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PARAMETRIC ANALYSIS OF IMPACT CONFIGURATIONS IN CRUTCH WALKING

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Abstract. *In this paper, sensitivity and parametric analyses of impact intensity in crutch walking are investigated. The selected parameters of the crutch for these analyses are chosen as the mass, the second moment of inertia, and the location of the center of mass. The intensity of impact can be characterized by the pre-impact kinetic energy of the constrained motion space. Consequently, this part of the kinetic energy is selected as the performance indicator for our parametric analysis. The crutch walking is modeled based on a four-segment rigid body model with joints in the hip, the shoulder, and the elbow. The Sensitivity and Parametric Analyses Toolbox (SPAT) developed in the Applied Dynamics Group at McGill University is employed for the analysis.*

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1 INTRODUCTION

Crutch walking gait is a primary means for rehabilitation applications for foot injuries. The crutch impact with ground causes significant energy loss. High impact intensity can also be harmful for body joints [6], [7]. Furthermore, required metabolism of the body is increased due to the energy loss by impact [8], [9]. Human configuration and crutch length can influence the intensity of crutch-ground impact and the energy loss due to this impact [1]. More generally speaking, it is not only the crutch length that can affect the impact intensity. Physical parameters of the crutch, namely, the mass, the second moment of inertia and the location of the center of mass can also affect this energy loss. One way to minimize the energy loss of crutch impact is to modify the nominal physical parameters of the crutch. Sensitivity and parametric analyses may be used to assist such an optimization.

The crutch parametric analysis is carried out by exploiting a Sensitivity and Parametric Analysis Toolbox (SPAT) developed by the Applied Dynamics Group in the Department of Mechanical Engineering at McGill University [2]. In order to use this toolbox, a dynamic model of the system as well as a performance indicator which reflects the behaviour of the system should be formulated. The output of the SPAT is the parameter variation distribution with minimum/maximum effect on the performance indicator. One of the main ideas of the parametric analysis method utilized in this toolbox is related to the interpretation of a linear space based on sensitivity derivatives, in which the associated parameter variations represent components of vectors. The associated eigenvectors represent how minimum/maximum effect can be achieved on the given performance indicator by changing the parameters. By exploiting the SPAT, the physical parameter variation distribution of the crutch with minimum/maximum effect on the impact intensity can be achieved.

2 GENERAL FORMULATION

In order to perform parametric analysis of the crutch walking gait, the dynamics of the system as well as a performance indicator reflecting behaviour of the system which is of interest for our analysis should be defined. The crutch walking is modeled based on a four-segment rigid body model. These segments are leg, torso, upper arm and forearm with three joints associated with hip, shoulder and elbow while the forearm and crutch is considered as a single rigid body. We choose generalized coordinates as $\mathbf{q} = [x, y, \theta_1, \theta_2, \theta_3, \theta_4]$ (Fig. 1) and write the dynamics equations as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{c}(\dot{\mathbf{q}}, \mathbf{q}, t) - \mathbf{f}_a - \mathbf{f}_c = \mathbf{0} \tag{1}$$

where $\mathbf{f}_a$, $\mathbf{c}$, and $\mathbf{f}_c$ respectively represent 6×1 generalized applied forces, Coriolis and centrifugal effects, and generalized constraint forces, and $\mathbf{M}$ is the 6×6 generalized mass matrix. The impulsive event can be characterized by inert constraints which imposes that the post-impact velocity of the crutch end point is zero and via that they also characterize the contact forces, $\mathbf{f}_c$, [3].

Furthermore, an appropriate performance indicator which determines the intensity of impact should be defined. Here, the performance indicator is formulated based on the decomposition of the tangent space of the configuration manifold to two subspaces: constrained motion space, and its complement, admissible motion space [4]. It has been previously shown that by this decomposition, the pre-impact kinetic energy of constrained motion space can characterize the intensity of the contact transition for cases where the impact can be represented by either single-point contact or multiple-point contact, if all contact points are in the same phase of transition

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[3]. Consequently, this part of the kinetic energy is considered as the performance indicator for our analysis and can be formulated as

$$T_c^- = \frac{1}{2}(\dot{\mathbf{q}}^-)^T \mathbf{P}_c^T \mathbf{M} \mathbf{P}_c (\dot{\mathbf{q}}^-) \tag{2}$$

where T_c^- is the pre-impact kinetic energy of the constrained motion space, $\dot{\mathbf{q}}^-$ is the pre-impact generalized velocity, and $\mathbf{P_c}$ is the projection matrix to the constrained motion space. It can be shown that this projection matrix can be represented as

$$\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}^T(\mathbf{A}\mathbf{M}^{-1}\mathbf{A}^T)^{-1}\mathbf{A} \tag{3}$$

where $\mathbf{A}$ is the constraint Jacobian matrix associated with the impulsive constraints representing the directions constrained by the impact of the end point of the crutch [4].

