Modelling Dynamic Walking Bipeds as Variable Topology Mechanical Systems

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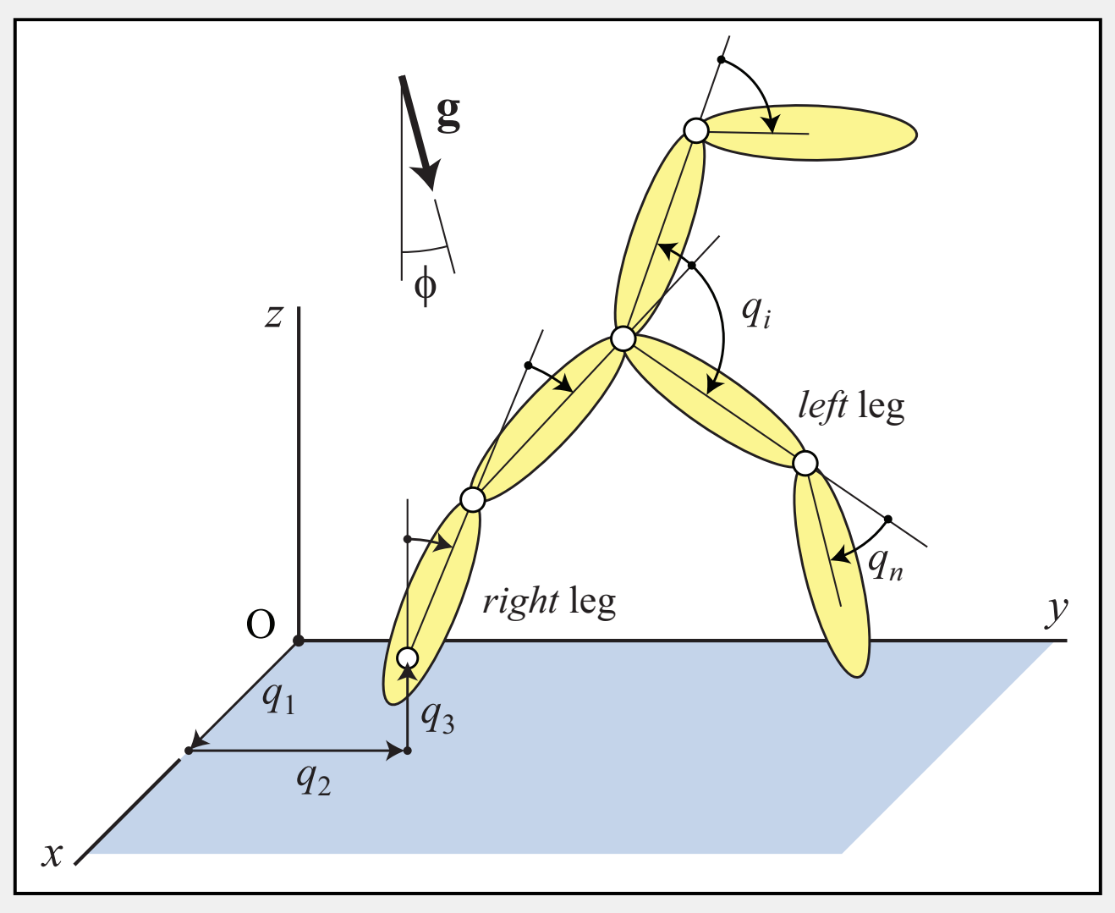
Introduction

- **Variable topology mechanical systems** frequently appear in biomechanics and robotics.
- Phases of walking: **Finite motion** single-support phase and **impulsive motion** at heel strike.
- The topology of the system changes at **heel strike** when some constraints are added and other become passive.
- The heel strike is a dominant event since it is the main cause of **energetic loss** during the walking motion.
- We present a novel **Lagrangian approach** to analyze the finite and impulsive motion dynamics of walking.

