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# IMC design for unstable processes with time delays

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**Abstract**

A modified IMC structure is proposed for unstable processes with time delays. The structure extends the standard IMC structure for stable processes to unstable processes and controllers do not have to be converted to conventional ones for implementation. An advantage of the structure is that setpoint tracking and disturbance rejection can be designed separately. A method is proposed to tune the modified IMC structure with an emphasis on the robustness of the structure. Design for some typical delayed unstable processes shows that the control structure can be tuned easily and achieve good tradeoff between time-domain performance and robustness.


*Keywords:* Internal model control (IMC); Unstable processes with time delay; Robustness

## 1. Introduction

Internal Model Control (IMC) (Fig. 1) has been shown to be a powerful method for control system synthesis [1]. However, for unstable processes the IMC structure cannot be implemented, since any input $d_1$ will make $y$ grow without bound if $P$ is unstable. Nevertheless, as discussed in [1], we could still use IMC approach to design a controller for an unstable process, if only the following conditions are satisfied for the internal stability of the closed-loop system:

(i) $Q$ stable.
(ii) $PQ$ stable.
(iii) $(1-PQ)P$ stable.

These conditions result in the well known standard interpolation conditions [1]:

- The RHP poles of $P$ must be canceled by the zeros of $Q$ [condition (ii)].
- The RHP poles of $P$ must be canceled by the zeros of $(1-PQ)$ [condition (iii)].

For example, consider an unstable process with the following transfer function:

$$P=\frac{1}{s-1} \tag{1}$$

Then an IMC controller for such a process will have the form as follows:

$$Q=(s-1)\frac{\alpha s+1}{(\lambda s+1)^2} \tag{2}$$

where $\lambda$ is a tuning parameter and $\alpha$ is a parameter to guarantee that $(1-PQ)$ cancels the RHP pole of $P$, namely,

$$(1-PQ)|_{s=1}=\left(1-\frac{\alpha s+1}{(\lambda s+1)^2}\right)\bigg|_{s=1}=0 \tag{3}$$

For actual implementation, the IMC law must be reduced to an equivalent conventional controller:

$$K=\frac{Q}{1-PQ}=\frac{\dfrac{(s-1)(\alpha s+1)}{(\lambda s+1)^2}}{1-\dfrac{\alpha s+1}{(\lambda s+1)^2}} \tag{4}$$

By Eq. (3) the RHP zero at $s=1$ is canceled by the pole at the same location, so $K$ is finally a controller

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**Nomenclature**

| | |
|---|---|
| $P$ | process transfer function |
| $P^{*}e^{-\tau s}$ | process model, with time delay |
| $P^{*}$ | process model, ignoring time delay |
| $y$, $u$ | process output, input |
| $r$, $d_1$, $d_2$ | setpoint, disturbance at input and output |
| $K_0$, $K_1$, $K_2$ | three controllers in the modified IMC structure |
| $u_1$, $u_2$ | output of controller $K_1$, $K_2$ |
| $G$ | stabilized plant model, with time delay |
| $G^{*}$ | stabilized plant model, ignoring time delay |
| $Q$ | IMC controller |
| $T_{yd_1}$ | transfer function from $d_1$ to $y$ |
| RHP | right half plane |

implementable. For example, if we choose $\lambda = 1$, then $\alpha = 3$ by Eq. (3), and

$$K = \frac{(s-1)(3s+1)}{(s+1)^2 - 3s - 1} = \frac{(s-1)(3s+1)}{s(s-1)} = 3 + \frac{1}{s} \tag{5}$$

The above procedure does not work for unstable processes with pure time delays. For instant, if the process has a delay of 1 s, i.e.,

$$P_d = \frac{1}{s-1}e^{-s} \tag{6}$$

then an IMC controller $Q_d$ for such a process is still given by Eq. (2) since both plants have the same minimum phase part. To make $(1 - P_dQ_d)P_d$ stable, the following conditions must be satisfied:

$$(1 - P_dQ_d)|_{s=1} = \left(1 - \frac{(\alpha s+1)e^{-s}}{(\lambda s+1)^2}\right)\bigg|_{s=1} = 0 \tag{7}$$

Now an equivalent conventional controller is

$$K_d = \frac{Q_d}{1 - P_dQ_d} = \frac{\dfrac{(s-1)(\alpha s+1)}{(\lambda s+1)^2}}{1 - \dfrac{(\alpha s+1)e^{-s}}{(\lambda s+1)^2}} \tag{8}$$

However, in this case the RHP pole and zero at $s = 1$ cannot be canceled explicitly as in Eq. (5), so the controller is not implementable in practice.

To overcome this problem we can use controller approximation to cancel the unstable factor. Usually the pure delay has to be approximated with some rational function. For example, Lee et al. [2] discussed how to use series expansion to approximate the final controller with a PID structure. Unfortunately there is no systematic approach to find the approximation, and it cannot retain the IMC structure.

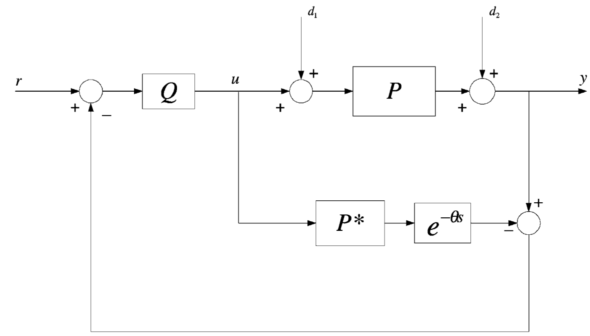


Fig. 1. Internal model control configuration.

