Dynamic and Energetic Analysis of Impacts in Crutch Locomotion

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ABSTRACT

Crutch walking is a widespread type of gait among elderly and injured people. The main problem of crutch locomotion is its high energy consumption which is between two and three times that of normal walking in terms of metabolic cost [1]. Furthermore, it is highly demanding for the upper-body muscles, which are not suited for such efforts. Despite these inconveniences, the psychological importance for the subjects of being able to stand and walk on their own makes this type of locomotion very common.

The swing-through crutch gait consists of four phases: crutch-stance phase, heel strike, foot-stance phase and crutch strike. In this work, we pay particular attention on the crutch strike that occurs at the end of foot-stance. This has been reported to be an important cause of energy loss during motion [2]. A four-segmental planar model of the human body is used to analyze various dynamic aspects of the crutch impact, e.g., kinetic energy redistribution, velocity change and the magnitude of contact impulses. The model and coordinates used are shown in Fig. 1, body segment parameters are based on anthropometric data.

Figure 1: Dynamic four-segmental model of the subject with crutches.

Let us consider that $t_i$ represents the time point when the crutch tip impacts the ground. This event takes place in the $[t_i^-, t_i^+]$ interval, where $t_i^-$ and $t_i^+$ represent the so-called pre- and post-impact instants. This interval is considered very short on the characteristic time scale of
the finite motion and, therefore, the configuration of the system can be assumed constant. This event can be characterized by bilateral impulsive constraints. These can be written as

$$\mathbf{A} \dot{\mathbf{q}}^+ = 0$$

(1)

where $\mathbf{q}^+$ is the post-impact array of generalized velocities, and $\mathbf{A}$ is the constraint Jacobian matrix. The above constraints define the required topology at $t_i^+$, i.e., the tip of the crutch to be in contact with the ground without slipping after the impact. Based on matrix $\mathbf{A}$, the tangent space of the system can be decomposed to two mutually orthogonal subspaces in terms of the mass metric, namely, the Space of Constrained Motion (SCM) and the Space of Admissible Motion (SAM). The decomposition to such subspaces can be accomplished via asymmetric projector operators, $\mathbf{P}_c$ and $\mathbf{P}_a$ [3]. These are used to project kinematic and kinetic quantities to the SCM and SAM, respectively. Based on them, the kinetic energy of the system can be decoupled as

$$T = T_c + T_a = \frac{1}{2} \mathbf{v}_c^T \mathbf{M} \mathbf{v}_c + \frac{1}{2} \mathbf{v}_a^T \mathbf{M} \mathbf{v}_a,$$

(2)

where $\mathbf{v}_c = \mathbf{P}_c \dot{\mathbf{q}}$ and $\mathbf{v}_a = \mathbf{P}_a \dot{\mathbf{q}}$. Assuming that muscular forces are finite, then the only impulses that need to be considered are the ones developed at the tip due to contact. These impulses are completely projected to the SCM [3] and thus, based on the dynamics analysis one can conclude that the pre-impact kinetic energy of the SCM is completely lost, $T_c^+ = 0$, and its counterpart associated with the SAM stays in the system after impact, $T_a^+ = T_a^- \leq T^-$. The previous energetic analysis was used to obtain guidelines for optimal crutch selection or crutch-use teaching. These can be applied to reduce energy consumption, which may also lead to a decrease of the muscular fatigue of the subject. The following points summarize some of the results obtained using the presented approach:

- A crutch longer than the conventional reduces the energy loss per unit distance and also reduces the magnitude of the contact impulses developed at the tip of the crutch.
- A crutch shorter than the conventional facilitates the separation of the feet at post-impact time and thus, less push-off effort is needed to start the crutch-stance phase.
- A small angle $\theta_1$ at impact, Fig. 1, is better to minimize the energy loss at crutch strike.
- A torso leaning forward ($\theta_2 > 0$, Fig. 1) yields less energy consumption per unit distance and reduces the magnitude of contact impulses.

References

