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Mechanical Analogies in Hybrid Position/Force Control

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Abstract *It is shown that in hybrid position/force control as well as in multi-robot cooperation and teleoperation, the desired behavior of the system can be defined in terms of a massless mechanism whose joints act as ideal position or force sources. Based on this remark, a simple and robust controller is proposed. Experiments validate the approach.*

1 Introduction

Many robotic tasks require mechanical interaction between a workpiece or tool and the environment, for example, in low clearance assembly, grinding, cutting, deburring, and so-forth. In this type of task, the motion of the manipulator is partially constrained and the interaction forces must be controlled. Hybrid position/force control (or simply hybrid control) has been proposed as an approach for simultaneously regulating some position and some force components [3, 6, 10, 15, 20]. Similar issues arise in in teleoperation with force feedback and in multi-robot cooperation.

The content of this paper is part of a research effort carried out at the Robotics and Teleoperation Department of the French Atomic Energy Commission to develop more performant, reliable, and user-friendly teleoperation systems for nuclear applications.

We propose an approach to hybrid control based on mechanical analogies. The analogies are used for producing specification as well as for control synthesis. We first show how, in hybrid control, the desired behavior of the robot can be described by an ideal mechanism (Part 2), and we generalize this idea to teleoperation and multi-robot cooperation (Part 3). Next, the notion of virtual mechanism is introduced (Part 4) and a simple control law is proposed (Part 5). Experimental results support the generality and the effectiveness of the approach (Part 6).

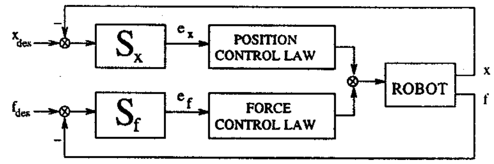

Figure 1: General structure of hybrid controllers.

2 Hybrid tasks specification

In this section, we show how most approaches to hybrid control consist in imposing to the robot a behavior that can be described by a mechanism.

The basic structure of most hybrid controllers is that of the original scheme [15] (Fig. 1). Coordinates transformations are not shown on the block diagram. The variables x and v denote the respective cartesian position and velocity of the robot, and f is the Cartesian force exerted on the environment. We assume that the same coordinates are used for v and f, so that $f^t v$ is the power supplied by the robot to its environment. (Joint coordinates could also be used [17]).

We now focus on the projection operators S_x and S_f, and show how they define the desired robot behavior. Three approaches are reviewed.

- If x and f are expressed in a specific coordinate frame, called *task frame* [2], S_x and S_f can be chosen as diagonal matrices, with 0 and 1 as diagonal elements. They are called *selection matrices*, and they satisfy $S_x + S_f = Id$.

- In more recent approaches, S_x and S_f are methodically deduced from the kinematic constraint imposed by the environment. Let A and B be two full column rank matrices spanning the twist and wrench spaces of the constraint. They satisfy $A^t B = 0$.

Weighted pseudo-inverses were proposed in place of the selection matrices [1, 4, 5, 13, 14, 11]. Then S_x

and S_f are given by

$$S_x = (A^t\psi A)^{-1} A^t\psi \quad (1)$$
$$S_f = (B^t\psi^{-1}B)^{-1} B^t\psi^{-1} \quad (2)$$

where ψ is a symmetric matrix, usually positive definite (excepted in [13] where ψ is symmetric, but not positive).

- The analysis in [16] leads to the following choice:

$$S_x = AD^t \quad (3)$$
$$S_f = BC^t \quad (4)$$

where C and D are full column rank matrices such that

$$rank([AC]) = 6 \quad (5)$$
$$rank([DB]) = 6 \quad (6)$$

and

$$[AC]^t [DB] = Id_6. \quad (7)$$

It is easily verified that with any of the cited approaches, S_x and S_f always satisfy

$$rank(S_x) + rank(S_f) = 6 \quad \text{and} \quad S_x^t S_f = 0. \quad (8)$$

The control laws are designed to drive the position and force errors e_x and e_f to zero. For ideal controllers, the behavior of the robot is defined by

$$S_x(v_{des} - v) = 0, \quad (9)$$
$$S_f(f_{des} - f) = 0. \quad (10)$$

The velocity constraint is the only one considered here. The position constraint is satisfied as a consequence.