Dynamics Modelling

General Description of the System Configuration



$\mathbf{q} = [q_1, \ldots, q_n]^T$ Non-minimum set of **generalized coordinates**

$$\left.\begin{aligned} \mathbf{v}_R &= \mathbf{A}_R\,\dot{\mathbf{q}} \\ \mathbf{v}_L &= \mathbf{A}_L\,\dot{\mathbf{q}} \end{aligned}\right\}$$ Velocities of points subjected to **physical constraints**

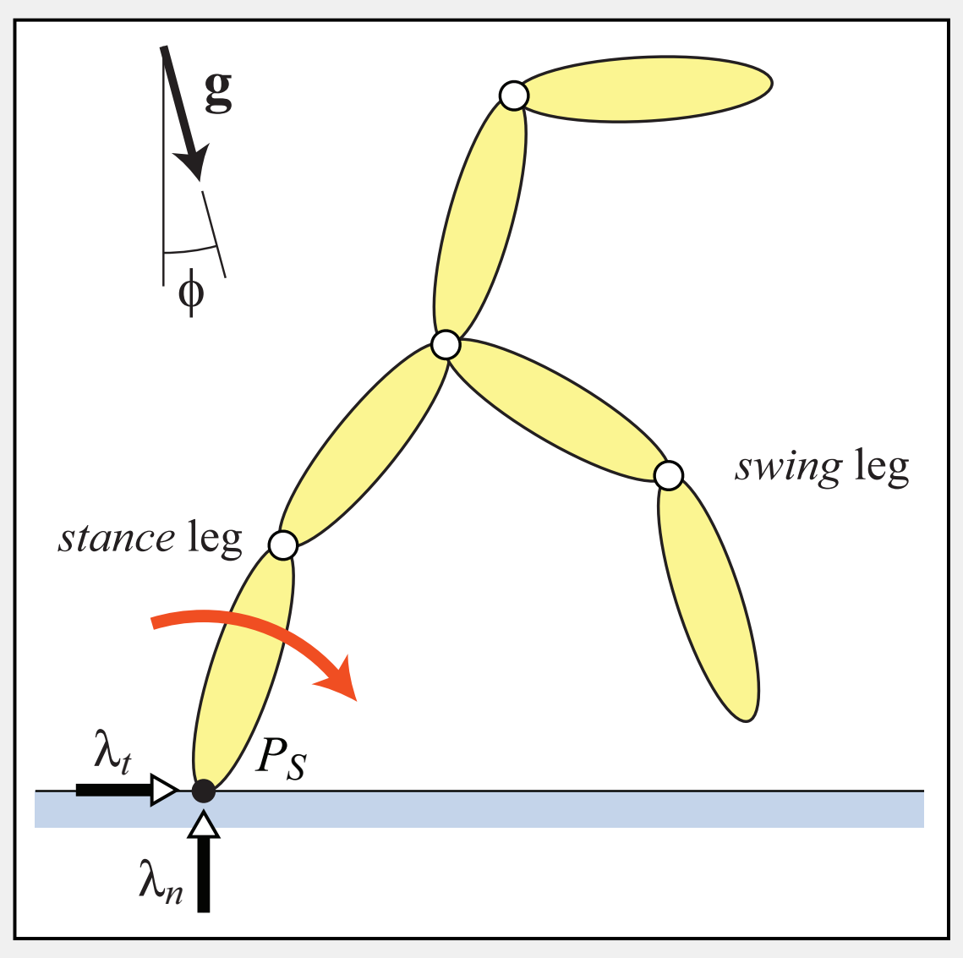
Finite Motion of the Single-Support Phase

Equations of Motion:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{u}(\mathbf{q}) = \mathbf{f}_A + \mathbf{A}_S^T\boldsymbol{\lambda}_S$$

$\mathbf{A}_S\,\dot{\mathbf{q}} = \mathbf{0}$ ***Bilateral*** Constraints (S is R or L)

$\mathbf{M}$: Mass matrix
$\mathbf{c}$: Coriolis and centrifugal effects
$\mathbf{u}$: Generalized conservative forces
$\mathbf{f}_A$: Generalized applied forces
$\mathbf{A}_S$: Constraint Jacobian
$\boldsymbol{\lambda}_S$: Contact forces on the stance foot



Impulsive Motion of Heel Strike (Topology Transition)

