

## HAPTIC SYNTHESIS

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Abstract: This paper discusses a set of algorithms to reconstruct interaction forces between objects in a physically accurate manner. They must be fast enough to minimize the creation of spurious energy resulting from the discrete-time realization of displacement-to-force relationships. The most fundamental is an algorithm to compute the force of friction. Another algorithm is then described for sharp cutting, a close cousin of friction because of its dissipative nature. Synthesis of the nonlinear deformation response of arbitrary bodies is then considered. Textural effect are discussed in terms of small perturbations to the nominal signal.  
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### 1. INTRODUCTION

The phrase “haptic rendering” was introduced by Salisbury *et al.* (1995) to designate a set of “algorithms for generating the force of interaction with virtual objects”. In this seminal paper many of the key issues associated with the implementation of virtual mechanical environments were first described. Here, I would like to comment on the concept of “haptic synthesis”, a set of algorithms designed to reduce the amount of online computations to a small and predictable amount, and yet able to synthesize signals which are physically accurate. The desire for a fixed, reduced amount of computation isn’t primarily motivated by the limitations of today’s microprocessors, but rather by basic facts about the physics of mechanical interaction between the macroscopic objects of interest in virtual reality simulations.

Long ago it was noticed that when simulating an elastic element with a haptic device where the manipulandum position is measured and the

returned force is commanded, the interaction has a tendency to either break into a limit cycle or to diverge when the stiffness of the virtual interaction exceeds a given value. A limit cycle may occur if there are nonlinear elements in the system, and divergence if the system is reasonably linear. Colgate and Schenkel (1994) attributed this to the delay introduced by the sampling of the virtual environment. By elegant application of the small gain theorem, they found a condition for passivity:  $\mathcal{B} > (\sigma\mathcal{T}/2) + b$ . In this expression,  $\mathcal{B}$  is the device viscous damping,  $\mathcal{T}$  the delay equated to one sample period, and  $\sigma$ ,  $b$  are the simulated stiffness and damping coefficients respectively. They concluded that achievable damping is not dependent on the sampling rate, nevertheless achievable stiffness is.

A commonly adopted approach to deal with this problem is the “virtual coupling” method described by Adams and Hannaford (1999) that limits the interaction impedance to an achievable value. Other approaches include deadbeat control ideas (Gillespie and Cutkosky, 1996) or predictive-sample-hold (Ellis *et al.*, 1997), methods which invariably increase the complexity and the amount of computations required from sample to sample.

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Suppose that the virtual environment to be simulated is a spring deflected by  $d$ . We may view sampling and reconstruction as a form of generative hysteresis where the force response of the computer simulation lags behind displacement. For a zero-order hold, we can evaluate the energy gained from sample to sample as the area described by the force trajectory branching off from the displacement trajectory until they meet again after one sample period, see Figure 1, that is  $1/2 \Delta f \Delta d \approx 1/2 \sigma (\Delta d)^2$ .

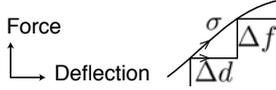


Fig. 1. Response Branching.

For energy to decrease at all times, the incremental potential energy gained by delaying the simulation of the spring by one period should be smaller than the energy lost in viscosity by the manipulum moving at average velocity  $v$  during the same period:  $\mathcal{B}v\Delta d \approx \mathcal{B}(\Delta d)^2/T$  which yields  $\mathcal{B} \gtrsim 1/2 \sigma T$ . This is equivalent to Colgate's expression. What is more, this reasoning does not require any particular assumption about the simulated environment so we can generalize this to  $\mathcal{B}(t) \gtrsim 1/2 \sigma(d, t)T$ .<sup>2</sup> In fact in (Mahvash and Hayward, 2005) a theorem is indicated that guarantees the existence of  $\mathcal{T}$  for the passive synthesis of a wide class of nonlinear, multidimensional virtual environments.

With haptic synthesis, the objective is to minimize the creation of spurious energy by increasing the sampling rate as much as required by the device used to produce force and read position. Of course, one special case is when the virtual environment is passive to start with, but it is also possible to consider environments which are not. In any case, what is needed is reduced complexity of the calculations in the closed loop. In the rest of this paper, we will discuss how a number of basic mechanical interactions can be synthesized at little cost. For consistency the notation may differ substantially from that used originally.

