Dynamics of Heel Strike in Bipedal Systems with Circular Feet

Josep Maria Font and József Kövecses

Abstract Energetic efficiency is a fundamental subject of research in bipedal robot locomotion. In such systems, the collision of the foot with the ground at heel strike is the main cause of energy loss during the gait. In this work, a Lagrangian framework to study the impulsive dynamics of collisions is presented. Based on the inert constraints of this event, a decomposition of the dynamics to the spaces of constrained and admissible motions is introduced. It is used to analyze the energy redistribution at heel strike in circular-feet bipeds. We present results that show the effect of the foot radius and the impact configuration on the energetic cost of walking.

Keywords Bipedal locomotion · Dynamics · Robotic walking systems · Collisions · Mechanism design

Introduction

In recent years, a large effort has been invested in the development of bipedal walking systems and in the control and analysis of gaits. In this field, energetic efficiency appears to be a major objective to increase the autonomy of those robotic systems [1]. Underactuated robots based on the idea of *passive dynamic walking* [2], appear to be energetically more efficient than anthropomorphic robots that use precise joint-angle control as in [3]. Passive dynamic walking refers to simple mechanical systems that are able to walk down a slightly inclined walkway with no external control or actuation, i.e., the gravity alone powers the motion [4–6].

The work on those walkers was primarily motivated by the drive for energy efficiency and showed that it was possible to obtain dynamically stable limit cycles, with remarkably human-like motion, without any kind of actuation. Although pure passive walking is in itself not very useful for many robotic applications, since the prototypes cannot walk on level ground, it has led to important insight regarding

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the dynamics of walking and its energetic aspects. Recently, a type of powered *passive-based* robot has appeared in the literature. These robots use minimal actuation, sensing and control to walk like a purely passive-dynamic walker but on level ground [7–9]. According to [7], these walkers use less energy and are easier to control than other anthropomorphic robots. The mechanical energy consumption of such systems is due to the loss of kinetic energy when the swing leg impacts the ground at heel strike [8]. This loss has to be compensated by actuation forces applied during the gait cycle.

The aim of this work is to gain a better understanding on the energetic aspects of such impacts that represent a change of topology. For this purpose, a Lagrangian framework to analyze the impulsive dynamics involved is introduced. Based on the post-impact inert constraints, we interpret a concept that completely decouples the dynamic characteristics of the system (including the kinetic energy) to the spaces of *constrained* and *admissible* motions [10, 11]. It will be seen that the kinetic energy decomposition at the pre-impact time yields powerful insight regarding how the kinetic energy is redistributed at heel strike.

The article is structured as follows. In Section "Dynamic Model of the Walking System" the dynamic model of the circular-feet walker is presented. Next, in Section "Lagrangian Impulsive Dynamics of Collisions" the dynamic equations for impulsive motion are introduced. Section "Energetic Analysis of Periodic Walking Motion" analyzes the energetics of walking from a novel perspective based on the decom-position of the dynamics at impact. Finally, Section "Results and Discussion" presents results regarding the influence of different parameters on the energetic losses.

Dynamic Model of the Walking System

In this work, we study a compass-gait biped with circular feet, which consists of two identical straight legs of length l and mass m, Fig. 1. Their centres of mass are at a distance $b(= l - a)$ from the hip. The radius of the feet is R and the hip is modelled as a point mass m_H located at the revolute joint between the two legs. Points P_R and P_L refer to the centres of the circumferences defining the *right* and *left* foot, respectively.

To study the general motion of the biped four generalized coordinates are required (Fig. 1). They define the vector of generalized coordinates, $\mathbf{q} = [q_1, q_2, q_3, q_4]^T$. Coordinates q_1 and q_2 give the (x, y)-position of P_R with respect to the inertial frame. Coordinate q_3 indicates the absolute orientation of the right leg, and q_4 denotes the relative angle between the two legs.

The kinetic energy of the walker can be expressed in the following form

$$T(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\, \dot{\mathbf{q}}, \tag{1}$$

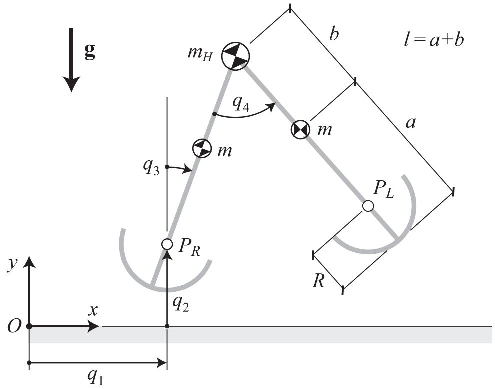


Fig. 1 Dynamic model of the compass-gait biped with circular feet

where $\dot{\mathbf{q}} = [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4]^T$ is the vector of generalized velocities, and $\mathbf{M}$ is the 4×4 symmetric mass matrix of the walker.

