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EFFECTS OF MASS DISTRIBUTION AND CONFIGURATION ON THE ENERGETIC LOSSES AT IMPACTS OF BIPEDAL WALKING SYSTEMS

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ABSTRACT

Understanding the dynamics of human walking is a complex task due to the interaction of the musculoskeletal and the central nervous systems. Nevertheless, the use of simple models can provide useful insight into the mechanical aspects of bipedal locomotion. Such models exploit the observations that human walking significantly relies on passive dynamics and inverted pendulum-like behaviour. The mechanical analysis of walking involves the study of the finite motion single support phase and the impulsive motion of the impacts that occur at heel strike. Such impacts are dominant events because they represent a sudden topology transition and moreover, they are the main cause of energy consumption during the gait cycle. The aim of this work is to gain insight into the dynamics and energetics of heel strike. We use a concept that decouples the dynamics of the biped to the spaces of admissible and constrained motions at the topology transition. This approach is then applied to a straight-legged biped with upper body. Detailed analysis and discussions are presented to quantify the effects of the mass distribution and the impact configuration on the energetics of walking.

NOMENCLATURE

$\mathbf{A}_I$ Jacobian matrix of the impulsive inert constraints.
$\mathbf{A}_S$ Jacobian matrix of the finite motion constraints.
$\mathbf{M}$ Mass matrix of the system.
$\mathbf{P}_a$ Projector associated with the space of admissible motion.
$\mathbf{P}_c$ Projector associated with the space of constrained motion.
$\mathbf{q}$ Generalized coordinates of the system.
$\dot{\mathbf{q}}$ Generalized velocities of the system.
$\mathbf{v}_a$ Generalized velocities of the space of admissible motion.
$\mathbf{v}_c$ Generalized velocities of the space of constrained motion.
L Step length.
T Kinetic energy of the system.
T_a Kinetic energy of the space of admissible motion.
T_c Kinetic energy of the space of constrained motion.
U Potential energy of the system.
ξ_I Ratio of the pre-impact kinetic energy that is lost.
ξ_L Energy loss due to impacts per unit distance.
$\bar{\lambda}_I$ Contact impulses on the colliding foot at heel strike.
λ_S Contact forces on the stance foot during finite motion.

INTRODUCTION

The mechanical analysis of walking is a fundamental task to understand the dynamics and energetics of human locomotion. This is a complex issue due to the interaction of the musculoskeletal and the central nervous systems and the fact that motion is actuated by numerous muscles. Therefore, obtaining detailed models to analyze the mechanics of walking is a difficult task. Nevertheless, it was observed that during the single support phase of the gait the motion of the stance leg is similar to that of an inverted pendulum, and the swing leg also performs a pendulum-like motion about the pelvis [1]. Electromyographic (EMG) data shows low muscular activity during this phase [2],

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which supports the fact that human walking is mainly passive between successive impacts (heel strikes). From these observations, simple models that provided useful insight into the dynamics of walking were developed [3].

Passive dynamic walking, first introduced in [4], is based on this idea of passive pendulum-like motion during the single support phase. It refers to simple mechanical systems that are able to walk down a slightly inclined walkway with no external control or actuation, i.e., gravity alone powers the motion [5–9]. The work on this type of bipeds was primarily motivated by the drive for energy efficiency and showed that it was possible to obtain stable limit cycles, with remarkably human-like motion, without any kind of actuation and control. The analysis of passive dynamic walking has also been useful to develop a better understanding of human locomotion. For instance, in [10] and [11] a simple passive-dynamic model was used to analyze the energetic cost of human walking. Useful conclusions were drawn based on combining experimental data with the dynamics of this biped.

In humanoid robotics, a type of powered passive-based robots (dynamic walking robots) have recently appeared in the literature [12–14]. These robots use minimal actuation, sensing and control to walk like a purely passive dynamic walker but on level ground. According to [12], the energy consumption of such robots is much lower than that of other anthropomorphic robots that use joint-angle feedback control to follow specified trajectories and, furthermore, it is similar to that of human walking.

The mechanical analysis of these systems involves the study of two phases: the *finite motion single support phase* in which the stance leg behaves like an inverted pendulum rotating about the foot, and the *impulsive motion phase* that occurs when the swing leg impacts the ground at heel strike. This impact is a very dominant event in the behaviour of the walking system since it is the main cause of energy consumption during the motion [13], and in turn, it gives rise to a sudden change of the topology of the system. Hence, the analysis of the impulsive dynamics of heel strike is a key issue in bipedal locomotion.

In most publications the mechanical analysis of walking is performed based on two different mathematical models developed separately [6–9], one for the finite motion of the stance phase, and the other for the impulsive motion of the heel strike. Conversely, in this work a unified formulation to analyze both phases of motion is presented. We use an augmented non-minimum set of coordinates that allows the study of the system, as a variable topology system, by changing the physical constraints during the successive phases of motion. We also interpret a concept that allows to decompose the kinetic energy and the dynamic equations of impulsive motion of the system to the spaces of *constrained* and *admissible* motions.

The article is organized as follows. First, the formulation to study the dynamics of the biped is introduced. Based on the impulsive constraints that characterize the topology change, next section introduces the decomposition technique which is used to analyze the energy redistribution at heel strike. The formulation is then applied to a biped with upper body and it is used to analyze how various dynamic parameters and the impact configuration influence the energetic losses at impacts. Finally, the main contributions of the work are summarized.