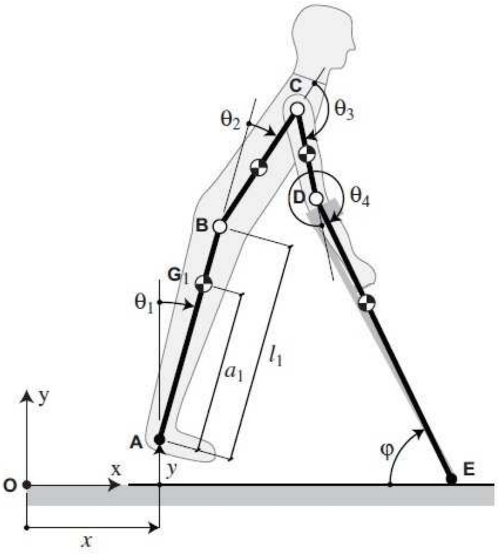

Figure 1: Crutch Walking Model

3 PARAMETRIC ANALYSIS APPROACH

Sensitivity and parametric analyses are performed via the SPAT toolbox. The main idea of the parametric analysis method implemented in this toolbox is related to the interpretation of a linear space based on sensitivity derivatives, in which the associated parameter variations represent components of vectors. Here, we are interested in analyzing three design parameters that can be collected in an array as $\mathbf{p} = [m_4, J_4, a_4]^T$, where these parameters are respectively the mass, the second moment of inertia, and the location of the center of mass of the crutch. If we expand the performance indicator (T_c^-) into a Taylor series, and by disregarding the higher order terms we can write

$$\Delta T_c^- = \frac{\partial T_c^-}{\partial \mathbf{p}} \Delta \mathbf{p} \tag{4}$$

The sensitivity derivatives of the performance indicator ($\frac{\partial T_c^-}{\partial \mathbf{p}}$) can be calculated via different existing analytical or numerical methods. In this work direct differentiation was used. The crutch walking analysis is performed in different body configurations. Consequently, we will consider that Eq. (4) is evaluated for a number of measurement points to characterize a finite range of motion for the system. We can then write

$$\mathbf{C}\mathbf{z} = \boldsymbol{\kappa} \tag{5}$$

where $\mathbf{C} = [(\frac{\partial T_c^-}{\partial \mathbf{p}})_1, ..., (\frac{\partial T_c^-}{\partial \mathbf{p}})_b]^T$ and $\boldsymbol{\kappa} = [\Delta T_{c1}^-, ..., \Delta T_{cb}^-]^T$, where b is the number of measurement points, and $\mathbf{z}$ is the parameter variation vector [3].

If we determine the eigenvectors of $\mathbf{C}^T\mathbf{C}$, then the characteristic eigenvectors $(\mathbf{z}_1^*, \mathbf{z}_2^*, \mathbf{z}_3^*)$ are obtained, which characterize the effects of parameter distributions on the performance indicator variation. The eigenvectors $\mathbf{z}_1^*$, and $\mathbf{z}_3^*$ are respectively associated with the smallest and largest eigenvalues. Considering Eq. (5), parameter changes along the largest eigenvector results in the largest performance indicator variations. This can be more clearly comprehended by considering the geometric representation of the equation $\mathbf{z}^T\mathbf{C}^T\mathbf{C}\mathbf{z} = \boldsymbol{\kappa}^T\boldsymbol{\kappa}$. We know from linear algebra that this quadratic form defines a generalized ellipsoid of which the eigenvectors of $\mathbf{C}^T\mathbf{C}$ are the principal axes. The eigenvectors associated with the smallest/largest eigenvalues are the minor/major axes of the ellipsoid. Consequently, changes along the smallest/largest eigenvector result in minimum/maximum effect on the performance indicator variation. These eigenvectors can be useful indicators of parameters with the lowest/largest effect on the performance indicator. The parameter distribution ratios as given in $\mathbf{z}_3^*$ have the most effect on the performance indicator. Similar statement is true about $\mathbf{z}_1^*$ for the minimum effect. In other words, these vectors represent directions in the space of parameter variation with the minimum and maximum effect on the performance indicator.

4 RESULTS AND DISCUSSIONS

Simulation was done for a male crutch walker with standard anthropometric parameter values shown in Table 1 [5]. The dynamic model of the system is formulated based on Eq. (1) and the pre-impact kinetic energy of constrained motion is selected as the performance indicator. The mass, the second moment of inertia, and the location of the center of mass of the crutch are the selected parameters. The aim of the analysis is to determine the parameter variation distribution of the crutch which minimizes the intensity of the crutch impact with the ground. Sensitivity and parametric analyses are performed via SPAT which has been implemented based on the formulations discussed in the previous section. For the purpose of the parametric analysis, matrix $\mathbf{C}$ in Eq. (5) is needed to be formed based on b measurement points. Consequently, 100 different pre-impact configurations are selected to cover as many impact configurations as possible to represent different possibilities of crutch walking gait associated with different walking patterns, and the sensitivity derivatives in Eq. (4) were determined. Finally, the sensitivity and parametric analyses are performed using this recorded information. The selected pre-impact configurations correspond to reported joint ranges in crutch gait [1] which are

$$\begin{aligned}
5^\circ \leq \theta_1 \leq 20^\circ \\
0^\circ \leq \theta_2 \leq 10^\circ \\
85^\circ \leq \theta_3 \leq 150^\circ \\
270^\circ \leq \theta_4 \leq 360^\circ
\end{aligned} \tag{6}$$

