A linear-by-part approach for dissipative multiple-point collisions in smooth multibody systems with perfect constraints

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Abstract

The dynamical analysis of multibody systems undergoing simultaneous multiple-point collisions is a relevant problem in various fields. One of the interesting aspects of such problem is that it shows high sensitivity to initial conditions (HSIC). Whether the bodies are treated as deformable bodies or rigid ones, the results must retain this feature.

Rigid-body models are in principle less demanding from the computational point of view. In those models, HSIC is the cause that small perturbations on the impact configuration result in different sequences of single-point collisions yielding different end conditions.

Most rigid-body models found in literature assume a sequence of nonoverlapping single-point collisions [3], and use Newton's or Poisson's restitution coefficients modified according to energy criteria. However, the real situation may imply partial overlapping, and thus the final results obtained under such an arbitrary hypothesis are not reliable.

In a previous work, we proposed a simple rigid-body linear approach retaining the high sensitivity to initial conditions without assuming any particular collision sequence and able to cope with redundancy [1] applicable to 3D multiple-point smooth and perfectly elastic collisions in rigid-body systems with perfect constraints. The main idea consists in assuming a finite linear normal stiffness (high enough to assume constant configuration throughout the process) at each impact point and solving a vibrational problem. Two different time and space scales are used. At the macro scale, the impact interval is negligible, and the overall system configuration is assumed to be constant. Consequently, the inertia and Jacobian matrices appearing in the formulation are also constant. The approach can cope with redundancy, that is, can be used to treat situations where the normal velocities of some colliding points are linearly related.

We now propose an extension including energy losses (ranging from low dissipation cases to perfectly inelastic cases). Energy losses associated with compression and expansion in percussive analysis is a matter as complex as the physical phenomena involved, at the nanoscale level, for different materials. Simplified models can be developed for specific purposes, which can retain the most relevant trends of internal damping and at the same time be suitable for a particular analytical approach of impact mechanics. In the context of this article, energy dissipation due to material deformation can be conveniently introduced through a bistiffness model as that shown in Figure 1 [2]. The model consists on an elastic force with stiffness coefficient k, $f_n = k\delta$, and a parallel dry friction proportional to f_n through a coefficient μ. For $\mu < 1$, the resultant normal forces during compression and expansion are proportional to the compression displacement through two different equivalent stiffness coefficients k^{comp} and k^{exp}:

$$\begin{aligned} f_n^{\text{comp}} &= k\left|\delta^{\text{comp}}\right| + \mu k\left|\delta^{\text{comp}}\right| = k(1+\mu)\left|\delta^{\text{comp}}\right| \equiv k^{\text{comp}}\left|\delta^{\text{comp}}\right|, \\ f_n^{\text{exp}} &= k\left|\delta^{\text{comp}}\right| - \mu k\left|\delta^{\text{comp}}\right| = k(1-\mu)\left|\delta^{\text{comp}}\right| \equiv k^{\text{exp}}\left|\delta^{\text{comp}}\right|. \end{aligned} \tag{1}$$

For $\mu \geq 1$ there is no expansion, the collision is perfectly plastic.

For a simple compression-expansion process, as the potential energy stored during compression and that released during expansion are proportional to k^{comp} and k^{exp} respectively, this hysteretic model is equivalent to using local energetic coefficients e_w^2:

$$\mathrm{W}_n^{\text{exp}} = \left(\frac{k^{\text{exp}}}{k^{\text{comp}}} \right) \left| \mathrm{W}_n^{\text{comp}} \right| \;\Rightarrow\; e_w^2 = \frac{k^{\text{exp}}}{k^{\text{comp}}} = \frac{1-\mu}{1+\mu} \,. \tag{2}$$

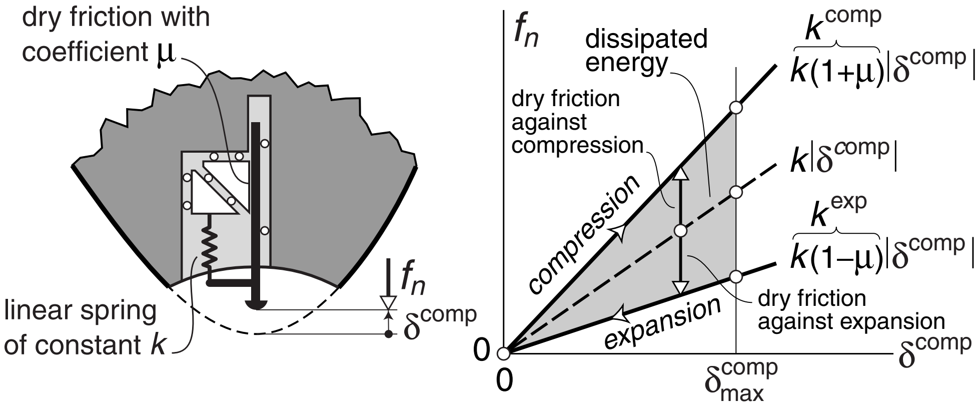

Figure 1: The bistiffness model for dissipation.

For the case of perfectly elastic collisions, changes in the vibrational modes and frequencies only happened when there was a change in the set of colliding points (that is, when there was a transition from zero indentation to positive indentation or vice versa). For the case of partial inelasticity ($\mu < 1$), the vibrational problem is formulated exactly in the same way, though the vibrational modes and frequencies change not only when there is a change in the colliding points sets but also when a colliding point changes from compression to expansion (or vice versa). In both cases, the total number of degrees of freedom (DOF) is constant throughout the process though the number of vibrational DOF may change.

The case of perfectly plastic collisions shows an important difference as compared to the previous cases: expansion phases disappear. Whenever a colliding point P reaches a zero compressing velocity (indicating a state of maximum compression with a spring force $(f_n^{comp})_{\max} \equiv F_{\max}$), an instantaneous new unilateral constraint at P appears thus reducing the number of DOF. The calculation of the corresponding normal constraint force N at that point is then necessary in order to detect whether the constraint holds ($\dot{\delta}^{\text{comp}}(P) = 0$) or disappears either because a new compression phase starts ($N > F_{\max}, \dot{\delta}^{\text{comp}}(P) > 0$) or because contact at P is lost ($N < 0$). This dissipative vibrational model will be applied to the case shown in Figure 2.

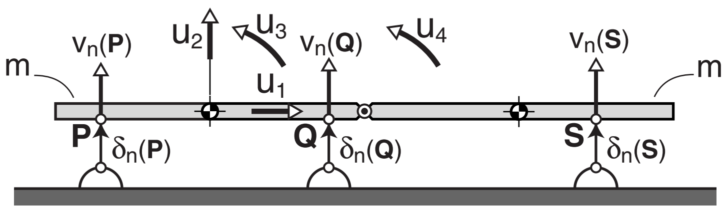

Figure 2: Application case: two-link chain.

References

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