Proceedings of the American Control Conference
Anchorage, AK May 8-10, 2002

Some Preliminary Results for IMC-Based Robust Tunable Control

Benoit Boulet, Yingxuan Duan and Hannah Michalska
Centre for Intelligent Machines, McGill University, 3480 University Street, Montreal, Quebec, Canada H3A 2A7

Abstract

This paper introduces the concept of robust tunable control and establishes theoretical guidelines for the design and on-line tuning of an SISO robust tunable control system. The tuning strategy is based on the performance robustness bounds of the system and knowledge of the plant uncertainty's weighting function, which may change with time. The IMC structure is adopted.

1. Introduction

Usually, a robust $\mathcal{H}_\infty$ or μ controller design is based on a set of weighting functions representing performance specifications and uncertainty sets to achieve the best robustness/performance tradeoff. Once the robust controller is implemented, its parameters are fixed and no tuning is possible. This may result in performance degradation or even instability as the plant dynamics change over time, e.g., from system component wear, or from changes in the feed properties of industrial processes. Tuning is required to trade-off performance with robustness on-line since an initial $\mathcal{H}_\infty$ or μ controller design rarely has the best possible weighting functions for the plant [3]. Internal model control (IMC) [2] is an interesting technique in that it allows tuning of the IMC filter Q while keeping the nominal closed loop stable. IMC is based on the parameterization of all stabilizing controllers for stable plants: $K=(I-Q\tilde{P})^{-1}Q$, where $Q\in\mathcal{RH}_\infty$. This paper establishes preliminary theoretical results on robust tunable control using IMC for minimum-phase SISO plants.

2. Problem Formulation

2.1 IMC Framework for Robust Tunable Control

The conventional setup of a unity-feedback feedback control system and its IMC equivalent is shown in Figure 1.

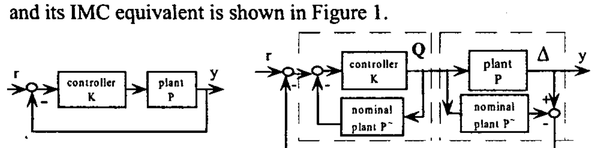

Figure 1 (a) Figure 1 (b)

Figure 1: (a) Unity feedback control system (b) Equivalent IMC diagram

The filter Q can be designed according to the optimal procedures outlined in [2]. Our focus here is to give tuning guidelines for the "best" robust performance tradeoff, given weighting functions representing the additive uncertainty, the performance specification, and the constraints on the control signals. When the plant uncertainty changes, the engineer has the opportunity to re-estimate its weight and adjust the IMC filter accordingly. Let the perturbed plant be given by $P:=\tilde{P}+W_a\Delta_a$, where $\tilde{P}$ is the nominal plant model, $\Delta_a\in\mathcal{RH}_\infty$ is the normalized additive perturbation with $\|\Delta_a\|_\infty<1$, $W_a\in\mathcal{RH}_\infty$ is the scalar weighting function bounding the perturbation at each frequency. Let $\Delta:=W_a\Delta_a$. Thus, the IMC framework boils down to the feedback interconnection of the IMC filter with the perturbation, as shown in Figure 1(b). Clearly, the robust stability of this system is determined by the small-gain theorem. Namely, the closed-loop system is well-posed and internally stable for all $\Delta\in\mathcal{RH}_\infty$ with $\|\Delta(j\omega)\|<W_a(j\omega)$ iff $\|Q(j\omega)\|\le|W_a^{-1}(j\omega)|$. Thus, the inverse of the size of the uncertainty yields a direct frequency-by-frequency constraint on the magnitude of the stable IMC filter to preserve robust stability. Based on this, we next tackle the problem of finding constraints on the IMC filter to obtain robust performance.

2.2 Robust Performance

For robust performance, we add a performance weight on the error signal (for sensitivity minimization) and a weight on the control signal (to satisfy actuator constraints) see Figure 2. We pull out the normalized additive perturbations and rearrange the system into a $G-\Delta_a$ linear fractional transformation (LFT) form, as shown in Figure 3(a).