Lagrangian Impulsive Dynamics of Collisions

The single support phase of walking ends when the swing foot impacts the ground at heel strike. At this time, the system undergoes a sudden change of topology. The dynamics of this impulsive motion phase can be characterized by the following impulse-momentum level dynamic equations [12, 13]

$$\left[\frac{\partial T}{\partial \dot{\mathbf{q}}}\right]_{-}^{+} = \mathbf{M}\left(\dot{\mathbf{q}}^{+} - \dot{\mathbf{q}}^{-}\right) = \bar{\mathbf{f}}, \tag{2}$$

where "−" and "+" denote the pre- and post-impact instants, and $\bar{\mathbf{f}}$ are the generalized impulses. Assuming that only constraints provide forces at the impulse-momentum level, then $\bar{\mathbf{f}} = \mathbf{A}_j^T \bar{\lambda}$, where $\bar{\lambda}$ is the vector of constraint impulses and $\mathbf{A}_j$ is the Jacobian matrix. The subscript j is either R or L depending on which is the colliding foot. Since the colliding foot is required to stay in contact with the ground without slipping (i.e., the collision is inelastic), the impulsive motion can be characterized by the following *inert* constraints [12, 13]

$$\mathbf{A}_j \dot{\mathbf{q}}^{+} = 0, \tag{3}$$

where the Jacobian matrix takes one of the following expressions

$$\mathbf{A}_R = \begin{bmatrix} 1 & 0 & -R & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{4}$$

$$A_L = \begin{bmatrix} 1 & 0 & -R & (l-R)\cos q_3 + R \\ 0 & 1 & -2\,(l-R)\sin q_3 & (l-R)\sin q_3 \end{bmatrix}, \tag{5}$$

depending on which foot collides the ground; $\mathbf{A}_R$ if the *right* foot collides and vice versa. Note that when collision takes place, q_3 completely defines the configuration of the system, since at this time $q_4 = 2q_3$.

Based on the inert constraints expressed in (3), the tangent space of the configuration manifold of the walking system can be decomposed to the spaces of *constrained* and *admissible* motions for the pre-impact instant. This will then also hold for the entire duration of the contact onset, since the configuration of the system is assumed to be unchanged during this short period of time. These two subspaces can be defined so that they are orthogonal to each other with respect to the natural, mass metric of the tangent space [10, 13]. The decomposition to the two subspaces can be accomplished via two projector operators [10, 11]. The projector associated with the space of constrained motion can be written as

$$\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}_j^T\left(\mathbf{A}_j\mathbf{M}^{-1}\mathbf{A}_j^T\right)^{-1}\mathbf{A}_j, \tag{6}$$

and the projector for the space of admissible motion as

$$\mathbf{P}_a = \mathbf{I} - \mathbf{P}_c = \mathbf{I} - \mathbf{M}^{-1}\mathbf{A}_j^T\left(\mathbf{A}_j\mathbf{M}^{-1}\mathbf{A}_j^T\right)^{-1}\mathbf{A}_j, \tag{7}$$

where $\mathbf{I}$ stands for the 4×4 identity matrix. These projectors are not symmetric, which is a direct consequence of the nature of the metric of the tangent space. The use of them allows the decomposition of the generalized velocities $\dot{\mathbf{q}}$ and impulsive forces $\bar{\mathbf{f}}$ as [10, 11]

$$\dot{\mathbf{q}} = \mathbf{v}_c + \mathbf{v}_a = \mathbf{P}_c\dot{\mathbf{q}} + \mathbf{P}_a\dot{\mathbf{q}}, \tag{8}$$

$$\bar{\mathbf{f}} = \bar{\mathbf{f}}_c + \bar{\mathbf{f}}_a = \mathbf{P}_c^T\bar{\mathbf{f}} + \mathbf{P}_a^T\bar{\mathbf{f}}. \tag{9}$$

Based on (1) and (8), it can be shown that T can also be decoupled as

$$T = T_c + T_a = \frac{1}{2}\mathbf{v}_c^T\mathbf{M}\mathbf{v}_c + \frac{1}{2}\mathbf{v}_a^T\mathbf{M}\mathbf{v}_a. \tag{10}$$