## 2. FRICTION

In its most basic aspect, friction relates a displacement to a force that tends to oppose it and has at least two distinct states: sticking or slipping. There are velocity dependent effects such as lubrication related effects (Armstrong-Hélouvy *et*

<sup>2</sup> Many similar conditions can be found depending on the assumptions made. For example in Bonneton (1994) approximating  $e^{-Ts}$  using the Padé approximation, it was found that conditions for stability were  $\mathcal{B} < b + 2M/T$  and  $\sigma T < b + \mathcal{B}$  among others.

*al.*, 1994), but these can be ignored. The relation between displacement and force, up to a factor, can be written in differential form using the original Dahl's (1976) model:

$$\frac{dd}{dp} = 1 - \zeta \operatorname{sgn}(dp) d. \quad (1)$$

This expression is particularly suitable for haptic synthesis since, once Eq. (1) is discretized, for each measured displacement  $\bar{p}$  it is easy to find an updated  $d$ . The "time free" governing dynamics makes it explicit that velocity is not required and, like real friction, gives a well defined value even if velocity is zero (Hayward and Armstrong, 2000). The state  $d$  represents an actual physical quantity: the elastic tangential deflection seen in any real contact. The tangential friction force is then a function of  $d$ , say proportionally to the normal force and to a coefficient  $\mu$  which embodies the properties of a contact (contact geometry, materials and other considerations). That the normal force also results from a deflection will allow us to realize haptic synthesis in general cases without ever to have to worry about interaction forces, as further discussed in Section 4.

However, in the course of implementation we realized that this model gave an unphysical behavior: small movements caused the simulated contact to drift, that is, some bounded inputs under the breakaway threshold gave unbounded net displacement (Hayward and Armstrong, 2000). In fact, Dahl's model does not admit a sticking phase as commented in (Dupont *et al.*, 2000). An improved model that retains much of the original simplicity is written:

$$\frac{dd}{dp} = 1 - \zeta(d) \operatorname{sgn}(dp) d, \quad (2)$$

where  $\zeta(d)$  now is a function that governs the transition from stick to slip according to the deflection. Referring to Figure 2, if  $\zeta(d) = 0$  for a range of values, then  $dd = dp$  and hence the contact is stuck. For any other case there will be a mix of elasticity (stick) and plasticity (slip).

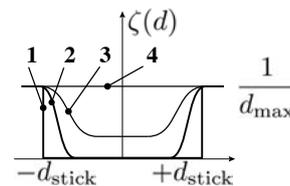


Fig. 2. Adhesion functions. 1) Adequate for haptics. 2) Better for control of machines (Dupont *et al.*, 2002). 3) Arbitrary mix of elasticity and plasticity. For 1, 2, and 3, we normally select  $d_{\max} = d_{\text{stick}}$ . If  $d_{\max} < d_{\text{stick}}$  additional solutions arise. 4) Dahl: an equal mix of elasticity and plasticity.

This model has many interesting properties but for haptic synthesis, an attractive feature is that it is easy to specify a vectorial extension and to compute its numerical solution. Using boldface notation to designate vectorial quantities, calling  $\bar{\mathbf{p}}_k$  the measured position of the manipulandum,  $\mathbf{d}_k$  the elastic component of the total displacement, and  $\mathbf{c}_k$  the plastic component, the online solution is

$$\mathbf{c}_k = \begin{cases} \bar{\mathbf{p}}_k - \frac{\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}}{|\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}|} d_{\max}, & \text{if } |\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}| > d_{\max}; \\ \mathbf{c}_{k-1}, & \text{otherwise,} \end{cases} \quad (3)$$

$$\mathbf{d}_k = \bar{\mathbf{p}}_k - \mathbf{c}_k$$

for the simplest version of  $\zeta(d)$ , the adhesion function 1 in Figure 2. Figure 3 illustrates this computation graphically.

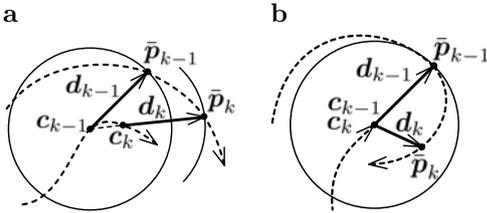


Fig. 3. a) Sliding state. b) Sticking state.