A much simpler method is to approximate the time delay with rational terms using Pade approximation. For example, consider a first-order unstable plant with delay:

$$\frac{k}{\tau s - 1}e^{-\theta s} \tag{9}$$

Suppose the delay is approximated by a first-order Pade approximation, then the nominal model will be:

$$\tilde{P} = \frac{k(1 - \frac{\theta}{2}s)}{(\tau s - 1)(1 + \frac{\theta}{2}s)} \tag{10}$$

So an IMC controller will have the form:

$$\tilde{Q} = \frac{(\tau s - 1)(1 + \frac{\theta}{2}s)(\alpha s + 1)}{k(\lambda s + 1)^3} \tag{11}$$

with $\lambda$ being the tuning parameter and $\alpha$ chosen to guarantee that $(1 - \tilde{P}\tilde{Q})$ cancels the RHP pole of $\tilde{P}$, namely,

$$\alpha = \tau\left(\frac{\left(\frac{\lambda}{\tau} + 1\right)^3}{1 - \frac{\theta}{2\tau}} - 1\right) \tag{12}$$

Now the final implementable feedback controller has the form:

$$\tilde{K} = \frac{\left(1 + \frac{\theta}{2}s\right)(\alpha s + 1)}{ks\left(\frac{\lambda^3}{\tau}s + \left(\alpha - 3\lambda - \frac{\theta}{2}\right)\right)} \tag{13}$$

Unfortunately the procedure works only for unstable processes with small delays. For unstable processes with large time delays, the approximation error sets a bound on the tuning parameter $\lambda$, thus the closed-loop performance will degrade. Examples below clearly shows the limitation of this method.

Two step controller design can also be used to overcome the implementation problem. The procedure consists of first designing a compensator to stabilize the plant, and then designing an IMC controller for the stabilized model [3]. The main shortcoming of the approach is complexity. Indeed, since the stabilized model is irrational, the IMC controller is very complex compared with the conventional IMC controller for stable processes.

Other extensions of the design and tuning methods from stable processes to unstable plants can be found in the literature, see for example, Ziegler–Nichols method [4], maximum-peak based method [5], gain-phase margin method [6], $H_\infty$-based method [7], GPC-based method [8] and integral error method [9]. Refs. [10,11] considered controller design for first-order unstable processes with time delays, and [12], for second order unstable processes. Setpoint weighting was considered in [13] to overcome the large overshoot of the setpoint response. [14,15] considered stabilizability conditions for unstable processes. Due to the existence of the RHP poles, control of the unstable processes is harder and more complicated than that for stable processes.

In this paper, we will propose a modified IMC structure for unstable processes with time delays. Compared with the IMC method in [2] and [3], and the IMC design using Pade approximation, the advantages of the modified IMC structure are:

- IMC structure can be retained for unstable processes. Controllers do not have to be converted to conventional ones for implementation.
- Setpoint tracking and disturbance rejection can be designed separately. The setpoint tracking design follows the standard IMC design for a stable plant.
- Robustness and disturbance rejection mainly rely on a controller in a feedback loop. Robustness of the whole structure can be considered by tuning this controller.

Examples show that the structure is easy to tune and can achieve better tradeoffs between time domain performance and robustness.

## 2. Modified IMC structure

It is easy to verify that the modified IMC structure shown in Fig. 2 is equivalent to that in Fig. 3. We have

$$y = \frac{PK_1\left(1 + P^*e^{-\theta s}K_2\right)}{(1 + P^*K_0)(1 + PK_2) + (P - P^*e^{-\theta s})K_1} r + \frac{P\left(1 + P^*K_0 + P^*e^{-\theta s}K_1\right)}{(1 + P^*K_0)(1 + PK_2) + (P - P^*e^{-\theta s})K_1} d_1 + \frac{1 + P^*K_0 + P^*e^{-\theta s}K_1}{(1 + P^*K_0)(1 + PK_2) + (P - P^*e^{-\theta s})K_1} d_2 \tag{14}$$

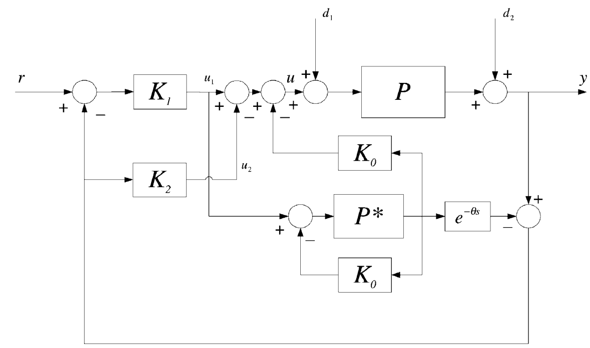


Fig. 2. Modified IMC structure for unstable processes with time delays.

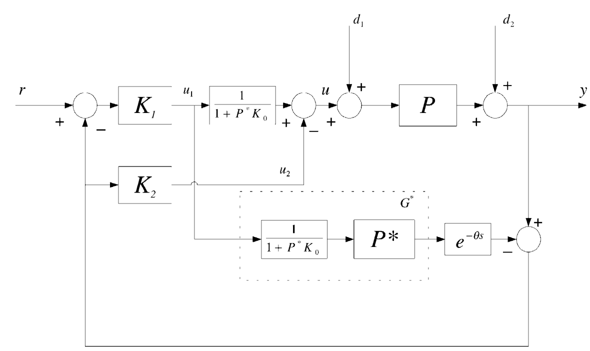


Fig. 3. An equivalent structure.