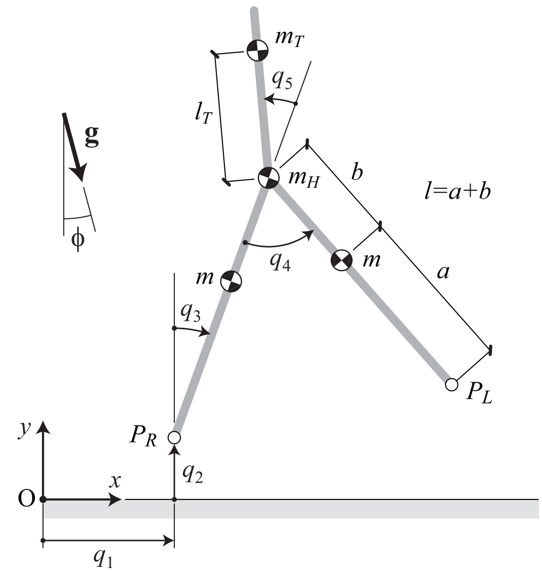

Figure 1. DYNAMIC MODEL OF THE BIPED.

DYNAMICS MODELLING OF THE BIPED

In this work we consider an extension of the so-called *compass-gait biped* [7, 8]. This has been widely studied in the literature since it is the simplest mechanism that models the dynamics of both legs in the sagittal plane during the gait cycle. The compass biped is a two-dimensional walking model that consists of two identical straight legs of length l and mass m. The centres of mass of the legs are at a distance a from each foot, i.e., at a distance $b = l - a$ from the hip. Each foot is modelled as a point and the hip as a point mass m_H located at the revolute joint between the two legs, Fig. 1.

To study how the mass and the position of the upper body influence the dynamics of walking, a third link that can rotate about the hip has been added. The mass of this link, which represents the upper body (or torso), is m_T and its centre of mass is at a distance l_T from the hip. In Fig. 1, points P_R and P_L refer to the *right* and *left* foot respectively.

The configuration of this system can be described by 5 generalized coordinates that are represented by the 5×1 dimensional array $\mathbf{q} = [q_1, q_2, q_3, q_4, q_5]^T$, Fig. 1. Coordinates q_1 and q_2 indicate the (x, y)-position of P_R with respect to the absolute inertial frame. Coordinate q_3 indicates the absolute orientation of the right leg, and q_4 and q_5 denote respectively the orientation of the left leg and the upper body with respect to the right leg. Angle q_4 will be referred to as *inter-leg angle*. It is important to point out that the angles are defined positive with the sense shown in Fig. 1.

Gait i is considered to take place between time points t_{i-1} and t_i. This is a variable topology system, where points P_R and P_L are subjected to physical constraints during the successive phases of walking. Their velocities can be expressed as

$$\mathbf{v}_R = \mathbf{A}_R \dot{\mathbf{q}} \quad \text{and} \quad \mathbf{v}_L = \mathbf{A}_L \dot{\mathbf{q}}, \tag{1}$$

where $\mathbf{v}_R$ and $\mathbf{v}_L$ represent the 2×1 velocity vectors of P_R and P_L; and $\mathbf{A}_R$ and $\mathbf{A}_L$ are the 2×5 related Jacobian matrices, which are given for this model in Appendix A. As said before, the gait has two characteristic phases: the single support phase with *finite motion* dynamics, and the *impulsive motion* phase that takes place when the swing foot collides the ground.

During the **finite motion** phase of walking, one of the feet (the *stance* foot) stays in contact with the ground without slipping. This can be modelled using the bilateral constraints

$$\mathbf{A}_S \dot{\mathbf{q}} = \mathbf{0}, \tag{2}$$

where S is either R or L depending on which of the feet is the stance foot. These velocity level constraints are holonomic, hence they can also be reduced for each step to configuration level constraints in the form of $\Phi(\mathbf{q}, k_i) = \mathbf{0}$, where k_i is a constant associated with gait i, representing a different offset along the direction of walking.

The instantaneous phase of heel strike represents **impulsive motion**. The swing foot impacts the ground at time t_i. This impact is required to be "inelastic", i.e., the colliding point of the swing foot must stay in contact with the ground. This is a reasonable and widely used assumption in the analysis of walking systems [6–8, 13]. This event can be characterized by *inert constraints*, which represent a class of impulsive constraints [15, 16]. Let us consider that $[t_i^-, t_i^+]$ is the interval representing the pre- and post-impact instants. This interval is considered to be very short on the characteristic time scale of the finite motion of the system. Therefore, during $[t_i^-, t_i^+]$ the configuration of the system remains unchanged. The inert constraints representing the contact transition and the change in topology can be written as

$$\mathbf{A}_I \dot{\mathbf{q}}^+ = \mathbf{0}, \tag{3}$$

where $\dot{\mathbf{q}}^+$ stands for $\dot{\mathbf{q}}$ at t_i^+, and I is either R or L depending on which foot is the impacting swing foot. Equation (3) represents the required topology (constraint configuration) at t_i^+ at the velocity level, i.e., the colliding foot in contact with the ground without slipping. In $[t_i^-, t_i^+]$ the contact of the stance foot should also undergo a transition. This is normally not modelled in other works, but simply assumed that this foot will lift from the ground. Here we will not make this assumption unconditionally, but always check its satisfaction. The stance foot contact and separation at impact is governed by the velocity-level unilateral constraint

$$v_{S_n}^+ = \mathbf{B}_S \dot{\mathbf{q}}^+ \geq 0, \tag{4}$$

where $S \equiv L$ if $I \equiv R$ and vice versa, $v_{S_n}^+$ is the component of the velocity of the stance foot normal to the ground at t_i^+, and $\mathbf{B}_S$ is a 1×5 array that gives the representation of the direction normal to the ground in terms of the generalized coordinates. The desired situation is that $v_{S_n}^+ > 0$, i.e., the non-colliding foot lifting up from the ground passively without extra actuation. If that happens, the pre-impact bilateral constraints (2) become passive and the topology of the system changes.