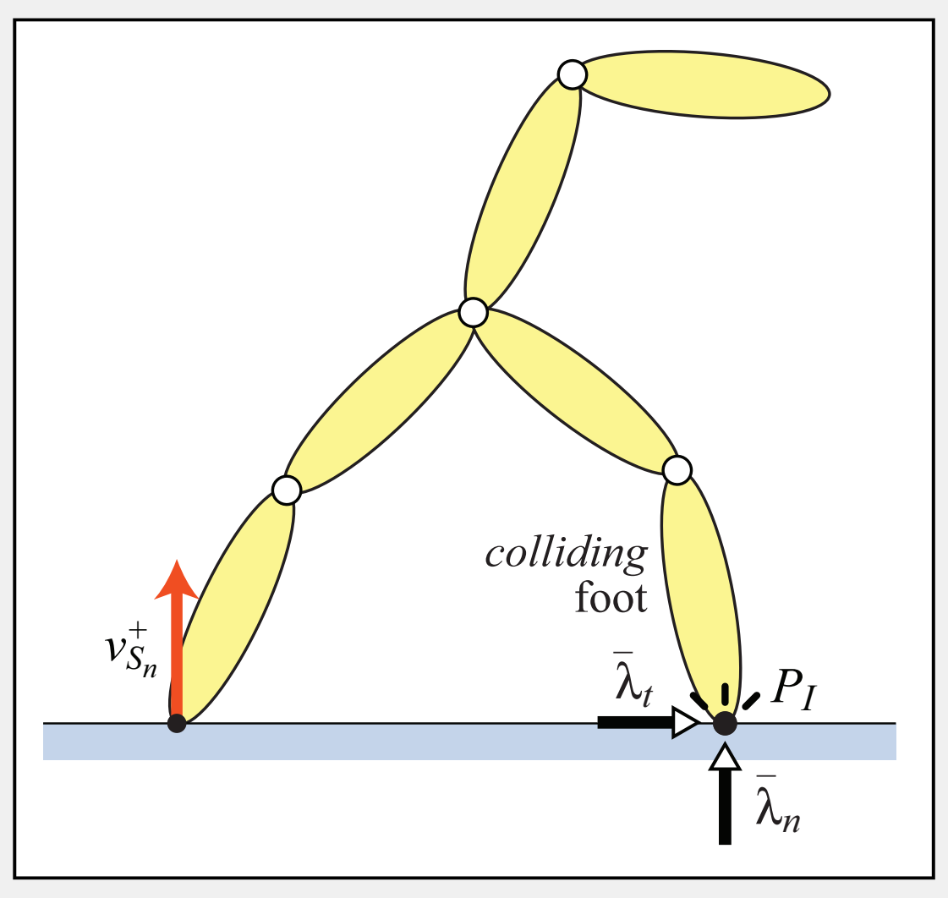
Dynamic Equations for Impulsive Motion:

$$\left[\frac{\partial T}{\partial \dot{\mathbf{q}}}\right]_-^+ = \mathbf{M}\left(\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-\right) = \mathbf{A}_I^T\,\bar{\boldsymbol{\lambda}}_I$$

$\mathbf{A}_I\,\dot{\mathbf{q}}^+ = \mathbf{0}$ ***Inert*** Constraints (S is R or L)

$v_{S_n}^+ = \mathbf{B}_S\dot{\mathbf{q}}^+ > 0$ ***Lift-off*** Condition

$\mathbf{M}$: Mass matrix at impact configuration
$\mathbf{A}_I$: Constraint Jacobian
$\bar{\boldsymbol{\lambda}}_I$: Contact impulses on the colliding foot



Decomposition and Energetics

- Based on the following projection operators

$$\mathbf{P}_c = \mathbf{M}^{-1}\mathbf{A}_I^T\left(\mathbf{A}_I\mathbf{M}^{-1}\mathbf{A}_I^T\right)^{-1}\mathbf{A}_I \quad \text{and} \quad \mathbf{P}_a = \mathbf{I} - \mathbf{P}_c$$

- The **generalized velocities and forces** can be decoupled as

$$\dot{\mathbf{q}} = \mathbf{P}_c\dot{\mathbf{q}} + \mathbf{P}_a\dot{\mathbf{q}} = \mathbf{v}_c + \mathbf{v}_a \quad \text{and} \quad \mathbf{f} = \mathbf{P}_c^T\mathbf{f} + \mathbf{P}_a^T\mathbf{f} = \mathbf{f}_c + \mathbf{f}_a$$

- This gives a complete decomposition of the **dynamic equations** and the **kinetic energy** of the system

$$\left[\frac{\partial T_c}{\partial \mathbf{v}_c}\right]_-^+ = \mathbf{M}\left(\mathbf{v}_c^+ - \mathbf{v}_c^-\right) = \mathbf{A}_I^T\,\bar{\boldsymbol{\lambda}}_I$$

