Configuration-dependent performance indicators for the analysis of foot impact in running gait

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Abstract

The dynamic analysis of human running gait is an important topic of research aimed at improving the performance of athletes and reducing the number of running-related injuries. Foot impact is a critical event during the running cycle, since the high contact forces developed on the foot are the main cause of energy loss during motion and may contribute to some injuries (e.g., tibial stress fractures or plantar fasciitis). The foot strike pattern may vary between runners, habitually barefoot runners tend to land on the fore-foot, while shod runners usually collide with the heel [1]. Experimental analysis presented in [1] showed that fore-foot strike (FFS) generates lower collision forces than rear-foot strike (RFS). In the same study, some model-based analysis was done using a simple 1-DOF impact model. In a further study [2], authors used a two-segment (tibia and foot) 4-DOF model to conduct model-based analysis and reached similar conclusions.

The main goal of this paper is to investigate the impact dynamics at foot-strike by using two performance indicators, that account for the intensity of the generated forces and the tendency of the runner to slide during the impact interval. Both indicators are evaluated for the configuration of the runner at impact, which will vary depending on the foot strike pattern. For this purpose, a planar whole-body model of the runner is used (Figure 1). It is composed of seven segments: head-arms-trunk (HAT), two thighs, two shanks and two feet. They are linked by revolute joints modeling the hip, knee and ankle joints. We use a set of $n = 9$ independent generalized coordinates that form vector $\mathbf{q} = [x_H \ z_H \ \theta_{HAT} \ \alpha_{Hr} \ \alpha_{Kr} \ \alpha_{Ar} \ \alpha_{Hl} \ \alpha_{Kl} \ \alpha_{Al}]^T$, defined in Figure 1. The first two correspond to the Cartesian coordinates of the hip, the third one is the absolute orientation of the HAT, and the remaining six correspond to the lower limb relative joint angles (hip, knee and ankle, both for the right and left legs).

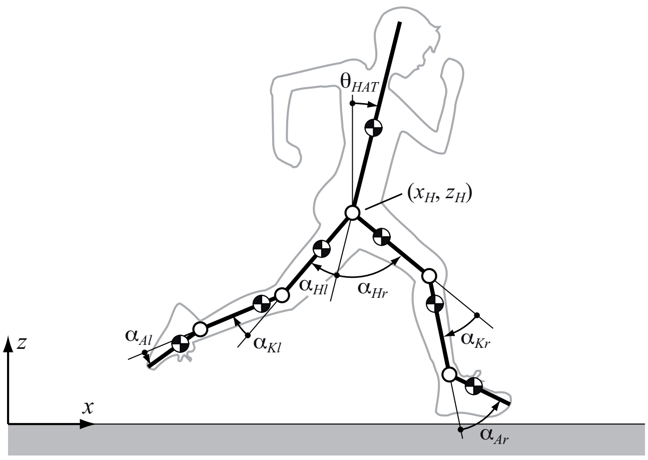

Figure 1. Biomechanical model of a runner in the sagittal plane.

The velocity of the foot colliding point Q can be related to the generalized velocities through the $2 \times n$ Jacobian matrix $\mathbf{A}$: $\mathbf{v}(Q) = \mathbf{A}(\mathbf{q})\,\dot{\mathbf{q}}$. At impact configuration, the Jacobian matrix can be decomposed into two $1 \times n$ arrays specific for the normal and tangential components of $\mathbf{v}(Q)$:

$$\mathbf{v}(Q) = \left\{ \begin{array}{c} v_n(Q) \\ v_t(Q) \end{array} \right\} = \left[\begin{array}{c} \mathbf{A}_n \\ \mathbf{A}_t \end{array} \right] \dot{\mathbf{q}}. \quad (1)$$

Denoting by t^- and t^+ the time instants just before and just after impact respectively, the collision end condition can be written as

$$v_n^+ (Q) = \mathbf{A}_n \dot{\mathbf{q}}^+ = 0, \tag{2}$$

which represents the new constraint condition of the system at post-impact time t^+. Using and impulsive approach, and considering that the only impulsive forces are the ground contact forces (both normal and tangential) at point Q, the system equations of motion to study the foot strike take the following form [3]

$$\mathbf{M}\, d\dot{\mathbf{q}} = \mathbf{A}_n^T dP_n + \mathbf{A}_t^T dP_t \equiv \mathbf{A}^T \left\{ \begin{array}{c} dP_n \\ dP_t \end{array} \right\}, \tag{3}$$

where $\mathbf{M}$ is the $n \times n$ mass matrix of the system, and dP_n and dP_t are the differential normal and tangential contact impulses at Q. Since the configuration is assumed to be constant during the whole impact interval, matrices $\mathbf{M}$, $\mathbf{A}_n$ and $\mathbf{A}_t$ are also constant.

Based on this formulation, we present two configuration-dependent performance indicators that can be used to predict impact behaviour. It was already said that when the foot collides the ground, a new constraint condition is established on the system, equation (2). The kinetic energy associated with this constrained motion T_c can be used as an indicator to represent the intensity of the impact. It was reported in [4] that the pre-impact value of T_c is proportional to the impulse of the constraint force. In the same work, some experimental measurements also showed that this quantity is also proportional to the peak constraint force. The pre-impact value of T_c is defined as

$$T_c^- = \frac{1}{2} \left(\dot{\mathbf{q}}^-\right)^T \mathbf{P}_c^T \mathbf{M} \mathbf{P}_c \dot{\mathbf{q}}^-, \tag{4}$$

where $\dot{\mathbf{q}}^-$ is the velocity vector at the instant just before foot strike and $\mathbf{P}_c$ is the following projection matrix [4]:

$$\mathbf{P_c} = \mathbf{M}^{-1} \mathbf{A}_n^T \left(\mathbf{A}_n \mathbf{M}^{-1} \mathbf{A}_n^T\right)^{-1} \mathbf{A}_n. \tag{5}$$

Another interesting indicator is the critical value of the friction coefficient μ_c, which can be obtained through the following expression [5]

$$\mu_c = \left| \frac{\mathbf{A}_t \mathbf{M}^{-1} \mathbf{A}_n^T}{\mathbf{A}_t \mathbf{M}^{-1} \mathbf{A}_t^T} \right|. \tag{6}$$

Depending on the value of μ_c, the value of the actual friction coefficient μ and the pre-impact tangential velocity of Q, there are four different $v_t\,(Q)$ evolutions during the impact. Those situations were discussed in [3] for the case of swing-through crutch gait. Note that the value of μ_c depends on the runner anthropometric parameters and the body configuration. In general, in order to avoid sliding during impact, the following condition should be satisfied: $\mu \geq \mu_c$. So, the runner should collide the ground in a way that minimizes μ_c to prevent foot sliding.

The presented performance indicators (μ_c and T_c^-) will be calculated for different modes of running (e.g., FFS or RFS), body configurations and pre-impact velocities. These conditions will be defined based on published studies and experimental measurements taken in a biomechanics laboratory. Based on the results, detailed conclusions will be drawn regarding the dynamic implications of running using different foot strike strategies.

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