If the plant model is perfect, i.e., $P = P^*e^{-\theta s}$, then

$$y = \frac{PK_1}{1 + P^*K_0} r + \frac{P}{1 + P^*K_0}\frac{1 + P^*K_0 - PK_1}{1 + PK_2} d_1 + \frac{1}{1 + P^*K_0}\frac{1 + P^*K_0 - PK_1}{1 + PK_2} d_2 \tag{15}$$

Let

$$G^* := \frac{P^*}{1 + P^*K_0} \tag{16}$$

$$G := \frac{P}{1 + P^*K_0} = G^*\mathrm{e}^{-\theta s} \tag{17}$$

then

$$y = \underbrace{GK_1r}_{y_r} + \underbrace{(1 - GK_1)\frac{P}{1 + PK_2}d_1}_{y_{d1}} + \underbrace{(1 - GK_1)\frac{1}{1 + PK_2}d_2}_{y_{d2}} \tag{18}$$

From Eq. (18) it follows that $K_2$ has no effect on the setpoint response $y_r$. The output of controller $K_1$ can also be computed:

$$\begin{aligned} u_1 &= K_1r - \frac{P}{1 + PK_2}K_1d_1 - \frac{1}{1 + PK_2}K_1d_2 \\ &= K_1r - \frac{G}{1 + PK_2}K_1\left(1 + P^*K_0\right)d_1 - \frac{1}{1 + PK_2}K_1d_2 \end{aligned} \tag{19}$$

If $K_2=0$, then $K_1$ can be shown to be an IMC controller for $G$. However, from Eq. (18), we see that the load response $y_{d1}$ is not stable if $K_2=0$. To overcome this problem we incorporate $K_2$ to stabilize $P$, the original delayed unstable model.

The structure thus proposed has three compensators, namely, $K_0$, $K_1$, and $K_2$, each having a distinctive use and influence on the overall closed loop response:

- $K_0$ is used to stabilize $P^*$, the original (unstable) plant, ignoring the time-delay.
- $K_1$ is an IMC controller for the new model $G$, defined in Eq. (17).
- $K_2$ is used to stabilize the original unstable system $P$, with the delay $\theta$. It is crucial for the internal stability of the structure.

The structure differs from the one proposed in [3] which takes a two-step design procedure in that $K_0$ stabilizes the delay-free unstable plant instead of the unstable plant with the time delay. An advantage with this is that the stabilized plant will not have delay in the denominator, and then the IMC design (for $K_1$) will become much easier.

## 3. IMC design

Having defined the modified IMC structure we now elaborate a design method. As we discussed above, $K_0$ can be tuned to be a stabilizer for an unstable plant without delay and $K_1$, an IMC scheme for a stable plant. Note that setpoint tracking does not depend on $K_2$, so we can first design for the setpoint tracking. It is well known that an IMC controller can be tuned to have good setpoint tracking response for a stable plant, thus the load response of the closed loop system depend mostly on $K_2$. We will discuss the tuning of this controller.

The transfer functions from $d_1$ ($d_2$) to $y$ ($u_2$) in the modified IMC structure can be found as

$$T_{yd_1} = (1 - GK_1)\frac{P}{1 + PK_2} \tag{20}$$

$$T_{u_2d_1} = \frac{PK_2}{1 + PK_2} \tag{21}$$

$$T_{yd_2} = (1 - GK_1)\frac{1}{1 + PK_2} \tag{22}$$

$$T_{u_2d_2} = \frac{K_2}{1 + PK_2} \tag{23}$$

To reject the load disturbance $d_1$ we can solve the following $H_\infty$ problem:

$$\inf_{K_2}\left\| \begin{bmatrix} W_1 T_{yd_1} \\ W_2 T_{u_2d_1} \end{bmatrix} \right\|_\infty \tag{24}$$

where $W_1$ and $W_2$ are weights for the input disturbance and the output of $K_2$. If we also consider rejecting the output disturbance $d_2$, then we can solve the following four-block problem:

$$\inf_{K_2}\left\| \begin{bmatrix} W_1 T_{yd_1} & W_3 T_{yd_2} \\ W_2 T_{u_2d_1} & W_4 T_{u_2d_2} \end{bmatrix} \right\|_\infty \tag{25}$$

where $W_i (i = 1, \cdots, 4)$ are weighting functions.

We see that the effect of $K_0$ and $K_1$ on disturbance rejection is through the term $1-GK_1$. Once $K_0$ and $K_1$ are designed, $K_2$ will not affect the term. So it can be absorbed in the weighting functions $W_1$ and $W_3$, and the above problem is simplified to:

$$\inf_{K_2}\left\| \begin{bmatrix} \dfrac{W_1 P}{1 + PK_2} & \dfrac{W_3}{1 + PK_2} \\ \dfrac{W_2 PK_2}{1 + PK_2} & \dfrac{W_4 K_2}{1 + PK_2} \end{bmatrix} \right\|_\infty \tag{26}$$

If all the weights are set to 1, the problem is equivalent to a robust stability problem in terms of coprime factors [16]:

$$\varepsilon_{\max}^{-1} = \inf_{K_2}\left\| \begin{bmatrix} K_2 \\ 1 \end{bmatrix} (1 + PK_2)^{-1}\tilde{M}^{-1} \right\|_\infty \tag{27}$$

where $\tilde{M}^{-1}\tilde{N}$ is a normalized left coprime factorization of $P$. This problem is equivalent to the gap metric [17] and the graph metric [18]. It was proved in [17] that $K_2$ will stabilize any plant such that the gap to $P$ is less than $\varepsilon_{\max}$.

The problem in Eq. (27) is easy to solve and only requires solving two Ricatti equations. To maximize the loop gain, we can design for $kP$, where the parameter $k$ is the desired loop gain. The quantity $\varepsilon_{\max}$ is then an indicator of how large a loop gain can be robustly achieved.

In summary, the design for the modified IMC structure takes three steps:

1. Delay-free part stabilizer. A controller $K_0$ is designed to stabilize the delay-free part of the model ($P^*$).
2. Setpoint tracking. A controller $K_1$ is designed as an IMC controller with respect to a new process $G = \frac{P}{1+P^*K_0}e^{-\theta s}$.
3. Load disturbance rejection and robust stability. A controller $K_2$ is designed to robustly stabilize the original delayed unstable model $P$ by solving Eq. (27).

Through simplification Step 3 can be done independent of Step 2, so for the modified IMC structure setpoint tracking and load disturbance can be designed separately.