To obtain this equation it was used that the projectors in (6) and (7) are orthogonal with respect to the system mass matrix, i.e., $\mathbf{P}_c^T\mathbf{M}\mathrm{P}_a = \mathbf{P}_a^T\mathbf{M}\mathrm{P}_c = \mathbf{0}$, with $\mathbf{0}$ denoting the 4×4 zero matrix. Based on the decompositions above, the equations in (2) can also be decoupled as

$$\left[\frac{\partial T_c}{\partial \mathbf{v}_c}\right]_-^+ = \mathbf{M}\left(\mathbf{v}_c^+ - \mathbf{v}_c^-\right) = \mathbf{A}_j^T\bar{\lambda}, \tag{11}$$

which are the dynamic equations for the space of constrained motion, and

$$\left[\frac{\partial T_a}{\partial \mathbf{v}_a}\right]_-^+ = \mathbf{M}\left(\mathbf{v}_a^+ - \mathbf{v}_a^-\right) = \mathbf{0}, \tag{12}$$

which gives the impulse-momentum level dynamics for the space of admissible motion. From (12) it comes immediately that $\mathbf{v}_a^+ = \mathbf{v}_a^-$, and therefore $T_a^+ = T_a^-$. Based on (3) and (8) it can also be concluded that $\mathbf{v}_c^+ = \mathbf{0}$, which means that the kinetic energy of constrained motion is completely lost at heel strike, i.e., $T_c^+ = 0$. Therefore, the total post-impact kinetic energy equals the pre-impact kinetic energy of admissible motion, $T^+ = T_a^+ = T_a^-$. Finally, from the results above, the following expression can be used to determine the post-impact velocities $\dot{\mathbf{q}}^+$

$$\dot{\mathbf{q}}^+ = \mathbf{v}_a^+ = \mathbf{v}_a^- = \mathbf{P}_a\dot{\mathbf{q}}^-, \tag{13}$$

where $\dot{\mathbf{q}}^-$ can be determined from the previous finite motion analysis of the single support phase of the gait.

Energetic Analysis of Periodic Walking Motion

Energetic aspects are very important for the optimal design and control of bipedal mechanisms [1]. In this section, the energetic balance of one complete *level-ground* and *periodic* gait is analyzed. The application of the energy theorem to the walking system between the instants after two successive collisions yields

$$\Delta T + \Delta U = W_{\text{act}} + W_{\text{col}} = 0, \tag{14}$$

where ΔT and ΔU represent the change of kinetic and potential energies respectively; W_{act} (> 0) is the work done by actuators during the walking cycle; and W_{col} (< 0) is the negative work done by the contact forces at heel strike. Note that in (14) $\Delta T = \Delta U = 0$, since the walking motion is periodic and on flat ground. The work W_{col} can be obtained from the energetic analysis of the impact. The application of the energy theorem between the pre- and post-impact instants yields

$$\left[\Delta T\right]_-^+ = T^+ - T^- = -T_c^- = W_{\text{col}} < 0, \tag{15}$$

which means that the magnitude of W_{col} equals the decrease in the kinetic energy at collision. Note that during the short impact period, the potential energy of the system does not change, $U^+ = U^-$. From (10), (13) and (15), it can be concluded that the loss of energy at collision in absolute value, as expected, is the constrained pre-impact kinetic energy, i.e., $-W_{\text{col}} = T_c^-$. We define the energetic loss per unit distance walked by the robot as

$$\eta = \frac{T_c^-}{L_S} = \frac{\frac{1}{2}\left(\dot{\mathbf{q}}^-\right)^T \mathbf{P}_c^T \mathbf{M} \mathbf{P}_c \dot{\mathbf{q}}^-}{2\,(l - R)\sin q_3 + 2Rq_3}, \tag{16}$$

where the denominator L_S accounts for the length of one step.

Results and Discussion

The aim of this section is to study the effect of the impact configuration and the foot radius on the energy dissipation per unit distance walked by the robot. The fixed dynamic parameters of the robot are $m_H = 2m = 10$ kg and $a = b = 0.5$ m ($l = 1$m). We assume that the right foot is in contact with the ground at pre-impact time, $q_2 = R$, and that the left foot collides the ground. The impact configuration is completely defined by angle q_3. We also assume that the system controls, by means of actuation, the pre-impact angular velocities: $\dot{q}_3^- = 1$ rad/s, $\dot{q}_4^- = 0$. This physically means that the pre-impact tangential component of the velocity of the colliding point is zero, which is often the case in walking motion to avoid slipping.

Figure 2 shows the pre-impact decoupling of the kinetic energy. Only the impacts that satisfy $\dot{q}_2^+ > 0$ are analyzed, this is the reason why the curves do not cover the same range of impact configurations q_3. This inequality means that the non-colliding foot is lifting from the ground after impact. If this condition is not satisfied, other constraints become active and the gait does not evolve in a natural way.

