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On the Linear Compensation of Hysteresis

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Abstract

Compensation for hysteresis often relies on a precise system model. This makes controller design complicated and time consuming. In this paper, the Preisach hysteresis model is interpreted in terms of phase shift. This leads to a simple linear compensator design methodology. The closed-loop connection of a compensator that we call a *phaser* with a nonlinear system with hysteresis is shown to produce an almost linear response in a given operating range. The present method is advantageous for its simplicity and robustness and requires the identification of only one parameter. This result is experimentally applied to the control of a piezoceramic actuator.

1 Introduction

The synthesis of controllers for nonlinear plants typically depends on a precise model. In the absence of such a model, an identification procedure is needed to obtain it. The motivation of the present work is to offer a method to design simple, yet effective, controllers for nonlinear systems with hysteresis using as little information as possible. In the present work, hysteresis in systems that can be identified by the classical Preisach model [2]—a geometric model—is considered. This model is by no means general, but is a convenient tool to analyze the behavior of such a system.

2 Model Formulation

There is little agreement in the literature for a definition of hysteresis, but for the purposes of this paper we take hysteresis to result from memory combined with rate independence [2].

2.1 The Model

It is important to remember that hysteresis is always part of a more complicated system [2]. This means that in practice, the input to the hysteresis model may not be accessible, and that the output may be hidden in the same way. The system will have the form:

$$y = \mathbf{L}[u] + \hat{\mathbf{\Gamma}}[u] \quad (1)$$

where $\mathbf{L}$ is the representation for a linear time invariant system, and $\hat{\mathbf{\Gamma}}$ represents the hysteresis of the system, e.g. the Preisach model [2]. This model can be seen as the superposition of weighted simplest hysteresis (relay) operators, $\hat{\gamma}_{\alpha\beta}[u(t)]$, represented through a double integral over all possible input values:

$$y(t) = \hat{\Gamma}[u(t)] = \int\int_{\alpha \geq \beta} \mu(\alpha, \beta)\hat{\gamma}_{\alpha\beta}[u(t)]d\alpha d\beta \quad (2)$$

As shown in section 4, piezoceramics exhibits rate independent hysteresis so its effect is the same at any frequency and the linear part acts only in the high frequency range.

3 Controller Synthesis and Analysis

A more detailed explanation of the controller synthesis and analysis can be found in [1]. Assume that the input varies periodically between two values. It is then possible to talk about phase shift between input and output. The Preisach model predicts that a loop is produced in the input-output graph. The loop and the output will have the same period as the input, and the loop can be considered as a phase lag (with angle ϕ) between input and output. We use the fact that the phase angle ϕ described above is independent of rate. Consider the following:

Definition. A *phaser* ($\mathbf{L}_{pa}$) is a frequency-domain operator that shifts a periodic input signal by a constant angle $\lambda \geq 0$ and leaves it with the same magnitude independent from frequency or magnitude, i.e. $|\mathbf{L}_{pa}(u)| = 1$.

By Fourier series expansion, all periodic signals can be approximated by combination of sinusoid signals with frequencies $\frac{1}{T}, \frac{2}{T}, \ldots, \frac{k}{T}$ where $\frac{1}{T}$ is the fundamental frequency. Because of rate independence, all harmonics are shifted the same way and all have to be corrected the same way. The system being nonlinear, the correction cannot be perfect, so an error term e (distortion) is formed for correction by feedback.

A *phaser* is an ideal system that cannot be implemented because no causal physical system can provide a constant phase shift over the infinite frequency range. Nevertheless, the phaser can be approximated by a linear filter. Since we are dealing with a nonlinear system, we will consider only the fundamental component of the output (as in the describing function analysis), without affecting the result as discussed above.