For any adhesion function, the solution can be found by Euler integration:

$$\mathbf{c}_k = \begin{cases} \bar{\mathbf{p}}_k - \frac{\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}}{|\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}|} d_{\max}, & \text{if } \zeta(\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}) |\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}| > 1; \\ \mathbf{c}_{k-1} + \frac{|\bar{\mathbf{p}}_k - \bar{\mathbf{p}}_{k-1}| \zeta(\bar{\mathbf{p}}_k - \mathbf{c}_{k-1}) (\bar{\mathbf{p}}_k - \mathbf{c}_{k-1})}{|\bar{\mathbf{p}}_k - \bar{\mathbf{p}}_{k-1}|}, & \text{otherwise.} \end{cases} \quad (4)$$

The solution can also be visualized by plotting the vector  $\mathbf{d}$  while tracing a trajectory with  $\mathbf{p}$  as input, see Figure 4.

From the perspective of haptic synthesis, this makes it clear that the simulation of realistic friction can be a considerable challenge since the characteristic distance  $d_{\max}$ —the ‘presliding’ distance—is measured in micrometers for hard objects. The resolution of the haptic device should be significantly higher than this number to simulate hard contact. The second challenge is related to the passivity of the simulation. While during sliding, the model is dissipative by construction, in the stick phase it is purely elastic. Of course, one might think of adding viscosity, but from the introductory discussion, we know that this approach has only limited value. To fix ideas, let’s assume that  $d_{\max} = 10^{-5}$  m and that the tangential sliding force is 1 N, thus the contact’s  $\sigma$  is  $10^5$  N/m. Therefore, viscosity, real or virtual,

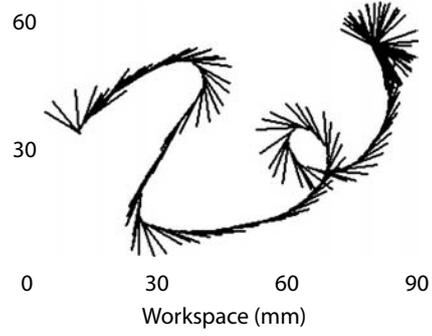


Fig. 4. Vector friction  $\mathbf{d}$  plotted with its origin at  $\mathbf{p}$ . Multiplying by a negative factor proportional to the normal force gives a friction force. The trajectory terminates at the upper right corner in a stuck state where  $\mathbf{c}$  is invariant, yet  $\mathbf{d}$  exists.

for a sampling frequency of  $10^4$  Hz should be of the order of  $\sigma T = 10$  N·s/m, a large value indeed. This limits how small  $d_{\max}$  can be for a given device.

### 3. DAMAGE

For haptic synthesis, damage is defined as the simulation of the creation of new surfaces in a solid. This may have many forms but we first looked at sharp cutting, basing our model, like that of friction, on basic physical properties (Mahvash and Hayward, 2001). Fracture mechanics indicates that the creation of new surfaces corresponds to the irreversible dissipation of energy proportionally to the area of a crack extension. Cutting is also preceded with storage of elastic energy. In that, it is quite similar to friction. Referring to Figure 5, consider an infinitesimal section of a solid of width  $dl$  cut by a sharp blade. As the blade moves by  $\Delta d^z$ , the crack surface is increased by  $\Delta s$  while the crack length extends from  $c$  to  $c + \Delta c$ . If the solid deforms, the solid element surrounding the crack changes from shape  $R_s$  to shape  $R_{s+\Delta s}$ . In the course of a complete cut, our model predicts a number of distinct events.

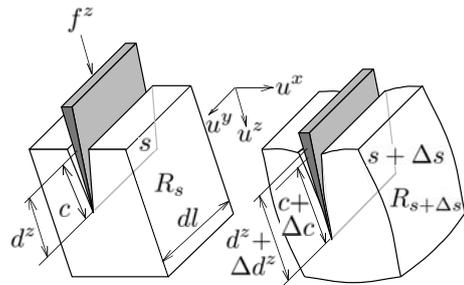


Fig. 5. Quantities defined for sharp cutting. A blade moves in an elementary block of width  $dl$  with a force  $f^z$ .

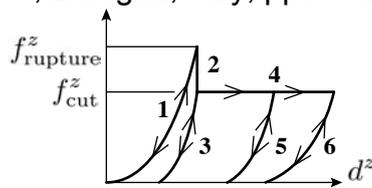


Fig. 6. Possible response branches.