We emphasize that $K_0$ can be chosen arbitrarily as long as it stabilizes the delay-free model and does not introduce any RHP zeros. The reason is that $K_1$, being an IMC controller, will invert the minimum phase part of the stabilized model $G$, thus if $K_0$ does not introduce additional RHP zeros in $G$, its effect will be canceled by $K_1$. The purpose of $K_0$ is simply to ensure internal stability of the structure and it has no impact on the final performance of the system.

## 4. Design for typical unstable processes with time delays

In this section we will illustrate our design procedure for some typical unstable processes with time delays. As discussed in the previous section, the proposed IMC structure is in fact a two-degree-of-freedom structure, and the setpoint response and the disturbance rejection can be considered separately, so we expect that better performance can be achieved using this structure.

### 4.1. First-order unstable processes with time delays

Consider a first-order unstable process with time delay of the form:

$$P(s) = \frac{k}{\tau s - 1} e^{-\theta s} \tag{28}$$

We will discuss the tuning of the three controllers as discussed in the previous section.

#### 4.1.1. Tuning of $K_0$

$K_0$ is intended to stabilize the delay-free unstable model:

$$P^* = \frac{k}{\tau s - 1} \tag{29}$$

A simple proportional gain $K_0$ can be used and the model defined in Eq. (16) will be

$$G^*(s) = \frac{k}{\tau s - 1 + kK_0} \tag{30}$$

Clearly $G^*$ is stable if $K_0 > 1/k$. We can choose $K_0 = \frac{2}{k}$ to make $G^*$ equal to $\frac{k}{\tau s+1}$.

#### 4.1.2. Tuning of $K_1$

$K_1$ is intended to be an IMC controller of the stabilized model:

$$G = G^* e^{-\theta s} = \frac{k}{\tau s + 1} e^{-\theta s} \tag{31}$$

So $K_1$ can be chosen as

$$K_1 = \frac{\tau s + 1}{k(\lambda s + 1)} \tag{32}$$

where $\lambda$ is a tuning parameter.

#### 4.1.3. Tuning of $K_2$

$K_2$ is intended to stabilize the original delayed unstable process. It is well known that the stabilizing proportional gain for a first-order delayed unstable process is bounded both from below and above [15]. In [4], a stabilizing proportional gain is found by maximizing the phase margin; while in [3] it is chosen to be the average of the two bounds. As we will see in the examples, a proportional controller is too conservative for disturbance rejection, while the PD controller used in [3] is too aggressive, so we need to solve problem defined in Eq. (27) directly to have a robust controller $K_2$.

Taking the average of the largest and the smallest gain that destabilize the plant $P$ as the desired normalized loop gain and solving problem in Eq. (27) against the normalized delay $\frac{\theta}{\tau}$, we get Fig. 4, showing the robust stability margin against $\frac{\theta}{\tau}$.

Note that we have approximated the delay with first-order Pade approximation in solving the $H_\infty$. This will simplify the design procedure and the final controller. Algorithms without approximation have been proposed in [19] and [20]. So if complex controllers are allowed, we can use the algorithms to find solutions and optimal controllers. It should be emphasized that in our design $K_2$ can be any stabilizing controller for the original plant, a robust controller is preferred here to account for the uncertainty of the plant model. Unlike in the IMC design with an approximated model, the model error here will not have so severe an effect on system performance. Our procedure works for unstable processes with large delays too.

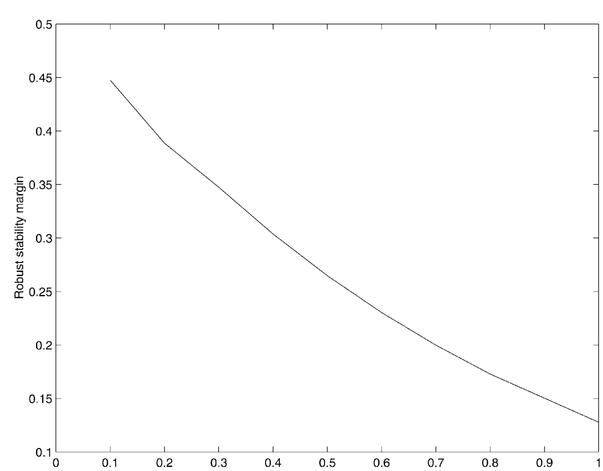


Fig. 4. Robust stability margin ($\varepsilon_{max}$) vs. normalized delay.

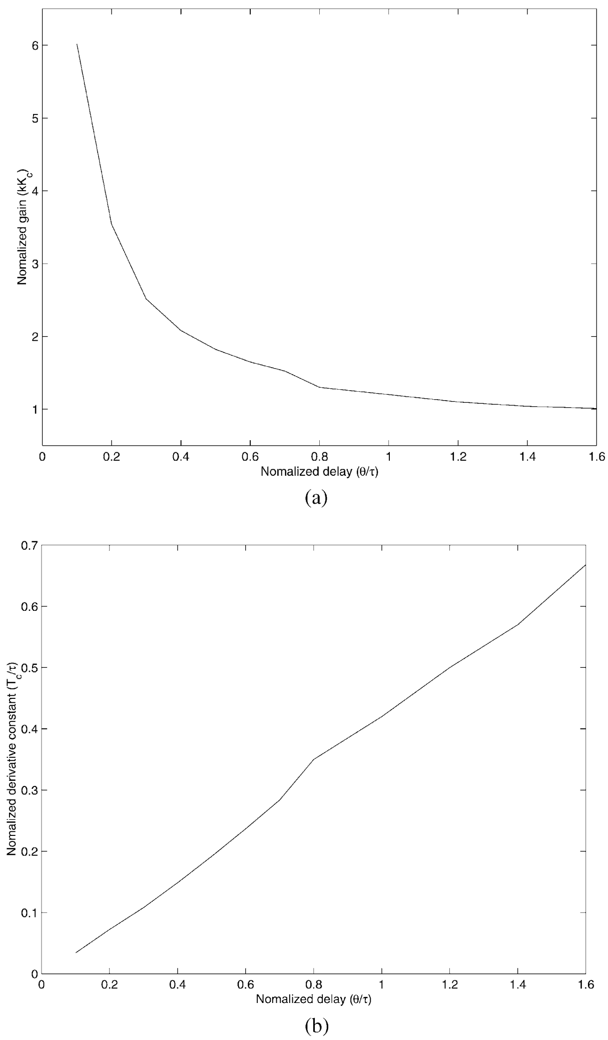


Fig. 5. Desired proportional gain and derivative time.