We now give a physical interpretation of equations (8, 9, 10). Consider the case illustrated in Fig. 2. Two massless rigid bodies, numbered 1 and 2, are connected by an ideal kinematic constraint. Let T and W be full column rank matrices spanning the twist and wrench spaces of the constraint. They satisfy

$$rank(W) + rank(T) = 6 \quad \text{and} \quad W^tT = 0. \quad (11)$$

Call v_{des} and v the respective velocities of bodies 1 and 2. Their relative velocity is in $Span(T)$, so

$$v_{des} - v = T\xi \quad (12)$$

where ξ is a free vector. Premultiplying by W^t gives, using (11),

$$W^t(v_{des} - v) = 0. \quad (13)$$

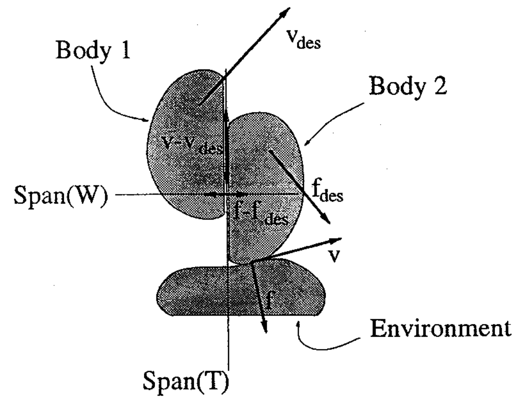

Figure 2: Mechanical interpretation of hybrid control.

Assume body 2 is in contact with any environment. Let f be the force exerted by body 2 on the environment. Apply an external force f_{des} on body 2. Since body 2 is massless, the equilibrium of forces is given by

$$f_{des} + f_r - f = 0 \quad (14)$$

where f_r is the reaction of body 1 on body 2. Since f_r is in $Span(W)$, premultiplying (14) by T^t gives

$$T^t(f_{des} - f) = 0. \quad (15)$$

Similarity between equations (8, 9, 10) and (11, 13, 15) leads to the conclusion that with hybrid control, the desired behavior of the robot end-effector is that of a massless rigid body subjected to an external force f_{des}, and connected through an ideal kinematic constraint to another body whose velocity is v_{des}.

3 Multi-Arm systems

In this section, we show by simple examples that in multi-arm cooperation and teleoperation, the ideal behavior of the system can be defined by equations similar to (13, 15).

3.1 Multi-arm cooperation

Consider the case of two coaxial one-DOF robots sharing a single load (Fig. 3). Intuitively, the control strategy might involve the specification of the load velocity and a desired internal force. This can be written as follows:

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \text{internal force}, \quad (16)$$
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \text{load velocity}. \quad (17)$$

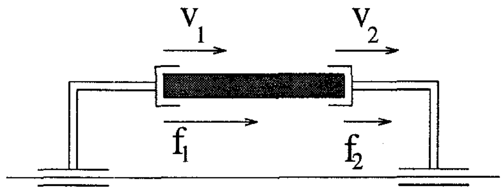

Figure 3: Two cooperating 1-DOF robots.

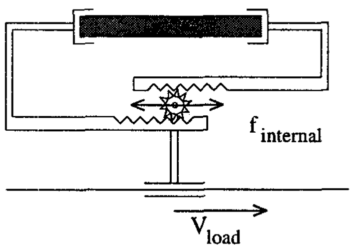

Figure 4: Mechanism describing the ideal behavior of the cooperating robots.

The desired behavior for the robots is that of the mechanism represented on Fig. 4. An approach to multi-robot cooperation, reported in [18], where internal forces are specified by means of "virtual linkages" may now be viewed as a particular case of the foregoing analysis.