Generalized velocities are chosen as $\dot{\theta}_1 = 1\frac{rad}{s}, \dot{\theta}_2 = \dot{\theta}_3 = \dot{\theta}_4 = 0\frac{rad}{s}$. These values are selected based on the values reported in [1].

The characteristic eigenvector, $\mathbf{z}_3^*$, can be determined as

$$\mathbf{z}_3^* = \begin{bmatrix} 0.0373 \ \ kg \\ 0.4167 \ \ kg.m^2 \\ 0.9083 \ \ m \end{bmatrix} \tag{7}$$

With the aid of this eigenvector, the maximum effect of parameter changes on the performance indicator can be investigated. Considering $\mathbf{z}_3^*$, it can be concluded that among the crutch parameters, the location of the center of mass has the biggest effect on the impact intensity. Furthermore, $\mathbf{z}_3^*$ is the direction in the parameter space along which the maximum effect on the performance indicator can be achieved. Consequently, if we select our parameter variation in this direction, we can achieve maximum effect on the impact intensity. A small change in the parameter vector, in this case a variation with magnitude of 0.1, can be selected as

$$\Delta\mathbf{p}^* = 0.1\mathbf{z}_3^* = \begin{bmatrix} 0.00373 \ \ kg \\ 0.04167 \ \ kg.m^2 \\ 0.09083 \ \ m \end{bmatrix} \tag{8}$$

This parameter variation results in a parameter distribution as

$$\mathbf{p}^* = \mathbf{p}_N - \Delta\mathbf{p}^* \tag{9}$$

where $\mathbf{p}_N$ is associated with the nominal parameter.

Parameter	Leg	Torso	Arm	Forarm and Crutch
$m_i \ \ [kg]$	28.6	34.4	4.2	3.4
$J_i \ \ [kg.m^2]$	0.6930	1.1672	0.04416	0.4026
$l_i \ \ [m]$	0.882	0.682	0.309	1.281
$a_i \ \ [m]$	0.564	0.381	0.148	0.301

Table 1: Anthropometric body segment parameters of the four-segment model

Simulations for the performance indicator variation associated with this crutch parameter distribution ($\mathbf{p}^*$) and the nominal crutch parameter values ($\mathbf{p}_N$) were performed. Based on the discussed parametric analysis formulation, $\Delta\mathbf{p}^*$ change should have a significant effect on the performance indicator. To show this, we consider the following index

$$\frac{T_{c_N}^- - T_c^{-*}}{T_{c_N}^-} \tag{10}$$

where $T_{c_N}^-$ and T_c^{-*} are respectively the performance indicators (pre-impact kinetic energy of constrained motion space) associated with $\mathbf{p}_N$ and $\mathbf{p}^*$.

This index can show the percentage of the change of the performance indicator value while using the new set of parameters in comparison with the nominal parameter values. Based on the results, the parameter distribution, $\mathbf{p}^*$, resulted in lower values for the pre-impact kinetic

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energy of the constrained motion space, up to 27% decrease in comparison with the nominal parameters. Also, the mean value of the pre-impact constrained motion space kinetic energy for those 100 different configurations showed 6% decrease after using the parameter distribution $\mathbf{p}^*$. These results indicate that via applying minor modifications to the nominal parameter values of the crutch, lower impulses can be achieved during the crutch walking gait. As a result, less impact energy loss will occur. Furthermore, smaller forces will be transmitted to the body joints as a result of the ground impact. These can significantly contribute to increase the level of comfort of the crutch walker while using this modified crutch design.

5 CONCLUSIONS

In this paper, sensitivity and parametric analyses of impact in crutch walking were investigated. Selected parameters of the crutch were the mass, the second moment of inertia, and the location of the center of mass. It has been previously shown that the intensity of impact can be characterized by the pre-impact kinetic energy of the constrained motion space. This was selected as the performance indicator. The crutch walking was modeled based on a four-segment rigid body model with joints in the hip, the shoulder, and the elbow. The variation in the parameter distribution with the maximum effect on the constrained motion space kinetic energy (and consequently the contact force) was obtained. Performance indicator values associated with the new parameters obtained were compared with the ones for the nominal crutch parameter values. This comparison verifies the importance of the parameter values determined in the reported analysis.

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