Now $K_2$ is of the form of a first-order lead controller. We can approximate it with a PD controller with the proportional gain and the derivative time shown in Fig. 5. For $\frac{\theta}{\tau} > 1$, the parameters are manually tuned with the goal of achieving acceptable response against at least 5% uncertainty on the delay. Since in such case the allowable upper and lower bounds are so close that it is not hard to find the acceptable parameters.

Using a curve fitting approach, we get the following tuning formula for $K_2$ for a first-order unstable process with time delay:

$$K_2 = K_c(T_c s + 1) \tag{33}$$

where

$$K_c = \begin{cases} \frac{1}{k}\left(\frac{0.533}{\theta/\tau} + 0.746\right) \text{if } \theta/\tau \leqslant 0.7 \\ \frac{1}{k}\left(\frac{0.490}{\theta/\tau} + 0.694\right) \text{if } 0.7 < \theta/\tau \leqslant 1.5 \end{cases} \tag{34}$$

$$T_c = (0.426\theta/\tau - 0.014)\tau \tag{35}$$

### 4.2. *Second-order processes with one unstable pole and time delay*

Consider a second-order process with one unstable pole and time delay,

$$P(s) = \frac{k}{(\tau_1 s - 1)(\tau_2 s + 1)} e^{-\theta s} \tag{36}$$

We will tune the three controllers as above.

#### 4.2.1. *Tuning of $K_0$*

Now the delay-free part of the plant has one unstable pole and one stable pole. We can choose $K_0$ as a PD controller with the form

$$K_0 = k_0(\tau_2 s + 1) \tag{37}$$

It cancels the stable pole of the plant model and the model defined in Eq. (16) would be

$$G^*(s) = \frac{k}{(\tau_1 s - 1 + k k_0)(\tau_2 s + 1)} \tag{38}$$

Again, if $k_0 > 1/k$, then $G^*$ is stable. To simplify, we choose $k_0 = \frac{2}{k}$ to make $G^*$ equal to $\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}$.

#### 4.2.2. *Tuning of* K$_1$

Now the stabilized model is

$$G = G^* e^{-\theta s} = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s} \tag{39}$$

So $K_1$ can be chosen as

$$K_1 = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k(\lambda s + 1)^2} \tag{40}$$

or simply

$$K_1 = \frac{\tau_1 s + 1}{k(\lambda s + 1)} \tag{41}$$

where $\lambda$ is a tuning parameter.

#### 4.2.3. *Tuning of* K$_2$

We can choose $K_2$ to stabilize $P$ with the maximum gap margin. For simplification, we choose $K_2$ of a PD form $K_c(\tau_2 s + 1)$, so we just need to tune a robust proportional gain $K_c$ for first-order delayed unstable process $\frac{k}{\tau_1 s+1} e^{-\theta s}$. With discussions above, we can combine the derivative time constant with $\tau_2$ and get the following tuning formula:

$$K_2 = K_c(T_c s + 1) \tag{42}$$

where

$$K_c \left\{ \begin{array}{ll} \frac{1}{k}\left(\frac{0.533}{\theta/\tau_1} + 0.746\right) & \text{if } \theta/\tau_1 \leqslant 0.7 \\ \frac{1}{k}\left(\frac{0.490}{\theta/\tau_1} + 0.694\right) & \text{if } 0.7 < \theta/\tau_1 < 1.5 \end{array} \right\} \tag{43}$$

$$T_c = (0.426\theta/\tau_1 - 0.014)\tau_1 + \tau_2 \tag{44}$$

### 4.3. Processes with two unstable poles and time delay

The process model is of the form

$$P(s) = \frac{k}{(\tau_1 s - 1)(\tau_2 s - 1)} e^{-\theta s} \tag{45}$$

#### 4.3.1. Tuning of $K_0$

To stabilize the delay-free part of the plant model, we choose $K_0$ as a derivative controller with the form $k_0 s$. Then we get

$$G^*(s) = \frac{k}{(\tau_1 s - 1)(\tau_2 s - 1) + k k_0 s} \tag{46}$$

If $k_0 = \frac{2(\tau_1 + \tau_2)}{k}$, then

$$G^* = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{47}$$

and $G^*$ is stable.

#### 4.3.2. Tuning of $K_1$

Now the stabilized model is

$$G = G^* e^{-\theta s} = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s}. \tag{48}$$

so an IMC controller $K_1$ can be chosen as

$$K_1 = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{(k(\lambda s + 1)^2} \tag{49}$$

where $\lambda$ is a parameter.

#### 4.3.3. Tuning of $K_2$

$K_2$ is chosen to stabilize $P$ with the maximum gap margin by solving problem defined in Eq. (27). If the delay is approximated with first-order Pade approximation, then the robust controller is second-order, which can be approximated with a PD controller. However, as will be shown in Example 4, the PD controller can only maintain the nominal stability; for robustness, the second-order controller is better. Unfortunately we haven't found a simple tuning formula as we did in the earlier cases.

## 5. Examples

In this section four examples are presented to illustrate the advantages of the modified IMC structure.