Space of
Constrained Motion

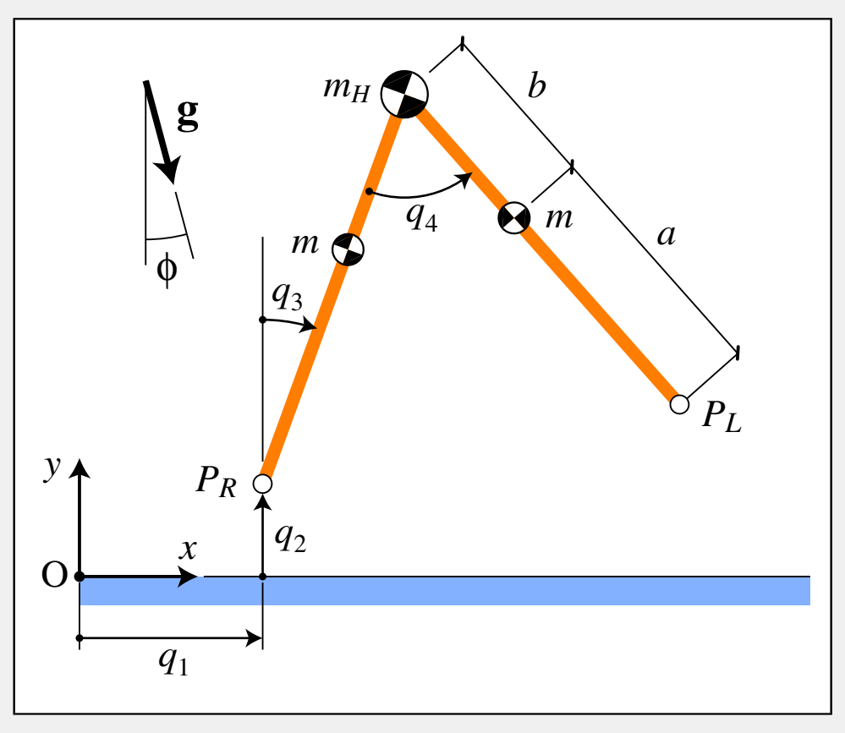
$$\left[\frac{\partial T_a}{\partial \mathbf{v}_a}\right]_-^+ = \mathbf{M}\left(\mathbf{v}_a^+ - \mathbf{v}_a^-\right) = \mathbf{0}$$

Space of
Admissible Motion

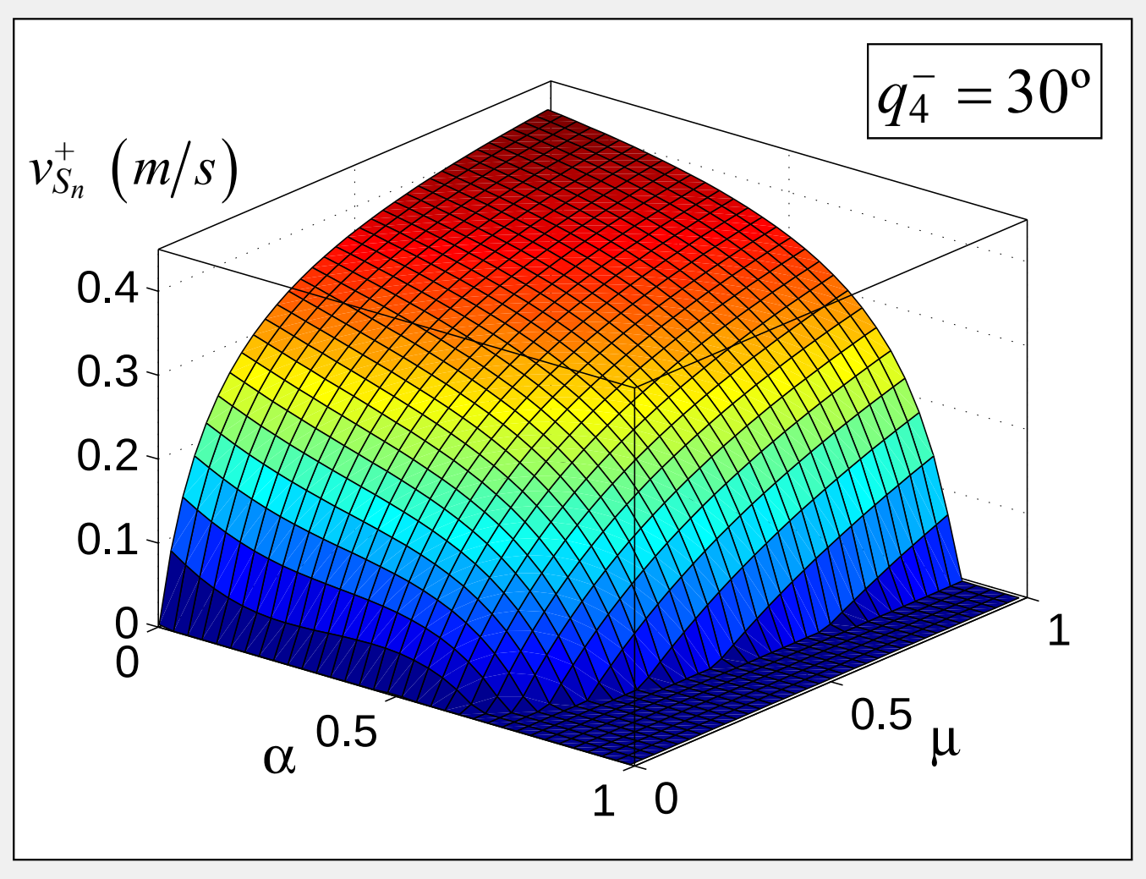
$$T^- = T_c^- + T_a^- = \underbrace{\frac{1}{2}\left(\mathbf{v}_c^-\right)^T\mathbf{M}\,\mathbf{v}_c^-}_{\textbf{LOST at Heel Strike}} + \underbrace{\frac{1}{2}\left(\mathbf{v}_a^-\right)^T\mathbf{M}\,\mathbf{v}_a^-}_{\textbf{STAYS in the System}}$$