3.2 Teleoperation

The ideal behavior of a teleoperation system is defined by the following equations [19]:

$$\begin{bmatrix} Id & Id \end{bmatrix} \begin{bmatrix} f_m \\ f_s \end{bmatrix} = 0, \tag{18}$$

$$\begin{bmatrix} Id & -Id \end{bmatrix} \begin{bmatrix} v_m \\ v_s \end{bmatrix} = 0. \tag{19}$$

where the subscripts m and s denote the master and slave arms respectively. The two arms should behave as ideally connected by an infinitely rigid massless mechanism. In [8, 9, 12], more complex command modes are defined using mechanical analogies.

4 The "virtual mechanism"

For some tasks, such as turning a crank, the desired behavior of the system cannot be defined using *fixed* matrices W and T. Position and force-controlled directions vary during the task execution. To deal with constraint nonlinearity, we now generalize the mechanical analogy to the nonlinear case. If the robotic system consists of several robots, x, v and f denote the concatenated vectors of Cartesian positions, velocities and forces respectively.

The desired behavior of the system is now that of a nonlinear massless mechanism, termed *virtual mechanism*, specifically designed to perform the task. On its joints, either position or force is imposed. The goal of this section is to translate this idea into mathematical statements.

Let q_{vm} be the VM joint coordinates, and τ_{vm} the associated force vector. These vectors can be partitionned and reassembled to separate position- and force-controlled joints:

$$q_{vm}^t = \left[q_p^t; q_f^t\right] \quad \text{and} \quad \tau_{vm}^t = \left[\tau_p^t; \tau_f^t\right]. \tag{20}$$

Let x_{vm} be the VM Cartesian coordinates (or a concatenated vector of positions corresponding to the several arms). We have

$$x_{vm} = K_{vm}\left(q_p, q_f\right) \tag{21}$$

where K_{vm} is the VM forward kinematics. The associated Jacobian J_{vm} can be partitionned and reassembled into position and force Jacobians. Then the Cartesian velocity of the virtual mechanism is

$$v_{vm} = J_p \dot{q}_p + J_f \dot{q}_f. \tag{22}$$

For the position-controlled joints, the position setpoint is a user-defined function of time:

$$q_p = q_{p,des}(t). \tag{23}$$

Similarly, force setpoints are applied to force-controlled joints:

$$\tau_f = \tau_{f,des}(t). \tag{24}$$

The force f_{vm} exerted by the VM satisfies

$$J_{vm}^t f_{vm} = \tau_{vm}. \tag{25}$$

Taking only the force-controlled components gives

$$J_f^t f_{vm} = \tau_f. \tag{26}$$

The robotic system ideal behavior is defined by

$$x = x_{vm} \tag{27}$$

$$f = f_{vm} \tag{28}$$

that is, using (21, 23, 24, 26)

$$x = K_{vm}\left(q_{p,des}, q_f\right) \tag{29}$$

$$J_f^t f = \tau_{f,des} \tag{30}$$

where q_f is a free vector. We assume that J_f is full column rank. Otherwise, (30) could have no solution.

Notice that (29) is the nonlinear analog of (12), and (30) corresponds to (15) with $\tau_{f,des} = T^t f_{des}$.

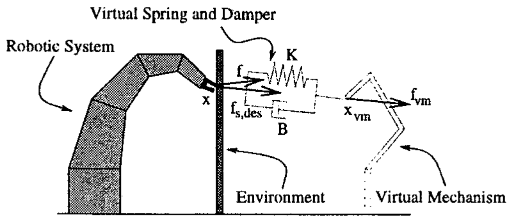

Figure 5: Physical analogy of the control law.