### 5.1. Example 1

Consider a process with transfer function

$$P(s) = \frac{1}{s - 1} e^{-0.4s} \tag{50}$$

It is a typical first-order delayed unstable process. For the modified IMC structure, the three controllers are tuned as discussed in the previous section to be:

$$K_0 = 2, \; K_1 = \frac{s + 1}{0.4s + 1}, \quad K_2 = 2.079(0.156s + 1) \tag{51}$$

To test the performance of the control system, suppose the setpoint has a step change of magnitude 1 at $t = 1$ and the load disturbance has a step change of magnitude 1 at $t = 20$, the time response is shown in Fig. 6. Also shown are responses of the three-element controller designed in [3], the PID controller with a filter designed by IMC methods in [2], and an IMC controller designed by approximating the the time delay in the plant with a first-order Pade approximation. The final implementable feedback controller is

$$\frac{(12.9348s + 7.4074)(s + 5)}{s(s + 23.9348)} \tag{52}$$

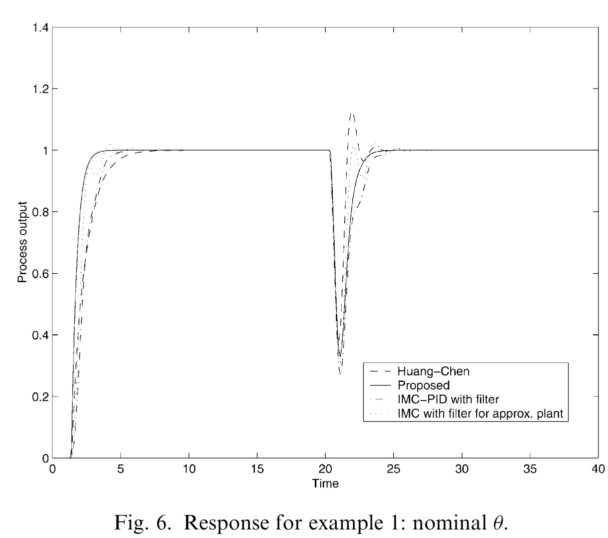


Fig. 6. Response for example 1: nominal $\theta$.

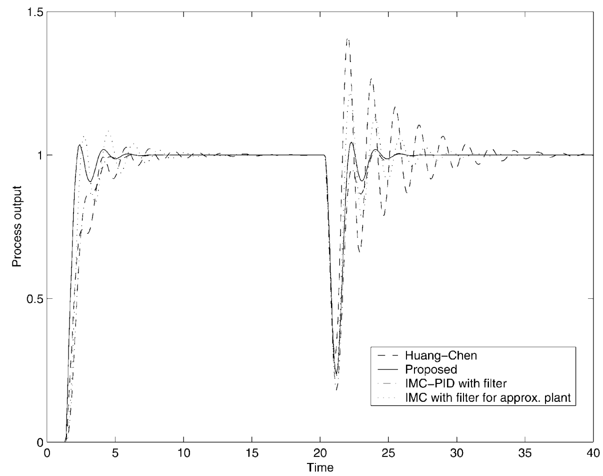


Fig. 7. Response for example 1: $\theta$ increases 10%.

with the tuning parameter $\lambda$ chosen as 0.3 and $\alpha$ computed as 1.7462. The setpoint filter is chosen as $\frac{1}{1.7462s+1}$.

We see that the proposed controller has the best setpoint response while the three-element controller has the best load response, which is achieved by sacrificing the robustness of the closed-loop system. For example, if the actual delay of the process is 10% larger, then the response of the three-element controller is the worst of the four controllers, see Fig. 7. The proposed controller has both good performance and robustness, and the IMC design for the approximated plant model works well enough since the delay is small.

If we choose $K_2$ as a proportional gain (1.5811) as suggested by [4] or a PD controller [2.58(0.12s + 1)] by [3], the responses for the nominal time delay and for the perturbed delay are shown in Fig. 8. It is easy to show that the proportional gain is too conservative while the PD by [3] is too aggressive. The proposed tuning parameters in the previous section have good compromise.

### 5.2. Example 2

Consider a process with transfer function

$$P(s) = \frac{1}{s-1}e^{-1.2s} \tag{53}$$

This process has a large time delay. For the modified IMC structure, the three controllers are tuned as discussed in the previous section to be:

$$K_0 = 2, K_1 = \frac{s+1}{2s+1}, K_2 = 1.1(0.49s+1) \tag{54}$$

To test the performance of the control system, suppose the setpoint has a step change of magnitude 1 at $t = 1$ and the load disturbance has a step change of magnitude 0.1 at $t = 40$, the time response is shown in Fig. 9 for the nominal

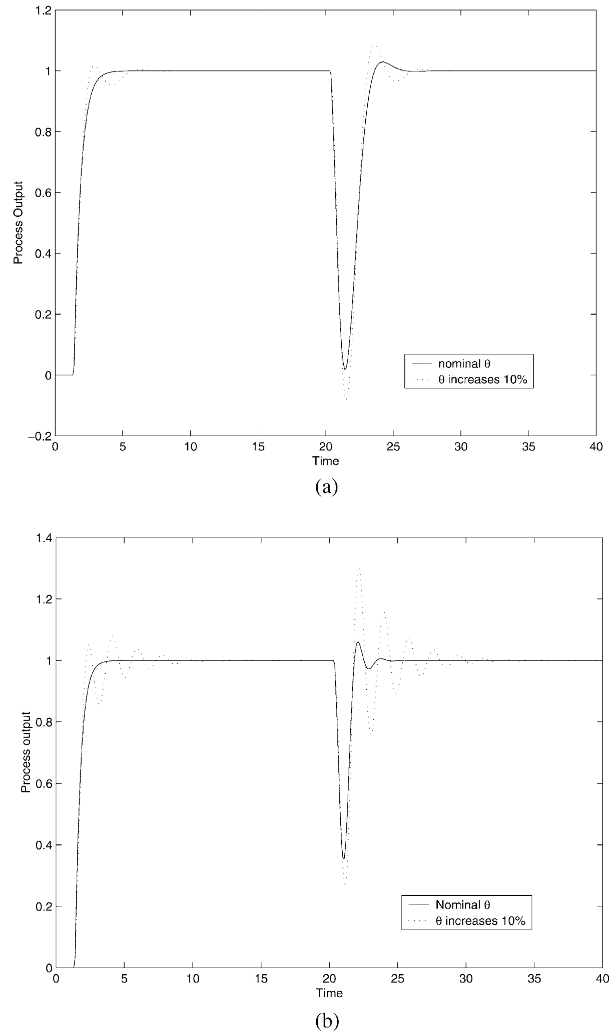


Fig. 8. Response for example 1: using different $K_2$.