These events can be described by reference to Figure 6. As the blade first touches the object, deformation occurs and the response follows path **1** where elastic energy is stored. Deflection continues until the cutting force  $f_{rupture}^z$  is sufficient to initiate a crack. Almost instantly, the stored energy is released, **2**, to create a crack whose size can be deduced from the energy stored during initial loading and from the fracture toughness of the material,  $J_c$ . If the blade retreats, the response follows another unloading curve **3**, owing to the existence of the crack. If the blade moves forward, sharp cutting occurs. The cutting force  $f_{cut}^z$  along **4** can be found from  $J_c$ , the movement of the blade and the width of the cut. If at any moment, the blade retreats as in **5** or **6** a new unloading/loading curve is created.

In all cases the force response can be determined from energy conservation considerations involving the work lost in extending a crack,  $J_c a(\Delta s)$ , as well as the work made by the moving blade,  $f^z \Delta d^z$  (Mahvash and Hayward, 2001).

Experiments carried out with liver and potato samples indicated good agreement between the model and experiments, see Figure 7. This was further applied to model cutting forces with scissors and other forms of cutting (Mahvash and Okamura, 2005; Mahvash, 2006).

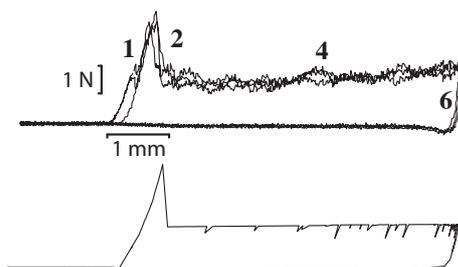


Fig. 7. Three overlaid responses from cutting a 20 mm wide potato prism with a sharp razor blade where the response branches are visible. Bottom panel: synthesized response.

#### 4. STATIC DEFORMATION

When a tool is used to interact with a body without causing damage, deformation occurs. Synthesizing the full, detailed response requires to account for the tool used, the body's shape, material, inhomogeneity, nonlinearity, small and large

deformations, support, and so on. These requirements seem to be in opposition with the fact that the fully detailed computational simulation of contact is a formidable computational problem. Experiments were carried out to highlight this (Mahvash and Hayward, 2002). Figure 8 shows a tool ready to indent a sample of liver well supported by a rigid plate. In this condition, the details of the contact mechanics dominate the response. Figure 9 shows the response for two different tools. Changing the tool size (same shape) by a factor 4 modifies the response by orders of magnitude for the same indentation.



Fig. 8. Testing a well supported sample of liver.

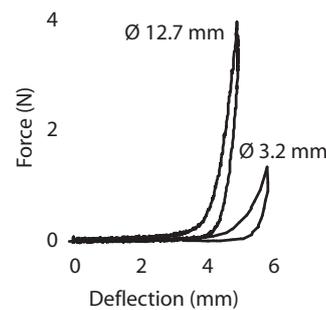


Fig. 9. Response to local deformation of biological tissue with two different tools. After a few millimeters of deflection the responses differ by orders of magnitude.

Similar significant differences would be observed in bodies which are homogenous or not, isotropic or not, whether deformation is small or large, local or global, etc. (Mahvash and Hayward, 2004). In this reference, we list four requirements for high-fidelity haptic simulation:

- (1) resemblance of virtual force responses with actual responses,
- (2) force continuity under all allowed maneuvers,
- (3) passivity of the virtual environment, and
- (4) high update rate.

Haptic synthesis techniques, however, allows one to account for the full complexity of mechanical interactions with deformable bodies while meeting these requirements. The basic observation is that when a given tool encounters a given body, no matter how complicated the interaction is, the subsequent response is entirely determined by the initial point of contact. If we consider that for a given tool each point of the surface determines a

different response—a vector function of a deflection vector—, then the entire response is nothing but a continuous field of functions. From physics, we know that each of these functions should be conservative and so must be the field. This observation allowed us to establish a synthesis method to reconstruct passively this field from a finite set of samples (Mahvash and Hayward, 2005).

Briefly, the method consists of interpolating a finite set of vector functions determined from first principles, from measurements, or from offline simulations. Referring to Figure 10, one approach is to store one function at each surface node of the synthesized body and interpolate a new response function for initial contact point  $c$  given a deflection  $d$ .

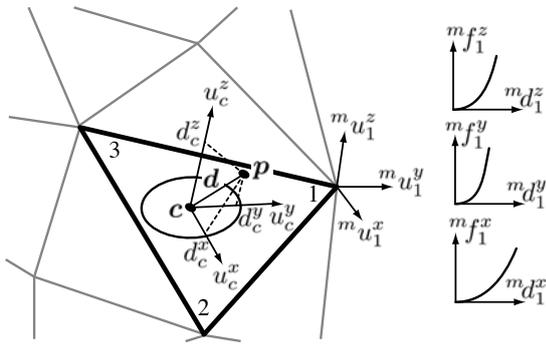


Fig. 10. Local response encoded as force deflection curves at each node. If the projection of point  $p$  is within a set bound, the contact is stuck.