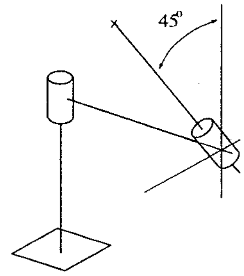

Figure 6: Model of the crank mechanism

5 Control

We propose a simple and robust control law to impose the virtual mechanism behavior to the robotic system. The following assumptions need to hold: The robotic system is composed of rigid bodies. Its joint positions and velocities are measured and transformed into Cartesian coordinates (x and v) using the robotic system forward kinematics and Jacobian. The joint forces are accurately controlled, either in open loop or in closed loop with force sensors. The input to the robotic system is then a Cartesian force $f_{rs,des}$, which is transformed into joint commands using the transposed Jacobian, ($\tau_{rs,des} = J_{rs}^t f_{rs,des}$). If gravity is not mechanically compensated for, a compensation term should be added to $\tau_{rs,des}$.

With the previous assumptions, imposing a command $\tau_{rs,des}$ is equivalent to applying the force $f_{rs,des}$ directly on the robotic system end-effector.

The control law is designed using the following analogy (Fig. 5). The robotic system end-effector(s) is (are) virtually connected to the VM by a spring and a damper (or springs and dampers). This corresponds to a proportionnal-derivative controller. The spring is chosen as stiff as possible, such that $x \approx x_{vm}$. In static situations, the dynamics of the robotic system are negligible so that $f \approx f_{vm}$.

The force $f_{rs,des}$ is given by

$$f_{rs,des} = K\left(x_{vm} - x\right) + B\left(v_{vm} - v\right) \qquad (31)$$

where K and B are symmetric positive definite matrices. The notation $(x_{vm} - x)$ may be abusive for rotations. More adequate angular error computation can be used without any restrictions.

In (31), x and v are measured, but x_{vm} and v_{vm} depend on q_f and $\dot{q}_f$, which are unknown. We now show that q_f follows a first order ordinary differential equation (ODE).

The force exerted by the VM on the spring-damper system is equal to $f_{rs,des}$, so

$$f_{vm} = K\left(x_{vm} - x\right) + B\left(v_{vm} - v\right). \qquad (32)$$

Using equations (22, 26, 32), we get

$$J_f^t B J_f \dot{q}_f = \tau_{f,des} + J_f^t f_{tmp} \qquad (33)$$

with

$$f_{tmp} \hat{=} K\left(x_{vm} - x\right) + B\left(J_p \dot{q}_p - v\right). \qquad (34)$$

The matrix $J_f^t B J_f$ is symmetric, positive, definite because J_f has full column rank and B is symmetric, positive, definite. So $J_f^t B J_f$ can be inverted in (33):

$$\dot{q}_f = \left(J_f^t B J_f\right)^{-1}\left(\tau_{f,des} + J_f^t f_{tmp}\right). \qquad (35)$$

This ODE should be integrated in real-time to compute $\dot{q}_f$ and q_f at each instant. Then, since q_p and $\dot{q}_p$ are known (23), x_{vm} and v_{vm} can be computed (21, 22). Finally, the driving force can be computed using (31).

Since the controller has a physical equivalent, it is passive. For a brief proof of this property, see [9] Since the robotic system is passive, the controlled system is passive. Then, stability is garanteed when the robotic systems interacts with any passive environment [7].

6 Experiments

6.1 Description of the task

This approach was first successfully applied to a teleoperation system at the Robotic and Teleoperation Department of the French Commission on Atomic Energy. Results are reported in [8, 9].

In this paper, other experiments carried out at McGill University are presented. The goal is to manipulate the two-DOF mechanism shown on Fig. 6. The two rotation axis are not parallel and do not intersect, so this task cannot be specified using selection matrices.

The controller has been implemented on a teleoperation system comprising two hydraulic seven-DOF anthropomorphic arms: the Sarcos Master Arm and the Sarcos Dextrous Arm.

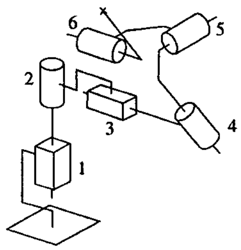

Figure 7: Virtual mechanism used in autonomous mode.

Both arms have torque sensors at the joints. An analog controller is used to compensate for the friction and nonlinearities of the actuators. The rest of the controller runs at 100 Hz on a C40 single computer board. Orientations were represented by Euler parameters (unitary quaternions). The orientation error computation is detailed in [8].