Dynamic Equations for the Finite Motion

The kinetic energy of the walking system can be written as

$$T(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}, \tag{5}$$

where $\mathbf{M}$ is the 5×5 mass matrix of the biped. This matrix is symmetric and positive definite, and it is given in Appendix A. The potential energy of the system $U(\mathbf{q})$ includes the effects of conservative forces. From the expressions of T and U, the dynamic equations for the finite motion of the system can be symbolically obtained as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial T}{\partial \mathbf{q}} + \frac{\partial U}{\partial \mathbf{q}} = \mathbf{f}_A + \mathbf{f}_R, \tag{6}$$

where $\mathbf{f}_A$ and $\mathbf{f}_R$ stand for the generalized non-conservative applied forces and the generalized constraint forces, respectively. These equations for the finite motion phase are associated with the constraints expressed in Eqn. (2). The generalized constraint forces can be expressed as $\mathbf{f}_R = \mathbf{A}_S^T \lambda_S$, where $\mathbf{A}_S$ is the constraint Jacobian in Eqn. (2) and $\lambda_S = [\lambda_{S_t}, \lambda_{S_n}]^T$ represents the constraint forces acting on the stance foot. The following conditions must be satisfied

$$\lambda_{S_n} > 0 \quad \text{and} \quad |\lambda_{S_t}| \leq \mu_s \lambda_{S_n}, \tag{7}$$

where μ_s is the coefficient of static friction, in order to assure that the stance foot does not lose contact with the ground and does not slip during the finite motion phase.

Dynamic Equations for the Impulsive Motion

When the swing foot impacts the ground at heel strike, the system undergoes a sudden change in topology, and part of the pre-impact energy of the system is lost. The dynamics of this impulsive motion phase can be characterized by impulse-momentum level dynamic equations. Based on Eqn. (6), these can be obtained in a general form as [15, 16]

$$\left[\frac{\partial T}{\partial \dot{\mathbf{q}}}\right]_{-}^{+} = \mathbf{M}\left(\dot{\mathbf{q}}^{+} - \dot{\mathbf{q}}^{-}\right) = \bar{\mathbf{f}}_A + \bar{\mathbf{f}}_R, \tag{8}$$

where "$-$" and "$+$" denote the pre- and post-impact instants t_i^- and t_i^+ for the transition between gaits i and $i+1$; $\bar{\mathbf{f}}_A$ and $\bar{\mathbf{f}}_R$ are the impulses of the generalized non-conservative applied forces and the generalized constraint forces; and $[(\partial T)/(\partial \dot{\mathbf{q}})]_{-}^{+} = -\bar{\mathbf{f}}_I$ is the negative of the impulse of the generalized inertial forces. If the applied forces have a non-impulsive nature, which is usually the case in dynamic walking, then $\bar{\mathbf{f}}_A = \mathbf{0}$. For this impulsive motion phase it is also assumed that $\dot{\mathbf{q}}^-$ can be determined based on the previous finite motion analysis of the single support phase.

The constraints in this impulsive motion phase were analyzed earlier in this section, they are expressed in Eqn. (3). The contact impulses are normally generated by these constraints, hence, $\bar{\mathbf{f}}_R = \mathbf{A}_I^T \bar{\lambda}_I$ where $\bar{\lambda}_I = [\bar{\lambda}_{I_t}, \bar{\lambda}_{I_n}]^T$ represents the impulse of the constraint forces generated at heel strike. If the unilateral constraint in (4), associated with the stance foot is passive (i.e., $v_{S_n}^+ > 0$), then this will not generate any impulse and the stance foot is lifting up, as it should for a natural walking gait.

Should the topology change cause a situation that $v_{S_n}^+ < 0$, then this would mean that the non-colliding stance foot does stay in contact. In this case, Eqn. (4) would also represent an active constraint that can generate impulses, since the equality sign would be valid. It would give a normal impulse $\bar{\lambda}_{S_n}$ which would be equivalent to the generalized constraint force component $\mathbf{B}_S^T \bar{\lambda}_{S_n}$.

DECOMPOSITION OF THE IMPULSIVE MOTION

Based on the Jacobian defining the inert constraints in Eqn. (3), the tangent space of the configuration manifold of the walking system can be decomposed to the spaces of *constrained* and *admissible* motions [17, 18] for the pre-impact instant. This will then also hold for the entire duration of the contact onset, since the configuration of the system does not change during this short period of time. The two subspaces can be defined so that they are orthogonal to each other with respect to the mass metric of the tangent space. This decomposition can be accomplished via two projector operators [17, 18]. The projector associated with the space of constrained motion can be written as

$$\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}_I^T\left(\mathbf{A}_I\mathbf{M}^{-1}\mathbf{A}_I^T\right)^{-1}\mathbf{A}_I, \tag{9}$$

and the projector for the space of admissible motion can be obtained as

$$\mathbf{P}_a = \mathbf{I} - \mathbf{P}_c = \mathbf{I} - \mathbf{M}^{-1}\mathbf{A}_I^T\left(\mathbf{A}_I\mathbf{M}^{-1}\mathbf{A}_I^T\right)^{-1}\mathbf{A}_I, \tag{10}$$

where $\mathbf{I}$ denotes the 5×5 identity matrix. These projectors are not symmetric, which is a direct consequence of the nature of the metric of the tangent space. Based on them, the generalized velocities of the system can be decomposed as

$$\dot{\mathbf{q}} = \mathbf{v}_c + \mathbf{v}_a = \mathbf{P}_c\dot{\mathbf{q}} + \mathbf{P}_a\dot{\mathbf{q}}, \tag{11}$$

which represent the two components associated with both subspaces. It is interesting to note that in general $\mathbf{v}_c = \mathbf{P}_c\dot{\mathbf{q}}$ and $\mathbf{v}_a = \mathbf{P}_a\dot{\mathbf{q}}$ are non-holonomic components. Any vector of generalized forces or generalized impulses can also be decomposed using the transpose of the operators given above [17]. Then, for the impulsive case we have

$$\bar{\mathbf{f}} = \bar{\mathbf{f}}_c + \bar{\mathbf{f}}_a = \mathbf{P}_c^T\bar{\mathbf{f}} + \mathbf{P}_a^T\bar{\mathbf{f}}. \tag{12}$$

