \eta_1, \eta_2, \eta_3, \eta_4, \dot{\eta}_1, \dot{\eta}_2, \dot{\eta}_3, \dot{\eta}_4]^T$ is the state vector which contains the angle of attack, the pitch rate, and the coordinates of the four flexible modes.

$u = [w_g, \dot{w}_g, \delta_{ele}, \delta_{cv}]^T$ are the gust vector, the elevator, and the canard actuators, respectively.

$y=\left[y_q, y_{acc}\right]^T$ are the pitch rate and the acceleration of the aircraft measured by the rate gyro and the accelerometer, respectively. $\rho =$ Mach number is the varying parameter. The following model gives the dynamics of the actuators:

$$\begin{bmatrix}\delta_{ele}\\ \delta_{cv}\end{bmatrix}=\left[\begin{array}{c|c}A_{act} & B_{act}\\ \hline C_{act} & D_{act}\end{array}\right]\begin{bmatrix}\delta_{ele}^{cmd}\\ \delta_{cv}^{cmd}\end{bmatrix} \quad (2)$$

where : $\begin{bmatrix}\delta_{ele}^{cmd} & \delta_{cv}^{cmd}\end{bmatrix}^T$ are the actuator commands, and

$$A_{act}=\begin{bmatrix}-10 & 0\\ 0 & -50\end{bmatrix}, B_{act}=\begin{bmatrix}10\\ 50\end{bmatrix}, C_{act}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, D_{act}=\begin{bmatrix}0\\ 0\end{bmatrix}. \quad (3)$$

The transfer function representing the turbulence acting on the aircraft is given as:

$$\begin{bmatrix}w_g\\ \dot{w}_g\end{bmatrix}=\left[\begin{array}{c|c}A_{wg} & B_{wg}\\ \hline C_{wg} & D_{wg}\end{array}\right]d_g \quad (4)$$

where:

$$A_{wg}=\begin{bmatrix}-2V_0/L_{wg} & 1\\ -\left(V_0/L_{wg}\right)^2 & 0\end{bmatrix}, B_{wg}=\begin{bmatrix}\sigma_{wg}\sqrt{3}\left(V_0/L_{wg}\right)^{0.5}\\ \sigma_{wg}\left(V_0/L_{wg}\right)^{1.5}\end{bmatrix},$$

$$C_{wg}=\begin{bmatrix}1 & 0\\ -2\left(V_0/L_{wg}\right) & 1\end{bmatrix}, D_{wg}=\begin{bmatrix}0\\ \sigma_{wg}\sqrt{3}\left(V_0/L_{wg}\right)^{0.5}\end{bmatrix}$$

and σ_{wg}, L_{wg}, d_g are the intensity, the turbulence scale length and a gaussian white noise, respectively.

The four flexible modes are characterized by their damping ratios $\varsigma_1=\varsigma_2=\varsigma_3=\varsigma_4=0.02$ and their modal frequencies:
$\omega_1 \; 12.57, \omega_2 \; 14.07, \omega_3 \; 21.17, \omega_4 \; 22.05\, rd/s$.
The flexible modes have an effect at high frequencies and thus can be separated from the rigid-body modes. To be as realistic as possible, we preferred to use all the flexible modes in the design rather than using techniques such as notch filters to cope with the reduction of flexible modes. The high level of flexibility in the B-1 bomber model acts negatively on the aircraft and exhibits high-frequency transient motions for the pitch rate open loop response. Thus a feedback controller design is needed to improve the pitch rate response and to decrease the acceleration of the aircraft due to turbulence effects.

2.2-Controller design

Linear parameter-varying model is used to generate frozen linear models corresponding to a set Λ of chosen Mach numbers (operating points). The flight envelope covers the variation of the Mach number from 0.2 to 0.8 at 5000ft. Hence, our choice of the frozen operating points corresponds to the set

$$\Lambda=\begin{Bmatrix}0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.50,\\ 0.55, 0.6, 0.65, 0.7, 0.75, 0.8\end{Bmatrix}.$$

The general setup of the frozen controller designs is given by the figure below:

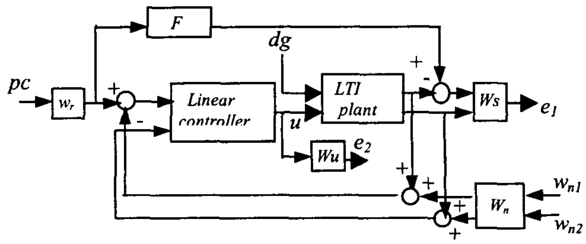

Figure 1: General setup for frozen controller design

Our control objectives are to track the pilot's pitch demand and to alleviate the gust load by reducing the acceleration of the aircraft due to the effect of the turbulence. For the tracking objective we used a filter $F=\dfrac{16}{s^2+5.2s+16}$ to shape the closed-loop pitch response. $W_s=\text{diag}\left\{\dfrac{800}{s+1}, \dfrac{10}{s+15}\right\}$ specifies levels of performance desired from both design objectives. $W_u=\text{diag}\left\{\dfrac{180}{15\pi}, \dfrac{180}{15\pi}\dfrac{(s+15)}{(s+0.01)}\right\}$ is used to constrain the effort of the actuators to reasonable and realistic limits which are 15° for the elevator and 15° with zero deflection at the steady state for the control vane actuator. $W_n=\begin{bmatrix}3\times10^{4} & 0\\ 0 & 0.01\end{bmatrix}$ are sensor noise levels and are used to ensure well-posedness of the $\mathcal{H}_\infty$ control design.

Note that the performance objectives were unified for all $\rho_i \in \Lambda$, by having the same weighting functions for the augmented plant along the flight envelope. The controller designs in Figure 1 can be recast into the general $\mathcal{H}_\infty$ controller design framework [7], where the augmented frozen linear models can be represented in the state space form as:

$$\begin{cases}\dot{x}=A_a(\rho_i)x+B_a(\rho_i)u\\ y=C_a(\rho_i)x+D_a(\rho_i)u\end{cases} \quad (5)$$

Such that: $\rho_i \in \Lambda$

With some simplifying assumptions, [7], we consider the state-space representation of the augmented system as:

$$G_a(s):=\left[\begin{array}{c|cc}A_a(\rho_i) & B_{a1}(\rho_i) & B_{a2}(\rho_i)\\ \hline C_{a1}(\rho_i) & 0 & D_{a12}(\rho_i)\\ C_{a2}(\rho_i) & D_{a21}(\rho_i) & 0\end{array}\right] \quad (6)$$

A suboptimal $\mathcal{H}_\infty$ controller [7], for the augmented plant, can be written in its state-space form as:

$$K_{sub}(s):=\left[\begin{array}{c|c}\hat{A}_\infty & -Z_\infty L_\infty\\ \hline F_\infty & 0\end{array}\right] \quad (7)$$

where:

$F_\infty := -B_{a2}^* X_\infty$

$\hat{A}_\infty := A_a + \gamma^{-2} B_{a1} B_{a1}^* X_\infty + B_{a2} F_\infty + Z_\infty L_\infty C_2,$

$L_\infty := -Y_\infty C_{a2}^*,\ Z_\infty := (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$

The assumption that the terms D_{a11}, D_{a22} are equal to zero is not verified in general, and in our case of study in particular. In this case, we use results from $\mathcal{H}_\infty$ control theory [7] that permit to resolve the general problem posed. These results are:

The controller in the case of $D_{a22} \neq 0$ is $K_{sub}(I + D_{a22} K_{sub})^{-1}$, if we suppose K_{sub} is the sub-optimal controller of G_a with the assumption that $D_{a22} = 0$. If $D_{a11} \neq 0$ and without loss of generality, by normalising D_{a12} and D_{a21}, we shall assume that the augmented plant can be written as:

$$G_a(s): \left[\begin{array}{c|cc} A_a(\rho_i) & B_{a1}(\rho_i) & B_{a2}(\rho_i) \\ \hline C_{a1}(\rho_i) & \begin{bmatrix} D_{a1111}(\rho_i) & D_{a1112}(\rho_i) \\ D_{a1121}(\rho_i) & D_{a1122}(\rho_i) \end{bmatrix} & \begin{bmatrix} 0 \\ I \end{bmatrix} \\ C_{a2}(\rho_i) & 0 \quad I & 0 \end{array}\right] \tag{8}$$

Such that: $\tilde{D}_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix}, \tilde{D}_{21} = \ 0 \quad I$.