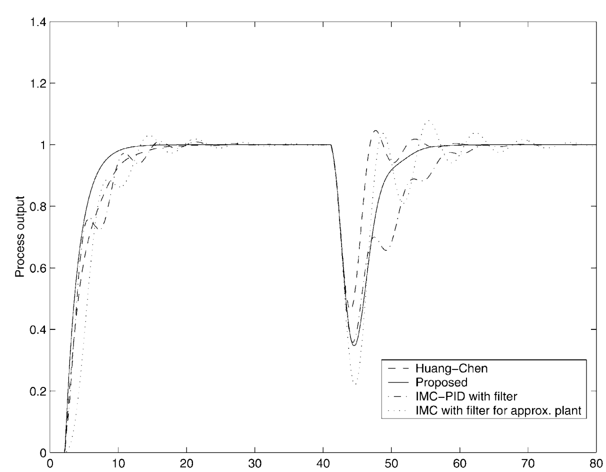


Fig. 9. Response for example 2: nominal $\theta$.

time-delay and in Fig. 10 for a delay 5% larger. Also shown are responses of the three-element controller designed in [3], the PID controller designed by IMC method in [2], and an IMC controller designed by approximating the delay with a first-order Pade approximation. For IMC-PID design, we choose $\lambda = 3$, $\alpha = 52.12$ (parameters used in [2]) and get the PID parameters as $K_p = 1.1728$, $T_i = 52.68$, $T_d = 0.5716$. The setpoint response of the PID controller with the suggested filter $\frac{1}{\alpha s+1}$ is too slow, since the desired closed-loop response with this filter is $\frac{e^{-\theta s}}{(\lambda s+1)^2}$. A better candidate for the filter is $\frac{\lambda s+1}{\alpha s+1}$, then the desired response will be $\frac{e^{-\theta s}}{\lambda s+1}$, much faster if $\lambda > 1$. In the simulation we use this filter. For the IMC design for the approximated plant, the final implementable feedback controller has the form:

$$\frac{(66.6667s + 1.7778(3s + 5)}{s(30s + 293)} \tag{55}$$

with the tuning parameter $\lambda$ chosen as 1.5 and $\alpha$ computed as 38.0625. The setpoint filter is chosen as $\frac{1}{38.0625s+1}$. Again, the proposed controller has good response and sufficient robustness. Since the delay is large, the IMC design for the approximated plant has the worst disturbance rejection performance and robustness. The bound on $\lambda$ also degrades the tracking performance.

Since the delay is greater than 1, the method by [4] does not apply. It can be shown the PD controller $[1.16(0.524s + 1)]$ by [3] is not good for perturbed case. The figure is omitted here for brevity.

### 5.3. Example 3

Consider a process with transfer function

$$P(s) = \frac{1}{(5s - 1)(2s + 1)(0.5s + 1)} e^{-0.5s} \tag{56}$$

It is a high-order unstable process. It can be approximated with a model with one unstable pole and time delay:

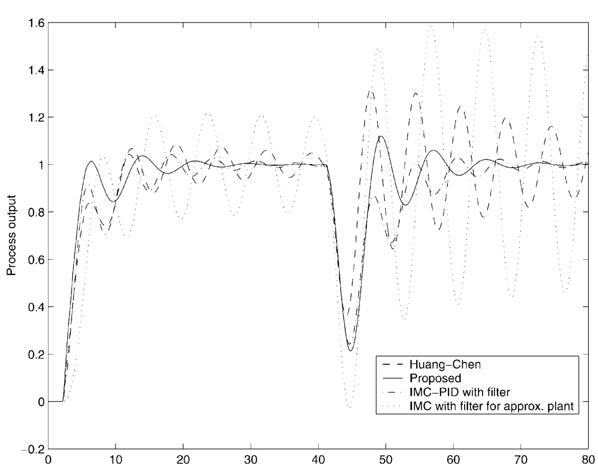


Fig. 10. Response for example 2: $\theta$ increases 5%.

$$\frac{1}{(5s - 1)(2.07s + 1)} e^{-0.939s} \tag{57}$$

The three controllers in the modified IMC structure are tuned as discussed in the previous section to be:

$$K_0 = 2(2.07s + 1),\ K_1 = \frac{s + 1}{0.2s + 1},\ K_2 = 3.584(2.4s + 1) \tag{58}$$

Suppose the setpoint has a step change of magnitude 1 at $t = 1$ and the load disturbance has a step change of magnitude 1 at $t = 40$, the time response is shown in Fig. 11 for the nominal time-delay and in Fig. 12 for a delay 10% larger, together with the responses of the three-element controller designed in [3] and the PID controller with a filter designed by the IMC method in [2]. Since the normalized delay is small, robustness is not a problem for all the three controllers. But the proposed controller has a smoother response.

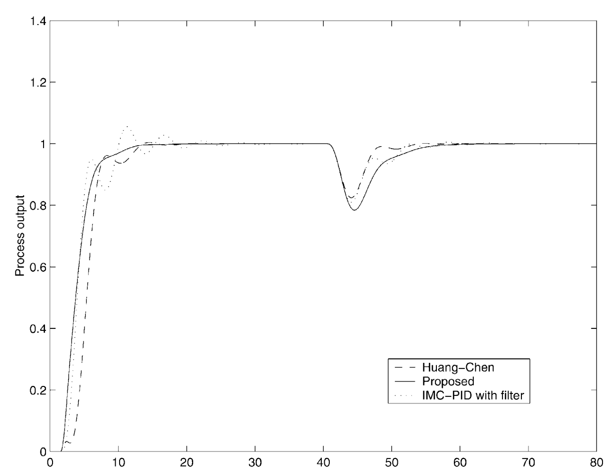


Fig. 11. Response for example 3: nominal $\theta$.