Based on Eqns. (5) and (11), it can be shown that the kinetic energy can also be decomposed as

$$T = T_c + T_a = \frac{1}{2}\mathbf{v}_c^T\mathbf{M}\mathbf{v}_c + \frac{1}{2}\mathbf{v}_a^T\mathbf{M}\mathbf{v}_a. \tag{13}$$

To obtain this equation it was used that the projectors in (9) and (10) are orthogonal with respect to the system mass metric, i.e., $\mathbf{P}_c^T\mathbf{M}\mathbf{P}_a = \mathbf{P}_a^T\mathbf{M}\mathbf{P}_c = \mathbf{0}$, with $\mathbf{0}$ denoting the 5×5 zero matrix. Therefore, $\mathbf{v}_c^T\mathbf{M}\mathbf{v}_a = \mathbf{v}_a^T\mathbf{M}\mathbf{v}_c = 0$. Any force or impulse arising in the space of constrained motion will change only T_c, leaving T_a unaffected and vice versa [19]. The impact characterized by the constraints in (3) gives rise to impulses which will influence quantities in the space of constrained motion only.

Based on the above decompositions, it can be seen that the impulse-momentum level dynamic equations in (8) can be decoupled as

$$\left[\frac{\partial T_c}{\partial \mathbf{v}_c}\right]_{-}^{+} = \mathbf{M}\left(\mathbf{v}_c^{+} - \mathbf{v}_c^{-}\right) = \mathbf{A}_I^T\bar{\lambda}_I, \tag{14}$$

which are the impulse-momentum level dynamic equations for the space of constrained motion, and

$$\left[\frac{\partial T_a}{\partial \mathbf{v}_a}\right]_{-}^{+} = \mathbf{M}\left(\mathbf{v}_a^{+} - \mathbf{v}_a^{-}\right) = \mathbf{0}, \tag{15}$$

which describes the impulsive dynamics associated with the space of admissible motion. From Eqn. (15) and using that $\mathbf{M}$ is positive definite, it is immediately visible that $\mathbf{v}_a^+ = \mathbf{v}_a^-$. Based on Eqns. (3) and (11) it can also be concluded that $\mathbf{v}_c^+ = \mathbf{0}$. Considering this and using Eqn. (14), we can write that

$$-\mathbf{M}\mathbf{v}_c^- = \mathbf{A}_I^T \bar{\lambda}_I, \tag{16}$$

and also, taking into account the velocity decomposition in (11), we obtain the following expression to solve for the post-impact generalized velocities

$$\dot{\mathbf{q}}^+ = \mathbf{v}_a^+ = \mathbf{v}_a^- = \mathbf{P}_a \dot{\mathbf{q}}^-. \tag{17}$$

Once the post-impact velocity vector $\dot{\mathbf{q}}^+$ is determined, it is important to verify that the normal component of the non-colliding foot, $v_{S_n}^+ = \mathbf{B}_S \dot{\mathbf{q}}^+$, is positive in order to leave the ground. If this is not the case that will mean that the unilateral constraint of ground contact becomes active and then forward walking does not materialize in a natural way.

Based on Eqns. (9), (11) and (16) we can also obtain the solution for the generalized constraint impulses as

$$\bar{\lambda}_I = -\left(\mathbf{A}_I \mathbf{M}^{-1} \mathbf{A}_I^T\right)^{-1} \mathbf{A}_I \dot{\mathbf{q}}^-, \tag{18}$$

which appear in order to set the velocity of the colliding foot to zero. The normal component of $\bar{\lambda}_I$ is the impulse perpendicular to the ground and it is usually associated with the deformation of the colliding bodies (foot-ground) in this direction. The other component is the impulse in the tangent direction which is more complex in nature, since it can come either from friction or from tangential compliance of the colliding bodies.

Energetic Aspects of the Heel Strike

Energetic aspects are very important for the optimal design and control of bipedal robots [20,21], and also to gain a better understanding on the energetic cost of human locomotion [10, 11]. In this section we use the kinetic energy decomposition introduced in (13) to analyze the energetic losses at heel strike. First of all, the application of the energy theorem between the pre-impact time t_i^- and the post-impact time t_i^+ yields

$$T(t_i^+) - T(t_i^-) \equiv T^+ - T^- = W_{HS} < 0, \tag{19}$$

where W_{HS} denotes the negative work done by the impulsive contact forces at heel strike. Then, based on the kinetic energy decomposition in Eqn. (13) and using that $\mathbf{v}_c^+ = \mathbf{0}$, we obtain

$$|W_{HS}| = -W_{HS} = \underbrace{\left(T_c^- + T_a^-\right)}_{T^-} - \underbrace{\left(0 + T_a^+\right)}_{T^+} > 0. \tag{20}$$

Using the fact derived from Eqn. (15) that $\mathbf{v}_a$ remains constant during impact, we can conclude that the kinetic energy of the space of admissible motion is also constant, $T_a^+ = T_a^-$. Then, from Eqn. (20) we have that the amount of energy lost at impact is exactly T_c^-,

$$|W_{HS}| = T_c^- = \frac{1}{2}\left(\dot{\mathbf{q}}^-\right)^T \mathbf{P}_c^T \mathbf{M} \mathbf{P}_c \dot{\mathbf{q}}^-, \tag{21}$$

whereas $T^+ = T_a^+ = T_a^-$ will stay in the system after impact. This is an important result, because it means that the energetic expenditure at impact can be predicted, via the presented decomposition approach, only with information of the pre-impact kinematics of the system. This can be an important tool for the design and control of dynamic walking systems, and to better understand the biomechanics of human locomotion.

Finally, based on Eqns. (11) and (16), and considering that $\mathbf{v}_a^T \mathbf{M} \mathbf{v}_c = 0$, it is also possible to write that

$$-(\dot{\mathbf{q}}^-)^T \mathbf{M} \mathbf{v}_c^- = -2T_c^- = (\dot{\mathbf{q}}^-)^T \mathbf{A}_I^T \bar{\lambda}_I, \tag{22}$$

which gives an explicit relationship between the impulses generated by the contact onset and the kinetic energy T_c^- that is lost in the contact event.