If we normalize $\gamma = 1$, then there exists a controller K that internally stabilises G_a and $\|\mathcal{F}_L(G_a, K)\|_\infty \leq 1$ if and only if there is $\tilde{K}$ that stabilizes M and $\|\mathcal{F}_L(M, \tilde{K})\|_\infty \leq 1$. where:

$$M := \left[\begin{array}{c|cc} \tilde{A} + \tilde{B}_1 R_1{}^{1}\tilde{D}_{11}^*\tilde{C}_1 & \tilde{B}_1 R_1{}^{1/2} & \tilde{B}_2 + \tilde{B}_1 R_1{}^{1}\tilde{D}_{11}^*\tilde{D}_{12} \\ \hline \tilde{R}_1{}^{1/2}\tilde{C}_1 & 0 & \tilde{R}_1{}^{1/2}\tilde{D}_{12} \\ \tilde{C}_2 + \tilde{D}_{12} R_1{}^{1}\tilde{D}_{11}^*\tilde{C}_1 & \tilde{D}_{21} R_1{}^{1/2} & \tilde{D}_{21} R_1{}^{1}\tilde{D}_{11}^*\tilde{D}_{12} \end{array}\right]$$

$$\tilde{D}_{11} := \begin{bmatrix} D_{1111} & D_{1112} \\ D_{1121} & D_{1122} + D \end{bmatrix},$$

$$D_\infty = \ D_{1122} \quad D_{1121}(I \quad D_{1111}^* D_{1111})^{\ 1} D_{1111}^* D_{1112}$$

$$R_1 := I - \tilde{D}_{11}^* \tilde{D}_{11},\ \tilde{R}_1 = I - \tilde{D}_{11} \tilde{D}_{11}^*.$$

Thus, an $\mathcal{H}_\infty$ controller can be calculated for our particular models where $D_{a11} \neq 0$ and $D_{a22} \neq 0$. Algorithms for this general $\mathcal{H}_\infty$ control problem are available in [8].

3- Scheduling technique

The $\mathcal{H}_\infty$ controllers designed for the frozen design points $\rho_i \in \Lambda$ depend essentially on the two solutions of the two Riccati equations needed in each $\mathcal{H}_\infty$ control design.

Let us define:

$K_i := f(\Im_i, X_i, Y_i)$ as the i[th] $\mathcal{H}_\infty$ controller for the ith design, where:

$$\Im_i := \left[\begin{array}{c|cc} A_a(\rho_i) & B_{a1}(\rho_i) & B_{a2}(\rho_i) \\ \hline C_{a1}(\rho_i) & D_{a11}(\rho_i) & D_{a12}(\rho_i) \\ C_{a2}(\rho_i) & D_{a21}(\rho_i) & D_{a21}(\rho_i) \end{array}\right]$$

is the state-space representation of the ith augmented plant. Matrices X_i, Y_i are the i[th] solutions of the i[th] two Riccati equations used in the ith frozen design. The advantage of our method is the online availability of a faithful linear model at any point of the flight envelope, using the LPV model. This considerably facilitates the implementation, in contrast to the procedure of linearization, which was used at each point of the flight envelope in [4]. The model $\Im_j$ can be evaluated for each j[th] parameter ρ_j $[0.2, 0.8]$. If we suppose that ρ_j $[\rho_i, \rho_{i+1}]$ we use the i[th] and the (i+1)[th] controllers designed, K_i and K_{i+1}, respectively, to calculate the controller K_j. The solutions of the two Riccati equations at any j[th] design point are linearly interpolated from the solutions $X_i, Y_i)$ and $X_{i+1}, Y_{i+1})$ as follows:

$$X_j = (1 - \mu_j) X_i + \mu_j X_{i+1},$$

$$\mu_j = (\rho_j - \rho_i)/(\rho_{i+1} - \rho_i) : \mu_j \in [0,1],$$

and similarly for Y_j. Once the j[th] solution pair of the Riccati equations is computed, and using the state-space matrices of $\Im_j$, the $\mathcal{H}_\infty$ controller at the j[th] point of the flight envelope is calculated as shown in the previous section as: $K_j = f(\Im_j, X_j, Y_j)$.

The use of this technique of controller scheduling has the advantage that it does not require any special structure for the frozen controller designs. They can be $\mathcal{H}_2$, $\mathcal{H}_\infty$ or $\mathcal{H}_\infty$ loopshaping controllers. All of these modern controllers have the property of being based on solutions of Riccati equations.