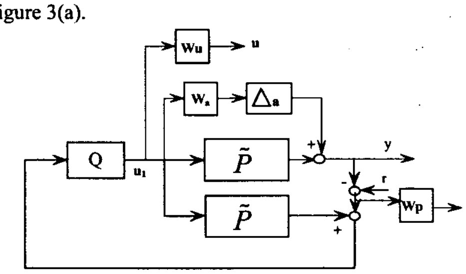

Figure 2: IMC block diagram with weighting functions

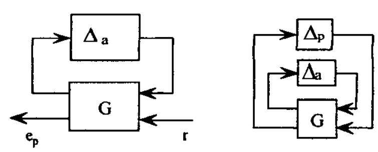

Figure 3(a) Figure 3(b)

Figure 3: (a) LFT form of system; (b) Setup for μ-analysis

Robust performance of the system requires that $\|\mathcal{F}_U\{G,\Delta_a\}\|_\infty\le 1,\ \forall\|\Delta_a\|_\infty<1$. This can be tested by computing the structured singular value $\mu_\Gamma[G(j\omega)]$ at all frequencies. Let $\Gamma:=\{\mathrm{diag}\{\Delta_a,\Delta_p\}:\Delta_a\in\mathbb{C},\Delta_p\in\mathbb{C}\}$ be the uncertainty structure. The transfer matrix G is given by:

$$G(s)=\begin{bmatrix}-W_aQ & W_aQ\\ -W_p(I-Q\tilde{P}) & W_p(I-Q\tilde{P})\end{bmatrix}=:\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix}.$$

The LFT form of the system is $e_p=\mathcal{F}_U\{G,\Delta_a\}r$, where $\mathcal{F}_U\{G,\Delta_a\}:=G_{22}+G_{21}\Delta_a(I-G_{11}\Delta_a)^{-1}G_{12}$. The Main Loop Theorem says that a robust performance problem is equivalent to a robust stability problem with an augmented structured uncertainty $\Delta_s\in\Gamma$ containing a fictitious perturbation Δ_p connecting e_p to r, see Fig. 3. The robust performance condition is satisfied iff $\sup_{\omega} \mu_\Gamma[G(j\omega)] \le 1$. This is satisfied iff, for each frequency [1],

$$\mu_\Gamma[G] = \inf_{d_\omega \in \mathbb{R}} \bar{\sigma}\left(\begin{bmatrix} \|G_{11}\| & d_\omega \|G_{12}\| \\ \frac{1}{d_\omega}\|G_{21}\| & \|G_{22}\| \end{bmatrix}\right) \le 1 .$$

The "two-block μ" has the following upper bound:

$$\mu_\Gamma[G] \le \sqrt{\|G_{11}\|^2 + \|G_{22}\|^2 + 2\|G_{12}\|\|G_{21}\|^2} = \|W_a Q\| + \|W_p(I - Q\tilde{P})\|$$

Thus, a sufficient condition for robust performance is as follows:

$$\mu_\Gamma[G] \le \|W_a Q\| + \|W_p(I - Q\tilde{P})\| \le 1, \ \forall\omega .$$

This sufficient condition is actually also necessary for an SISO plant, i.e., the upper bound on $\mu_\Gamma[G(j\omega)]$ is tight [1].

Since we want to keep the IMC filter Q tunable, there are two questions that arise: Given the weighting functions, what is the optimal Q that would minimize the upper bound on $\mu_\Gamma[G]$? And, what is the range and best "direction" of tuning for Q so that the upper bound remains less than one?

Suppose that the plant is square, $n \times n$. To answer the first question, a minimization problem is set up frequency by frequency:

$$\min_{\|Q\|<|W_u|^{-1}, Q \in \mathbb{C}^{n \times n}} \|W_a Q\| + \|W_p(I - Q\tilde{P})\| .$$

3. Tuning for minimum-phase SISO plants

In this section, we provide answers to the two questions posed above for minimum-phase SISO plants. The above minimization problem can be reformulated as follows:

$$F = \min_{q_1 \in \mathbb{C}, |q_1| \le \gamma^{-1}} \{\gamma|q_1| + \beta|1 - q_1|\} = \gamma \min_{q_1 \in \mathbb{C}, |q_1| \le \gamma^{-1}} \{|q_1| + \beta\gamma^{-1}|1 - q_1|\}$$

where $q_1 := q\tilde{p}^{-1} \in \mathbb{C}$, $\gamma := |w_a||\tilde{p}|^{-1}$, $\beta = |w_p|$. The quantity $|q_1| + \beta\gamma^{-1}|1 - q_1|$ is best viewed as the sum of lengths of vectors q_1 and $\beta\gamma^{-1}(1 - q_1)$. in the complex plane. The vector $\beta\gamma^{-1}(1 - q_1)$ starts from the tip of vector q_1 and passes through the point 1, as shown in Figure 4. It is clear that the minimum sum of vector lengths is obtained when the two vectors are aligned on the positive real axis. This means that the best direction for tuning the IMC filter q is along the inverse of the nominal plant.

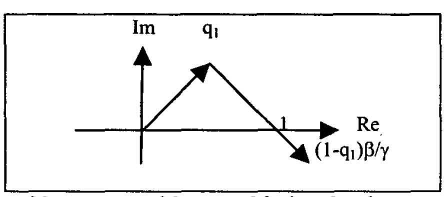

Figure 4: Minimization of the sum of the lengths of two vectors

Case $\gamma = \beta$: Optimal $\hat{q}_1 \in [0, \min\{1, \gamma^{-1}\}] \subset \mathbb{R}$, and $F = \gamma$.