The redundancy was not fully exploited. On the slave arm, the third joint was servoed on a user-defined position. Then, only six joints were used to perform the task. On the master arm, only the end-point position was controlled. As a consequence, the internal motion was totally free. The operator was able to place her or his elbow in the most comfortable position to perform the task.

Two experiments were conducted. The first one consists in manipulating the mechanism with the slave arm in autonomous mode. In the second one, the operation is teleoperated with an assistance to control the efforts.

6.2 Autonomous mode

The VM used in autonomous mode is represented on Fig. 7. Joints 2 and 4 (corresponding to the joints of the real crank) are position-controlled. The other joints are force-controlled, with a desired force/torque equal to zero.

This specification enabled to turn the crank successfully. The system behavior was stable and smooth. Because of the force control, the task could be carried out even when the position of the crank was moved a few centimeters away from the initial position, without any need for parameter adjustments.

Moreover, the trajectory generation was very easy. The desired position on the two axis of the crank were entered on a keyboard and simply low-pass filtered before being input in the controller.

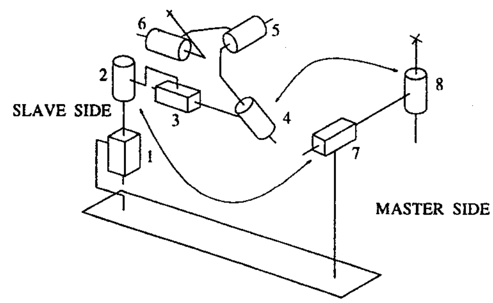

Figure 8: Virtual mechanism used in master-slave mode.

6.3 Master-Slave mode

In the master-slave mode, the VM (Fig. 8) is used to provide an assistance to the operator. In this case, it has two end-effectors: one for the slave and one for the master.

On the slave side, 4 effort components — the same as in the previous case — are automatically controlled, independently of the master side.

The master workspace is reduced to a two-dimension space. Only a horizontal translation and a rotation around a vertical axis are possible (VM joints 7 and 8). These two DOF are "mechanically coupled" to VM joints 2 and 4 (slave side). This is done by using the same parameters to describe the positions on VM joints 2-7 and 4-8. In fact, the resulting VM has only six DOF. Two of them affect both the master and slave positions. All the joints are force-controlled, with a null desired force.

The resulting behavior was completely satisfactory. When the operator moved the master arm along the horizontal line, the slave moved the first joint of the crank. When the master end-effector was turned around a vertical axis, the second joint of the crank was moved by the slave arm. Along these two motions, the operator had force feedback. If the crank was directly manipulated and the master arm was left free, then the master arm moved according to the motion of the crank.

Any attempt on the part of the operator to drive the arm away from its two-dof-workspace produced a repelling force. This force did not affect the force exerted on the crank by the slave arm.

7 Conclusion

It was first shown that all hybrid control approaches share the objective of imposing a behavior to a robot robot system, that can be desribed by a mechanism.

Two examples illustrated how this can be applied to multi-robot cooperation as well as teleoperation. The VM concept generalizes this idea in the nonlinear case. It can be viewed as a method for describing the desired behavior needed to accomplish a given task. A simple and robust control law was derived so that when applied, it caused the robotic system to exhibit the desired behavior. Due to mechanical analogy, passivity of the controller is ensured. This garanties the stability of the controlled system when it interacts with any passive environment. Finally, experimental results were presented to validate the theoretical conclusions.

In summary, the principal advantages of this approach can be listed as follows:

- The VM joint coordinates describe the task in a natural way. This simplifies trajectory generation.

- The mechanical equivalence clearly shows how the system will react to geometrical uncertainties. This is not the case for approaches using pseudo-inverses.

- VM nonlinearity provides a means to deal with the constraint nonlinearity. It is specially effective in teleoperated tasks were the trajectory in not known in advance.

- Physical equivalence garanties control robustness.

- The VM concept applies to hybrid control, teleoperation and multi-robot cooperation whithin a unified framework.

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