Because these functions are nonlinear, the choice of coordinates is crucial and a new set must be interpolated at  $c$  from the coordinates used for each node. For the case indicated in Figure 10, there are three coordinates,  $\nu \in \{x, y, z\}$ . For any patch  $m$ , with the interpolation weights  ${}^m n_i(c)$  the interpolation formulae are:

$$u_c^\nu = \sum_{i=1}^3 {}^m n_i(c) {}^m u_i^\nu, \quad (5)$$

$$f_c^\nu(d^\nu) = \sum_{i=1}^3 {}^m n_i(c) {}^m f_i^\nu(d^\nu). \quad (6)$$

The synthesis of the nonlinear response is a simple process which can be decoupled from the other processes in a complete simulation system. In particular, interference detection which reduces to the determination of an ‘active patch’, can be performed asynchronously. The algorithmic details are in (Mahvash and Hayward, 2005). Moreover the storage required for many cases of practical interest is quite modest owing to necessity to store data proportionally to the surface of the body, but not to its volume. Now, if the interaction has a lateral component, then slip can occur and therefore the point  $c$  could be moving.

In Section 2, we developed a synthesis model for the dynamics of sliding contacts. Following this model, point  $c$  should move such that the projection of point  $p$  on the envelop of the undeformed body remains within bounded lateral deflections. We have seen earlier that for hard objects, this lateral deflection could be as small as a few micrometers, but for deformable bodies such as organs, it can be as large as centimeters. The basic phenomenon is nevertheless the same so the synthesis method outlined here can be viewed an extension of the simple model of Section 2, but accounting for shape, normal deflection, tool and material properties. It is also possible to synthesize a difference response for different manners in which a tool can contact a body. If  $m$  is a patch on the body and  $j$  a specific response:

$$u_c^\nu = \sum_i {}^j n_i(c) \left( \sum_l {}^m n_l(c) {}^j f_{il}^\nu \right), \quad (7)$$

$$f_c^\nu(d^\nu) = \sum_i {}^j n_i(c) \left( \sum_l {}^m n_l(c) {}^j f_{il}^\nu(d^\nu) \right) \quad (8)$$

The techniques described up to now can be combined in a unified framework for the haptic synthesis of a wide range of effects (Mahvash, 2006).

## 5. TEXTURE

Texture refers to small-scale modifications of mechanical interaction response during scanning. In Champion and Hayward (2005) we observed that textural synthesis could be viewed as a small oscillatory component superposed to a low frequency nominal response component, see Figure 11. This small oscillatory component can be combined with any synthesized signal, for example, adding it to the synthesized response of Figure 7 would increase realism. Thus, texture synthesis is amenable to ‘small signal analysis’. Using the analogy between scanning a texture and a wave traveling at a variable speed, we used the Nyquist and the Courant conditions to derive relationships that state the conditions under which a texture can possibly be synthesized by a haptic device—a mechanical system which no longer should be approximated by a rigid body.

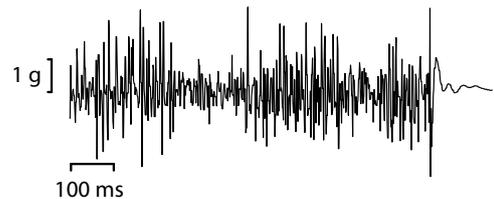


Fig. 11. Acceleration of a stylus dragged on a wooden surface.

6. CONCLUSION

Haptic synthesis bears some analogy with realtime audio synthesis where a computational loop must be able to reconstruct physically and perceptually relevant aspects of the original signal. What our experience has shown is that in many cases, unlike the case of audio synthesis, the limits due to the performance characteristics of currently available devices far exceed the limits due to computation (Hayward and Astley, 1996).

This state of affairs calls for new approaches in the design of devices, e.g. (Harwin and Wall, 1999; Gosline *et al.*, 2006) among others, with significantly improved performance characteristics that can take full advantage of the currently available computational techniques of haptic synthesis in addition to those presently under development. In our laboratory, these are specifically targeted at accurately synthesizing dynamics effects such as impact, viscosity and others.

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