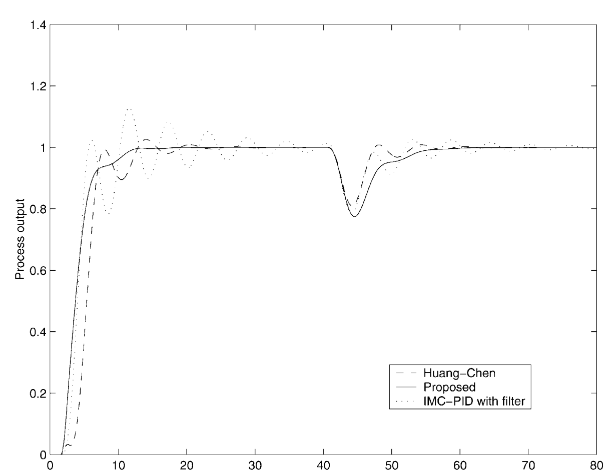


Fig. 12. Response for example 3: $\theta$ increases 10%.

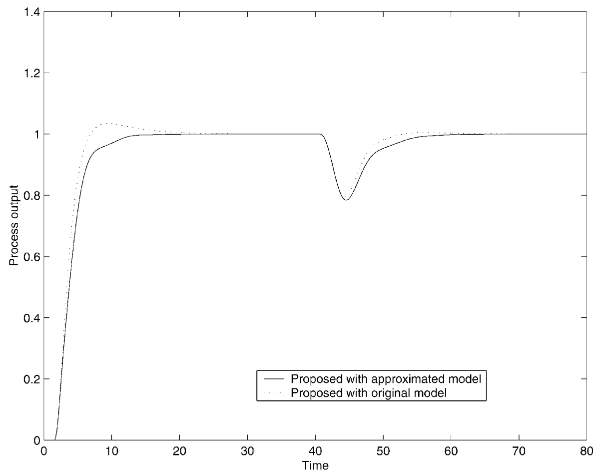


Fig. 13. Response for example 3: using high-order model.

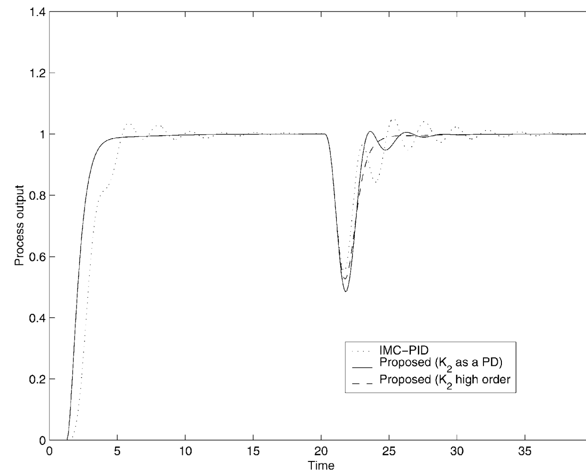


Fig. 14. Response for example 4: nominal $\theta$.

If we choose $P^*$ as the original high-order model instead of the approximated second-order model for simulation, the responses are shown in Fig. 13. The response with the original high-order model is slightly better.

### 5.4. Example 4

Consider a process with transfer function

$$P(s) = \frac{2}{(3s-1)(s-1)} e^{-0.3s} \tag{59}$$

It has two unstable poles. For the modified IMC structure, the three controllers are tuned as discussed in the previous section to be:

$$K_0 = 4s, \quad K_1 = \frac{(3s+1)(s+1)}{2(0.5s+1)^2}, \quad K_2 = \frac{66.8(s+0.27)(s+6.667)}{s^2+14.0s+121.31} \tag{60}$$

Here $K_2$ is the robust controller for the delayed unstable process with a desired gain chosen to be 10. If we insist on using a PD controller for $K_2$, we can approximate it by $3.7s+1$.

Suppose the setpoint has a step change of magnitude 1 at $t=1$ and the load disturbance has a step change of magnitude 1 at $t=20$, the time responses are shown in Fig. 14. [3] did not discuss this situation, and the PID by [2] has an oscillatory response. The proposed controllers with $K_2$ as a high-order controller and $K_2$ as a PD both work well for the nominal case. But if the delay increases by 10%, then the controller with $K_2$ as a PD is unstable. However, the controller with $K_2$ as a high-order controller is quite robust, as shown in Fig. 15. This suggests that for high-order delayed unstable process high-order controller should be used to achieve good performance.

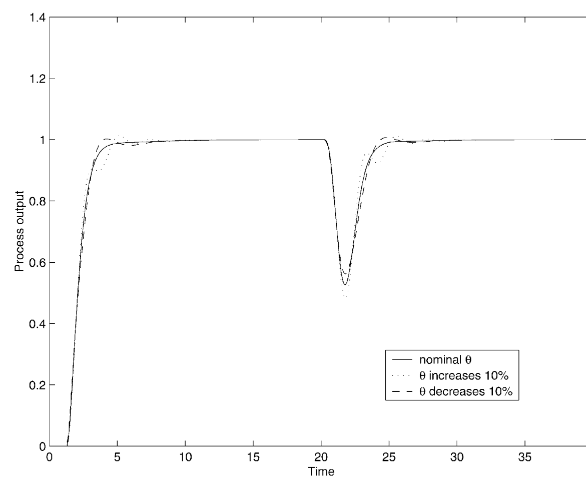


Fig. 15. Response for example 4: $\theta$ perturbed by 10%.

## 6. Conclusions

A modified IMC structure was proposed for unstable processes with time delays. Advantages of the structure were discussed and a design method aiming at system robustness was proposed. Examples showed that the control structure can be tuned easily and achieve good compromise between time-domain performance and robustness.

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