Results. Compass-Gait Biped

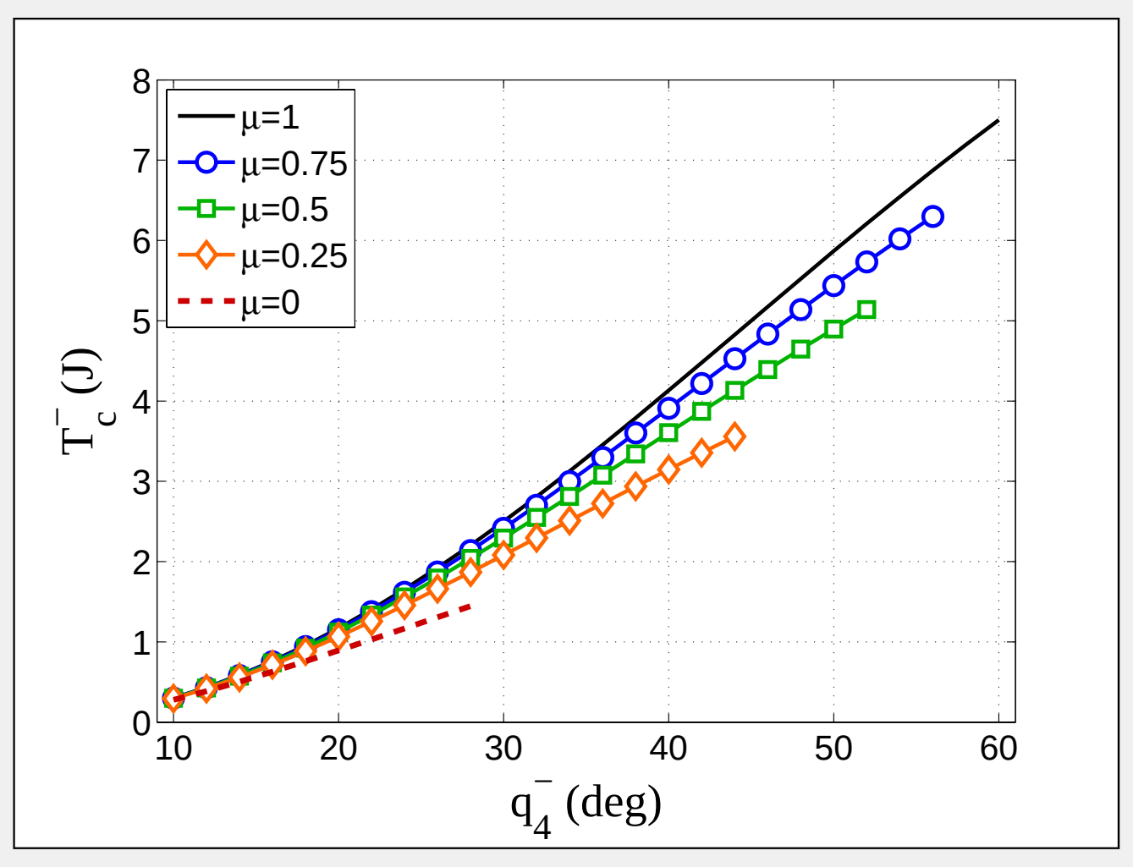
- We applied the method to a **compass-gait biped**.