4- Results and nonlinear simulation

The frozen designs made are based on a mixed-sensitivity $\mathcal{H}_\infty$ setup. The performance objectives were unified for all frozen design points. The performance indices γ obtained were equal to {1.1,1.09,1,0.93,0.86,0.80,0.74,0.76,0.80,0.84,0.88,0.93,0.98} for ρ {0.2,0.25,0.3,0.35,0.4,0.45,0.5,0.55,0.6,0.65,0.7,0.75,0.8}, respectively. Thus, the performance specifications on pitch demand tracking, the gust load alleviation and the controller output constraints, were satisfied for all frozen designs. Figure 2 shows closed-loop responses and actuator deflections for 0.55 Mach. It confirms the good pitch tracking obtained for pilot pitch demand pc $3\pi/180$ and the reduction of the acceleration with the $\mathcal{H}_\infty$ design [9] using reasonable actuator deflections.

The full flight envelope controller using the technique presented in this paper is shown by Figure 3. Notice the good performance level for the pitch-tracking objective. For the gust load alleviation objective, we get a degradation of performance with respect to the frozen design results.

However, the worst level of acceleration obtained is equal to $-1.5m/s^2 = -0.15g$ which is still reasonable, taking into account that we are operating over the full flight envelope. We have to comment that the gust load alleviation problem is in general not easy to deal with for just one operating point. Dealing with that problem over a full flight envelope is presented in this paper for the first time, with good performance results obtained. The actuator responses are still in the acceptable range of 15° for both the elevator and the control vane actuator. We notice some bumps for the control vane actuators. These bumps are due essentially to the high degree of change in the dynamic of the aircraft. This change is noticed even between two adjacent operating points. Although these are undesirable effects on the deflection actuator responses, these bumps can be supported by the real actuators, which are bounded in magnitude and rate of variation. We went further in our research trying to avoid those undesirable bumps for the actuator response. We used a safety technique that consists in constraining the actuator deflections to safe ranges. Figure 4 shows respectively the closed loop pitch response, the acceleration of the aircraft and the constrained actuator deflections. We could reduce the actuator bumps in magnitude and variation rate into more homogenous deflections without significantly affecting the performance obtained in the unconstrained actuator case.

5- Conclusion

A novel method, combining a nonlinear parameter-varying model an interpolating technique based on the solutions of Riccati equations of frozen $\mathcal{H}_\infty$ control designs, was applied successfully to a B1 flexible bomber. Although reasonable actuator deflection responses were obtained, further research is needed to get rid of the effect of bumps.

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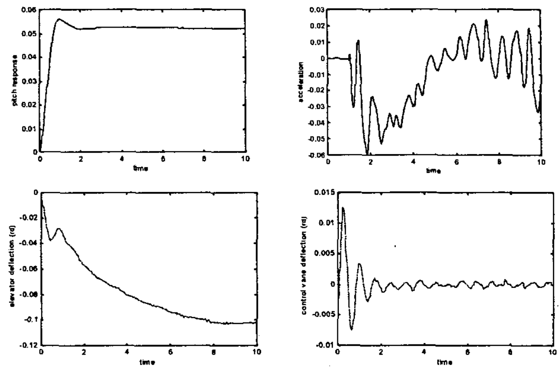

Figure 2: Closed-loop responses and actuator deflections, 0.55 Mach

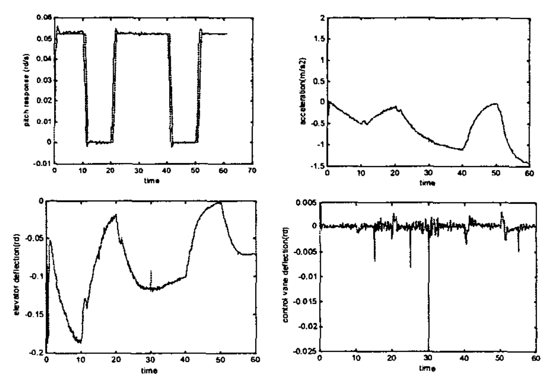

Figure 3: Responses with full flight envelope controller

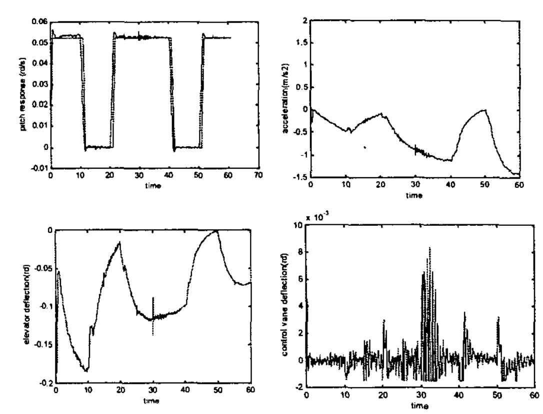

Figure 4: Responses with full flight envelope controller and constrained actuator deflections