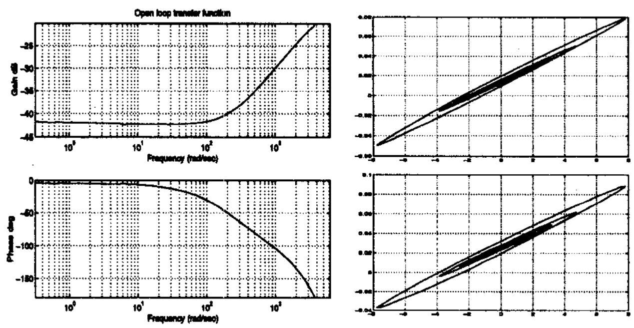
For a system with hysteresis the phase angle Bode plot is nonzero or near a multiple of $\frac{\pi}{2}$ deg. The phase angle ϕ is the difference between the nearest multiple of $\frac{\pi}{2}$ to the quasistatic phase angle, and the quasistatic phase angle itself. Thus, the design method is as follows:

Algorithm
1. Obtain the experimental transfer function estimate of the system.
2. Find the angle ϕ.
3. Select a frequency range where the *phaser* is needed.
4. Design the linear controller (series of leads controllers) making the magnitude as constant as possible over the bandwidth selected in 3.

The wider the frequency range, the greater the magnitude "distortion" given by the approximation to a *phaser* is produced. Also the controller should be connected in closed loop to minimize the distortion of the output signal.

4 Linear Control of a Piezoceramic Actuator

In this section, the control strategy of section 3 is applied to a piezoceramic actuator. The hysteresis characteristic of the system is shown in Figure 1.



a) b)

Figure 1: a) Empirical open loop Bode plot (top: amplitude, bottom: phase), b) Branching in open loop. Abscisa: input voltage, ordinate: output displacement (top: 0.3 rad/s, bottom: 6.14 rad/s)

The phase plot is shifted down over the whole frequency range by approximately 5 degrees. This angle is used to design a *phaser*. We chose the range to be from 1 to 10 rad/sec. A 6th order lead approximation of a *phaser* is presented in Figure 2.

The closed loop configuration of the system with the *phaser* was implemented, and the phase plot for the same frequency range as in Figure 1.a) is now shown in Figure 3.a). Figure 3.a) shows the response for a

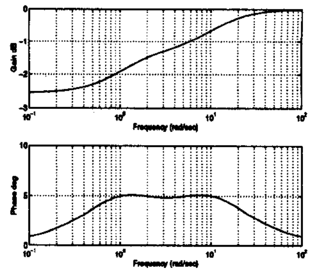
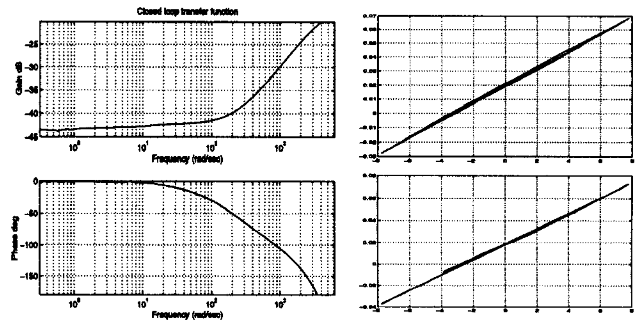


Figure 2: Controller Bode plot.

frequency of 0.31 rad/sec and 6.14 rad/sec. At 0.31 rad/sec, the result is not as good as at 6.14 rad/sec because this frequency falls outside the action range of the compensator which acts in the $1-10$ rad/sec range. Observe that the phase angle of the closed loop, in the quasistatic region, is closer to zero as a result of the compensation.



a) b)

Figure 3: Same presentation as Fig. 1 and same operating conditions, but in closed loop.

5 Conclusions

A partial linearization for certain nonlinear systems with hysteresis has been obtained by the use of what we have called a *phaser*, specifically, for systems that have a constant phase angle ϕ between input and output for a certain frequency range. Most importantly, the compensator is linear and can be designed using only the phase angle ϕ, reducing considerably the time and the computational load in designing and implementing the controller.

References

[1] Cruz-Hernandez, J.M. and Hayward, V. (1997). On the linear compensation of hysteresis. *Technical Report TR-CIM-97-08*, Centre for Intelligent Machines, McGill University, Montreal, Canada.

[2] Mayergoyz, I.D. (1991). *Mathematical Models of Hysteresis*. Springer-Verlag.