We define the following indexes to quantify the energetic aspects of the gait

$$\xi_I = \frac{T_c^-}{T^-} = \frac{(\dot{\mathbf{q}}^-)^T \mathbf{P}_c^T \mathbf{M} \mathbf{P}_c \dot{\mathbf{q}}^-}{(\dot{\mathbf{q}}^-)^T \mathbf{M} \dot{\mathbf{q}}^-} \quad \text{and} \quad \xi_L = \frac{T_c^-}{L}, \tag{23}$$

where ξ_I (non-dimensional) represents the ratio of the pre-impact kinetic energy that is lost at heel strike, i.e., the local energetic efficiency of the impact. Regarding ξ_L (with units J/m), it accounts for the energetic losses per unit distance walked by the biped. This index will be useful when analyzing different step lengths. Distance L in Eqn. (23) stands for the step length which can be expressed as a function of the configuration of the system at heel strike. For the system at hand, we have $L(\mathbf{q}^-) = 2l \sin\left(q_4^-/2\right)$.

RESULTS AND DISCUSSION

In this section, the decomposition of the kinetic energy at the pre-impact time of the heel strike event is used to analyze how different configurations and mass distributions of the considered system influence the energetic aspects of that event. The dynamic parameters of the biped are given in Table 1.

We will vary the mass parameters m, m_H and m_T to analyze how different mass distributions of the system affect the energetics of heel strike. We will use the following non-dimensional

Table 1. DYNAMIC PARAMETERS OF THE BIPED.

Param.	**Value**	**Description**
m_B	30 kg	Mass of the biped $(= 2m + m_H + m_T)$
l	0.8 m	Length of the leg
l_T	0.4 m	Distance from hip to torso centre of mass
b	0.4 m	Distance from hip to leg centre of mass

parameters, $\mu = \frac{2m}{m_H}$ and $\mu_T = \frac{m_T}{m_H}$, to take into account the effect of the lower body and the upper body mass distributions, respectively.

In this study, we assume that the right foot is in contact with the ground at the pre-impact time, i.e., $q_2^- = 0$, and that the left foot impacts the ground. For this case, matrix $\mathbf{B}_S$ representing the normal direction of the contact between the stance foot and the ground is the second row of Eqn. (24) given in Appendix A. This is used to check the validity of $v_{S_n}^+ = \mathbf{B}_S \dot{\mathbf{q}}^+ > 0$, according to Eqn. (4). Matrix $\mathbf{A}_I$, defining the inert constraints at the colliding foot (left foot), takes the expression of Eqn. (25). Regarding the pre-impact generalized velocities $\dot{\mathbf{q}}^-$, we make the following assumptions:

(i) The right foot is in contact with the ground without slipping, i.e., $\dot{q}_1^- = 0$ and $\dot{q}_2^- = 0$.
(ii) The tangential component of the velocity of the colliding point at pre-impact time is zero. This is a reasonable assumption in order to avoid slipping in the beginning of the contact onset. This assumption implies that $\dot{q}_4^- = 0$, i.e., there is no relative angular velocity between the legs just before impact.
(iii) The angular velocity of the torso with respect to the inertial frame is zero, i.e., $\omega_T^- \equiv \dot{q}_3^- - \dot{q}_5^- = 0$ (defined positive clockwise). From that, we obtain the following kinematic relationship $\dot{q}_5^- = \dot{q}_3^-$.

The assumptions above imply that all the pre-impact kinematics depend on the absolute angular velocity of the right leg, $\dot{q}_3^-$. We will consider for all the analyzed impacts $\dot{q}_3^- = 1$ rad/s, which is a typical value for compass walkers.

Effects of the Lower Body Configuration and Mass Distribution

In this subsection, we study the effect of the lower body mass distribution and the inter-leg angle at impact q_4^-. It is important to point out that this angle completely defines the geometry of the two legs of the biped at heel strike, since for that event the following geometric relationship holds, $q_3^- = q_4^-/2$. We consider the following range of possible values for the inter-leg angle at impact $q_4^- \in [10°, 50°]$.

As for the lower body mass distribution, it will be accounted for using different values of parameter μ between 0 and 2. A value of $\mu = 0$ means that all the lower body mass is concentrated at the hip, and $\mu = 2$ means that $m = m_H$. We consider the following mass of the torso, $m_T = 10$ kg. We will also consider that the upper body is placed perpendicular to the ground, therefore its absolute angle with respect to the vertical (y-axis of the inertial frame) is $q_3^- - q_5^- = 0$, which establishes that $q_5^- = q_3^-$.

Figure 2 shows the value of $v_{S_n}^+$ as a function of q_4^- and μ. It can be seen that for high values of q_4^- and μ, there are situations that do not satisfy the aforementioned condition of natural stance foot separation ($v_{S_n}^+ > 0$). The area for the satisfaction of this condition is indicated with a white dashed line in the bottom graphic of Fig. 2. It can be concluded that a low value of μ (mass concentrated at the upper part of the leg) is better in order to have a larger range of valid configurations for which the stance foot lifts up. For $\mu = 0$ (mass concentrated at the hip), all the considered inter-leg angles are valid.