Case $\beta > \gamma$: $\hat{q}_1 = \min\{1, \gamma^{-1}\}$, $F = \min\{\gamma, 1 + \beta(1 - \gamma^{-1})\}$.

Case $\beta < \gamma$: $\hat{q}_1 = 0$, for which $F = \beta$.

We now use these results to find the optimal robust performance level and optimal IMC filter. Referring back to Figure 2, $T_{ru} := r \mapsto u = w_u q(1 + q w_a \Delta_a)^{-1}$. The actuator constraint $|T_{ru}| \le 1$ at frequency ω can be expressed as: $|qw_u| \le |(1 + qw_a\Delta_a)^{-1}|$, $\forall|\Delta_a| < 1$. Note that normally $\gamma = |w_a||\tilde{p}|^{-1} < 1$, and from the above result, $|qw_a| = |q_1\tilde{p}^{-1}w_a| = |q_1\gamma| < 1$. Then, the tightest constraint is: $|qw_u| \le 1 - |qw_a|$. Therefore, the resulting constraint on the IMC filter is given by $|q| \le (|w_u| + |w_a|)^{-1}$.

Theorem 1:

$$\mu_\Gamma[G] = \min_{q_1 \in \mathbb{C}, |q_1| \le |\tilde{p}|(|w_u|+|w_a|)^{-1}} \underbrace{|w_a\tilde{p}^{-1}|}_{\gamma}|q_1| + \underbrace{|w_p|}_{\beta}|1 - q_1| =$$

$$\begin{cases} \left\{\begin{array}{l} \min\left\{1, \frac{|\tilde{p}|}{|w_u|+|w_a|}\right\}\left|\frac{w_a}{\tilde{p}}\right| \\ +\left(1 - \min\left\{1, \frac{|\tilde{p}|}{|w_u|+|w_a|}\right\}\right)|w_p| \end{array}\right. , & \text{for } |w_p| > \left|\frac{w_a}{\tilde{p}}\right|, \text{ and } \begin{array}{l} \hat{q} = \hat{q}_1\tilde{p}^{-1} \\ = \min\left\{1, \frac{|\tilde{p}|}{|w_u|+|w_a|}\right\}\tilde{p}^{-1}. \end{array} \\ |w_p|, & \text{for } |w_p| < \left|\frac{w_a}{\tilde{p}}\right|, \text{ and } \quad \hat{q} = \hat{q}_1\tilde{p}^{-1} = 0. \\ |w_p| = \left|\frac{w_a}{\tilde{p}}\right|, & \text{for } |w_p| = \left|\frac{w_a}{\tilde{p}}\right|, \text{ and } \begin{cases} \hat{q} = \hat{q}_1\tilde{p}^{-1}, \\ \hat{q}_1 \in \left[0, \frac{|\tilde{p}|}{|w_u|+|w_a|}\right] \end{cases} \end{cases}$$

Where $\hat{q}$ is the optimal IMC filter that minimizes $\mu_\Gamma[G]$ while satisfying the actuator constraint.

Remarks:

1. The quantity $|w_a||\tilde{p}|^{-1}$ is the relative size of the additive uncertainty with respect to the nominal plant;
2. When the performance weight exceeds the relative size of the uncertainty, $\mu_\Gamma[G]$ cannot go below the latter;
3. When the performance weight is smaller than the relative size of the uncertainty, $\mu_\Gamma[G]$ is limited by the former.

4. Conclusion

Theorem 1 allows us to draw some conclusions as guidelines for controller design and on-line tuning:

- The most favorable direction of the IMC filter q for improving robust performance is along $\tilde{p}^{-1}$.
- If the uncertainty level $|w_a|$ and/or the control weight $|w_u|$ is increased, q_1 has to decrease for the best tradeoff.
- The robust performance level degrades linearly, i.e., $\mu_\Gamma[G]$ increases, with an increase in $|w_p|$.

References

[1] K. Zhou and J.C. Doyle(1998)., *"Essentials of Robust Control".* Prentice Hall, Upper Saddle River, New Jersey.

[2] M. Morari and E. Zafirious (1989)., *"Robust Process Control".* PTR Prentice Hall, Englewood Cliffs, New Jersey.

[3] B. Zhu, H. S. Lee, L. Guo and M.Tomizuka. "Robust tuning of Fixed-Structure Controller for Disk Drives Using statistical Model and MOGA". *Proc. 2001 American Control Conference*, pp. 2773~2778. June 2001.