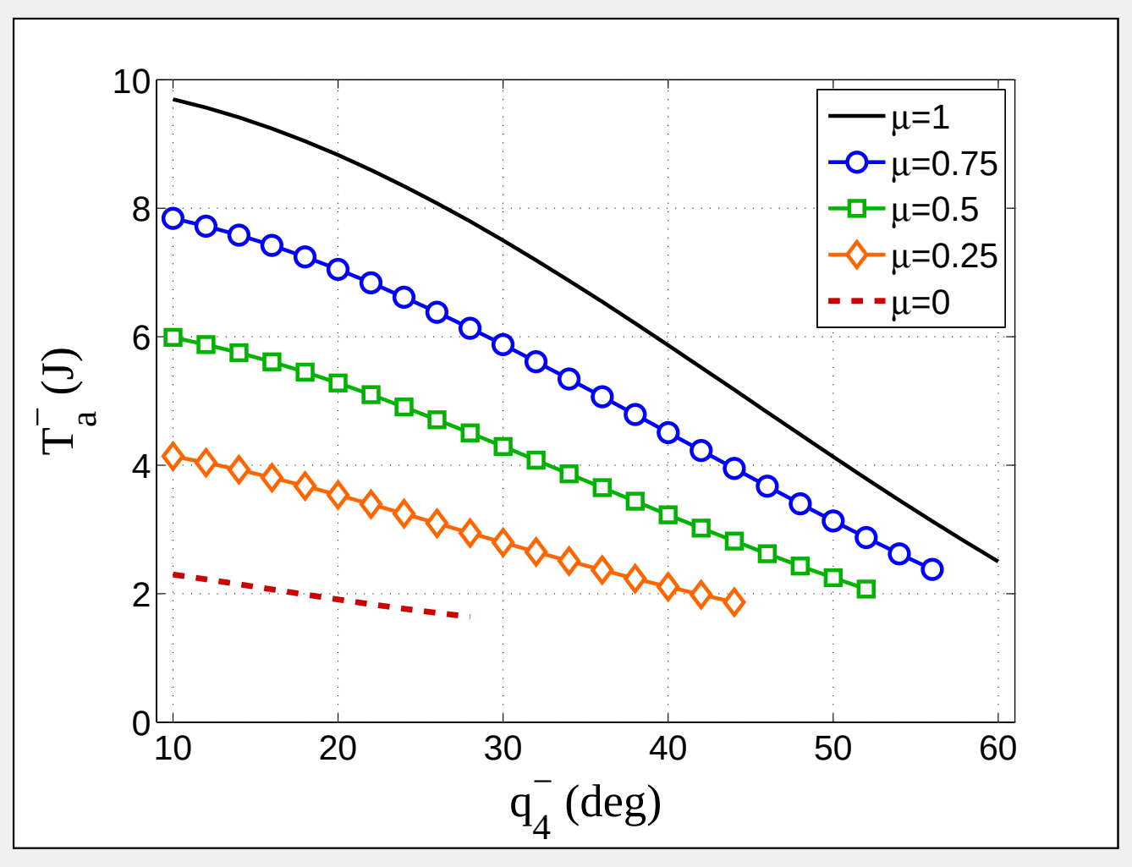


$$\alpha = \frac{a}{a+b}, \qquad \mu = \frac{m_H}{m_H + 2m}$$



- **Kinetic energy decomposition** at pre-impact time.





Contributions of the Work

- The formulation holds for a **general walking system** and makes it possible to analyze **different types of bipedal locomotion** (walking, sliding, running).
- The **dynamics decoupling** at heel strike gives insight into the **velocity change** and **energy redistribution** that occurs when the topology of the system changes.