Figure 3 shows the pre-impact decoupling of the kinetic energy for the conditions that satisfy $v_{S_n}^+ > 0$, this is the reason why the curves of the figure do not cover the same range of angles. The graphics of this figure indicate the part of the pre-impact ki-

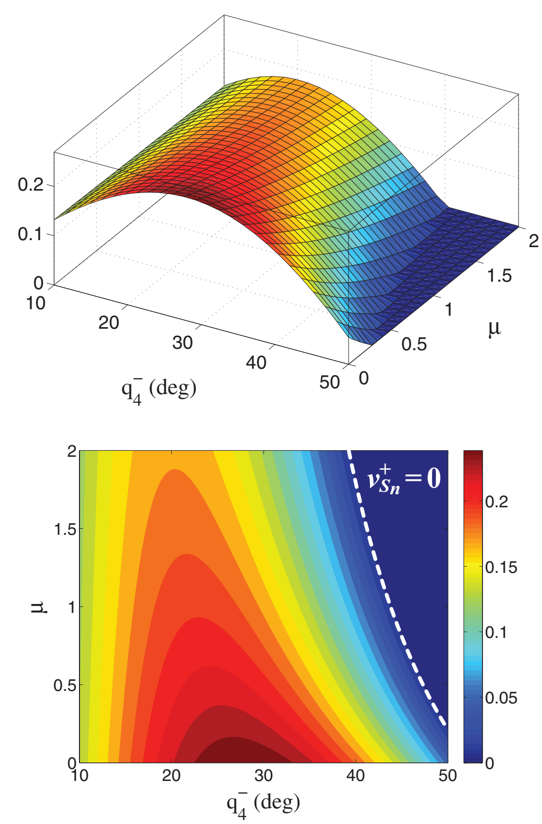

Figure 2. POST-IMPACT VELOCITY $v_{S_n}^+$ (m/s) AS A FUNCTION OF THE INTER-LEG ANGLE AND PARAMETER μ.

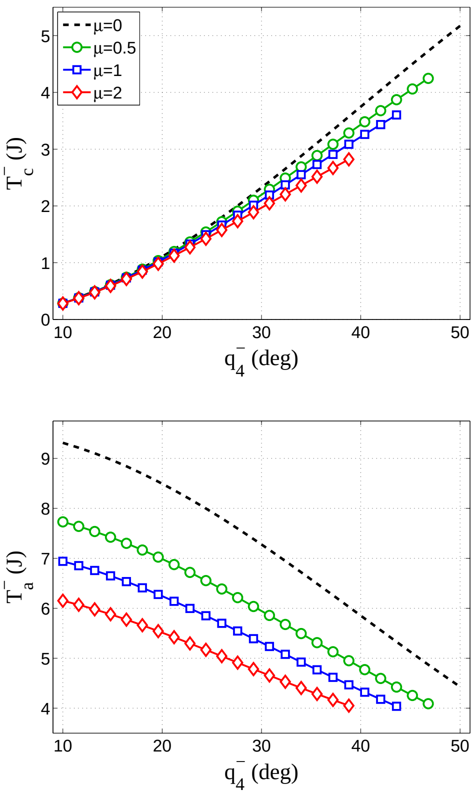

Figure 3. DECOMPOSED PARTS OF THE KINETIC ENERGY AS FUNCTIONS OF THE INTER-LEG ANGLE AND PARAMETER μ.

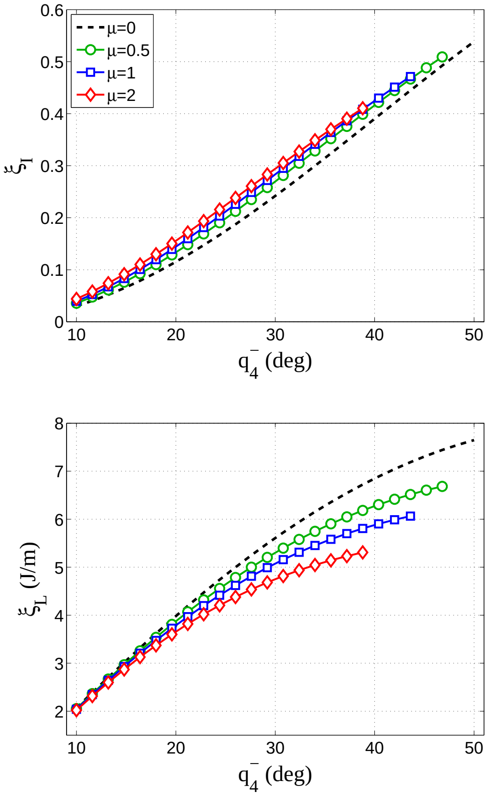

Figure 4. ENERGETIC INDEXES AS FUNCTIONS OF THE INTER-LEG ANGLE AND PARAMETER μ.

netic energy that is lost, T_c^-, and the part that stays in the system $T_a^- = T^+$.

Figure 4 shows the two indexes defined in Eqn. (23). The first of them, ξ_I, shows the ratio of the total pre-impact kinetic energy that is lost at impact. It can be seen that for a given inter-leg angle a low value of μ yields better results in terms of impact efficiency, since this index is lower. Also, from the same figure it can be noticed that short steps (low value of q_4^-) are better than longer steps in terms of energetic efficiency. Nevertheless, it has to be taken into account that in such a case more steps, and also heel strikes, are required to walk a certain distance.

Index ξ_L can be used to evaluate energetic aspects for a given finite distance the walker should travel and its evolution can be seen in the bottom graphic of Fig. 4. It allows to conclude that for a given value of μ, it is better to walk a certain distance with more short steps than with less long steps. This completely agrees with the conclusions of [13] and [11], but looking at the problem from a different point of view. In [13], the energetics of walking of a powered straight-legged walker was analyzed using two different types of actuation (impulse at toe-off and torque applied on the stance leg). The work concluded that in either type of actuation shorter steps required less actuation energy. The same conclusion was reached in [11], but in this case the metabolic cost of walking was measured from real subjects performing different step lengths.

From Fig. 4, it can also be seen that a higher value of μ is better to reduce the energetic losses per unit distance for a given configuration. This may seem to be in contradiction with the conclusions reached from ξ_I. However, the reason is that the higher μ is, the lower the position of the centre of mass of the system becomes, and therefore, the total kinetic energy at the pre-impact time is less (for a fixed angular velocity $\dot{q}_3^-$). Therefore, although a low value of μ provides locally more efficient impacts (ξ_I lower), a high value of μ would be better in order to reduce the energetic expenditure per unit distance, since in such a case less energy is required to walk.