From Fig. 2 it can be observed that for a given configuration, the kinetic energy of the constrained motion T_c^- decreases with the radius R. Consequently, the kinetic energy of the admissible motion T_a^- grows with R for a fixed q_3. Since T_c^- is completely lost at heel strike, a design with a radius $R = l = 1$m is the best because in that case no energy is lost at impact. This completely agrees with [2], in which McGeer showed that a circular-feet walker with $R = l$, which he called "synthetic wheel", could walk on flat ground with no energy expenditure. From Fig. 2 it can

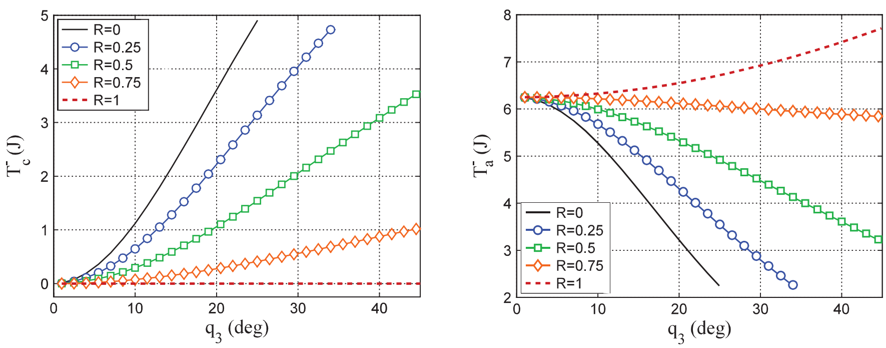
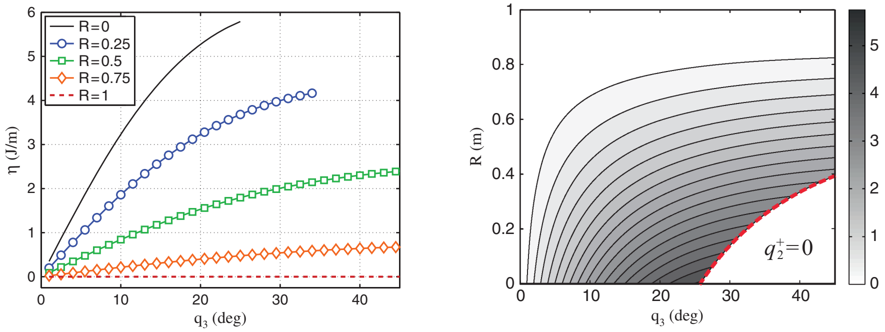


Fig. 2 Decomposed parts of the kinetic energy T_c and T_a as functions of q_3 for different values of R



(a) (b)

Fig. 3 Energy lost per unit distance η (J/m) as a function of q_3 and R

also be noticed that a low impact angle q_3 also reduces the amount of kinetic energy lost for a given radius R.

Measure η can be used to evaluate the energetic expenditure per unit distance. Figure 3a represents its value as a function of the impact angle for five values of R. Figure 3b represents η as a function of q_3 and R. From the results obtained, it can be concluded that a high value of R and low impact angles are better to reduce the energetic expenditure per unit distance. Then, in terms of energetic cost, it is better to walk a certain distance with more short steps than with less long steps.

Figure 3b is also interesting because the white area delimited by the dashed line ($\dot{q}_2^+ = 0$), shows conditions for which the non-colliding foot does not lift up, and therefore natural walking does not materialize. As it can be seen, for values of $R >$ 0.4 m the non-colliding foot always lifts up at the post-impact instant for angles $q_3 \in [0, 45°]$. The results obtained in this work are in accordance with the conclusions of previous studies of circular-feet walkers, purely passive [14] or actuated [15]. Both studies concluded that bipeds with circular feet are energetically more efficient than point-feet walkers, because the energetic losses at collision are lower.

Conclusions

In this work, we presented a Lagrangian formulation applicable to the study of the impulsive dynamics of heel strike. Based on the inert constraints that characterize this event, we introduced a decomposition of the tangent space of the system to the spaces of constrained and admissible motions. This concept, which is novel in the field of bipedal locomotion, totally decouples the impulse-momentum level dynamic equations and the kinetic energy into two independent parts. It is useful in the analysis of velocity change and energy redistribution at heel strike.

The formulation was applied to the compass-gait biped with circular feet, and we analyzed the energetic losses at collision for different impact configurations and values of the foot radius. From the obtained results, some important conclusions were drawn regarding the energetic aspects of walking in bipeds with circular foot.

We believe that the presented framework can be of considerable value to understand the dynamics and the energetics of bipedal locomotion machines. Furthermore, its generality makes it possible to analyze more complex systems like for example walkers with knees and torso. The presented work can also be of interest in the analysis, design and control of energetically efficient humanoid robots.

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