Effects of the Upper Body Configuration and Mass Distribution

In this subsection, we analyze how the upper body of the walker affects the dynamics of heel strike. For this purpose, we will study impacts for a usual inter-leg angle of $q_4^- = 30°$, and therefore for a fixed step length. The influence of the configu-

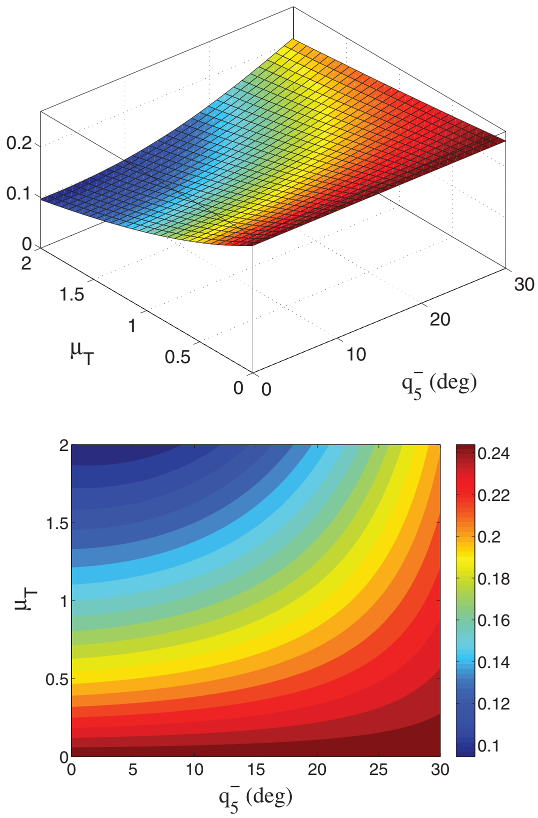

Figure 5. POST-IMPACT VELOCITY $v^+_{S_n}$ (m/s) AS A FUNCTION OF THE TORSO ANGLE AND PARAMETER μ_T.

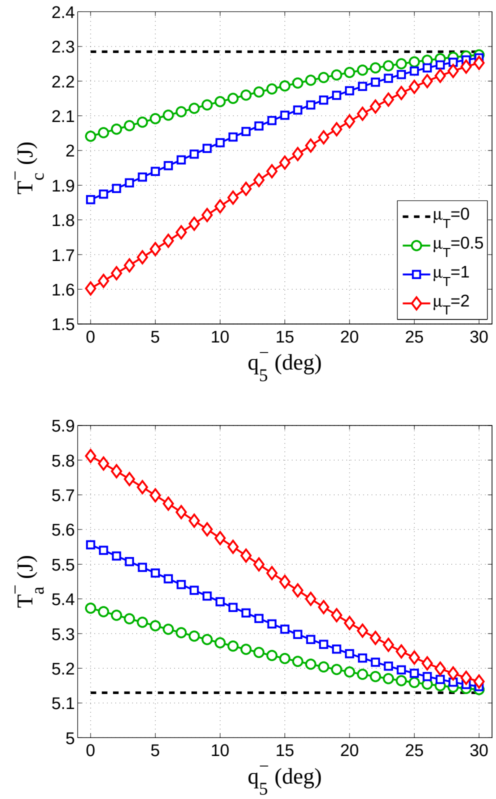

Figure 6. DECOMPOSED PARTS OF THE KINETIC ENERGY AS FUNCTIONS OF THE TORSO ANGLE AND PARAMETER μ_T.

ration of the upper body will be studied by varying angle q_5^- in the following range $[0°, 30°]$. That is, configurations between upper body completely aligned with the pre-impact stance leg ($q_5^- = 0°$) and upper body completely aligned with the colliding leg ($q_5^- = 30°$). As for the mass distribution, it will be accounted for using different values of parameter μ_T between 0 and 2, with a constant mass of the legs $m = 5$ kg. A value of $\mu_T = 0$ means that all the upper body mass is concentrated at the hip, and $\mu_T = 2$ means that $m_T = 2m_H$.

The post-impact normal velocity of the stance foot $v^+_{S_n}$ is shown in Fig. 5. In such a case it can be seen that the condition $v^+_{S_n} > 0$ is satisfied regardless of q_5^- and μ_T. It can be seen that the higher values of $v^+_{S_n}$ are obtained when the value of μ_T is low ($m_T \approx 0$) and q_5^- is high.

Figure 6 shows the kinetic energy decoupling at heel strike. Two main conclusions arise regarding the influence of the upper body. First, for a given mass distribution μ_T it can be seen that the minimum pre-impact kinetic energy associated with the space of constrained motion T_c^- is achieved for $q_5^- = 0°$. Then, T_c^- grows with higher values of angle q_5^-. As opposed to this, the pre-impact kinetic energy of the space of admissible motion has its maximum at $q_5^- = 0°$ and then it decreases. This holds for all the values of μ_T with the exception of $\mu_T = 0$, which would mean that all the mass is concentrated at the hip (this is the case of a compass-gait walker without upper body).

Based on this, it can be concluded that in order to reduce the energetic losses at heel strike, the upper body should be leaning forward when the system undergoes heel strike impact. It is worth noting that this conclusion is also supported by [22], in which the complete gait of a straight-legged walker with upper body was optimized. The obtained optimal joint-angle evolution shows an upper body configuration inclined forward aligned with the non-colliding leg at the end of the gait (heel strike).

The second conclusion that can be drawn form the results in Fig. 6 is that increasing m_T with respect to m_H (higher μ_T) yields better results in terms of energetic cost, since T_c^- is lower for an specific biped configuration. It can also be seen that the influence of μ_T is smaller when q_5^- increases. For $q_5^- = 30°$, the values of T_c^- and T_a^- are almost equal for any value of μ_T.

Figure 7 shows the energetic indexes ξ_I and ξ_L. The obtained curves are proportional to T_c^- since both the pre-impact kinetic energy T^- and the step length L are constant for all the upper body configurations and values of μ_T. Therefore, the con-

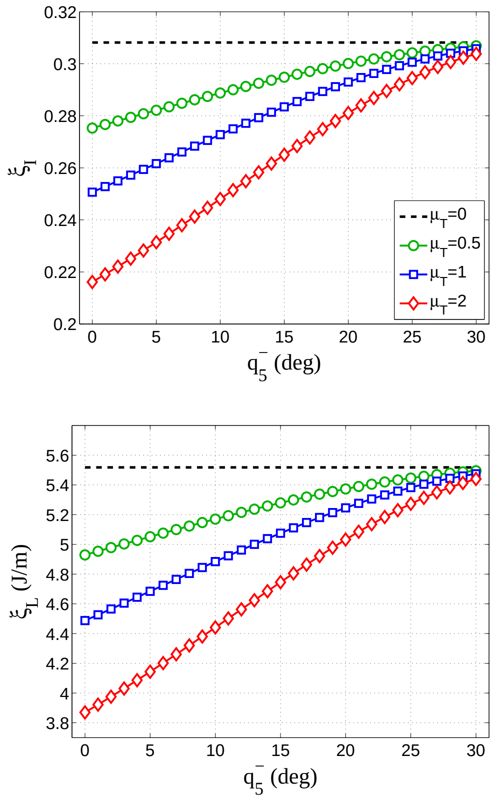

Figure 7. ENERGETIC INDEXES AS FUNCTIONS OF THE TORSO ANGLE AND PARAMETER μ_T.

clusions that can be drawn have the same nature as was already discussed above for T_c^- (Fig. 6).

CONCLUSIONS

In this paper we presented a novel formulation to analyze the impulsive dynamics of the heel strike event in dynamic walking systems. Based on the impulsive constraints that characterize this event, the formulation allows to decouple the impulse-momentum level dynamic equations and the kinetic energy of the system to the spaces of constrained and admissible motions. The decomposition of the pre-impact kinetic energy appears to be a useful concept to study the energetics of heel strike. The kinetic energy of the space of constrained motion is completely lost at impact and, conversely, the part associated with the space of admissible motion stays in the system.

The formulation was applied to a straight-legged biped with point-foot and an upper body. The effects of the mass distribution and the impact configuration of the system on the kinetic energy decomposition at impact were analyzed. The obtained results can give rise to useful information regarding the energetic aspects of impacts in walking, both for the design of efficient walking machines and to get insight into the mechanical aspects of human locomotion. Among the conclusions reached from the results, the following deserve being highlighted:

(*i*) Concentrating the mass of the lower body at the hip increases the range of inter-leg angles for which the stance foot passively lifts up from the ground.
(*ii*) The energetic losses are lower if the biped walks a certain distance with short steps rather than long steps.
(*iii*) The upper body has a significant effect on the energetics of walking. Leaning it forward aligned with the stance leg reduces the energetic losses at heel strike.

The presented technique can provide a better understanding of the velocity change and energy redistribution at heel strike. Furthermore, this approach is valid for any walking system whose configuration is described by a general non-minimum set of generalized coordinates. We believe that it can be of considerable value in the areas of dynamic analysis, mechanical design and control of dynamic walking systems. Its application can also be of interest to analyze the dynamics of physiological multibody models of the human musculoskeletal system.

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Appendix A: Elements of the Dynamic Model

For the biped considered in Fig. 1, the Jacobian associated with the right foot P_R can we written as

$$\mathbf{A}_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \tag{24}$$

and the one for the left foot P_L has the expression

$$\mathbf{A}_L = \begin{bmatrix} 1 & 0 & l\cos q_3 - l\cos(q_4 - q_3) & l\cos(q_4 - q_3) & 0 \\ 0 & 1 & -l\sin q_3 - l\sin(q_4 - q_3) & l\sin(q_4 - q_3) & 0 \end{bmatrix}. \tag{25}$$

The elements of the mass matrix $\mathbf{M}$ can be derived by expanding Eqn. (5). Its expression is known to have the following symmetric form

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ & M_{22} & M_{23} & M_{24} & M_{25} \\ & & M_{33} & M_{34} & M_{35} \\ & & & M_{44} & M_{45} \\ \text{Sym.} & & & & M_{55} \end{bmatrix}, \tag{26}$$

where the fifteen independent elements have the expressions

$$\begin{aligned}
M_{11} &= 2m + m_H + m_T, \\
M_{12} &= 0, \\
M_{13} &= ((m + m_H + m_T)l + ma)\cos q_3 - mb\cos(q_4 - q_3) + \\
&\quad \ldots + m_T l_T \cos(q_5 - q_3), \\
M_{14} &= mb\cos(q_4 - q_3), \\
M_{15} &= -m_T l_T \cos(q_5 - q_3), \\
M_{22} &= 2m + m_H + m_T, \\
M_{23} &= -((m + m_H + m_T)l + ma)\sin q_3 - mb\sin(q_4 - q_3) + \\
&\quad \ldots + m_T l_T \sin(q_5 - q_3), \\
M_{24} &= mb\sin(q_4 - q_3), \\
M_{25} &= -m_T l_T \sin(q_5 - q_3), \\
M_{33} &= m\left(a^2 + b^2 + l^2\right) + m_H l^2 + m_T\left(l^2 + l_T^2\right) - 2mbl\cos q_4 + \\
&\quad \ldots + 2m_T l_T l\cos q_5, \\
M_{34} &= -mb^2 + mbl\cos q_4, \\
M_{35} &= -m_T l_T\left(l_T + l\cos q_5\right), \\
M_{44} &= mb^2, \\
M_{45} &= 0, \\
M_{55} &= m_T l_T^2.
\end{aligned}$$