Consistent Triangulation for Mobile Robot Localization
Using Discontinuous Angular Measurements

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• The university is located in **Barcelona** (Spain).
• The university is located in **Barcelona** (Spain).
Lines of research of the Department:

• Kinematics and Localization of Mobile Robots.

• Dynamics and Impact of Multibody Systems.

• Technologies for Disabled People.

• Mechanical Design and Manufacturing.

• Automotive Engineering.

• Acoustics and Vibrations.
Introduction

Geometric Localization Using Triangulation

Error Analysis for the Robot Position

Dynamic Localization From Discontinuous Measurements

Experimental Results

Conclusions

Design of an Automatic Omnidirectional Wheelchair
Research on Mobile Robotics at UPC

- Projects supported by the Research Network in Advanced Production Techniques:
Sensors used for the localization of the robot

- **Laser Positioning System (LPS):** laser scanner and reflective landmarks (external measurements)
- **Angular encoders** at the motor axes (internal *odometric* measurements)
Laser Positioning System

Introduction
Laser Positioning System

Position of landmarks $R_i$

Angular measurements $\theta_i$

Geometric Triangulation

Position of the scanner centre

Orientation of the robot

Introduction
Robot in motion: *Dynamic Localization Problem*

- The landmarks are detected from different robot poses.
- The triangulation methods **cannot be consistently applied** using the direct sensor measurements.
Objectives of the work

- Development of a **dynamic angular estimation algorithm** in order to guarantee the consistent use of triangulation when the robot moves.
  - Adequate accuracy (mm, mrad)
  - Robust to outliers
  - Dynamic error filtering

- **Test the accuracy of the proposed localization method.**
  
  Computer simulations
  Experiments
Structure of the localization approach

(1) Data Fusion Problem

(2) Geometric Problem
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Experimental Results

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Definition of the geometric problem

Determine the position and the orientation of the robot, from the angular measurements $\theta_1, \theta_2, \theta_3$ (relative to the robot frame) associated with the lines from P to three known landmarks R1, R2, R3.
Definition of the geometric problem

- Determine the position and the orientation of the robot, from the angular measurements $\theta_1$, $\theta_2$, $\theta_3$ (relative to the robot frame) associated with the lines from P to three known landmarks R1, R2, R3.

- Position of R1, R2, R3
- Angular measurements $\theta_1$, $\theta_2$, $\theta_3$

- Position of P: $p = \{x, y\}^T$
- Robot orientation: $\psi$

Geometric triangulation
Triangulation based on circle intersection

- The angle $\alpha_{ij} (= \theta_j - \theta_i)$ between the lines connecting P and any two landmarks $R_i$ and $R_j$, constrains the robot position to be on a circular arc.

\[ \alpha_{ij} = \theta_j - \theta_i \]
Triangulation based on circle intersection

- The angle $\alpha_{ij} (= \theta_j - \theta_i)$ between the lines connecting P and any two landmarks $R_i$ and $R_j$, constrains the robot position to be on a circular arc.
- The use of **three landmarks** $(R_1, R_2, R_3)$ allows to determine the position of P from the intersection of the arcs associated with each pair of landmarks.

\[\alpha = \theta_2 - \theta_1 \quad \Rightarrow \quad f_\alpha(x, y, \alpha) = 0\]
Triangulation based on circle intersection

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\[ \alpha \equiv \theta_2 - \theta_1 \quad \rightarrow \quad f_\alpha(x, y, \alpha) = 0 \]

\[ \beta \equiv \theta_3 - \theta_2 \quad \rightarrow \quad f_\beta(x, y, \beta) = 0 \]
Triangulation based on circle intersection

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\[
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\gamma \equiv \theta_3 - \theta_1 \quad \rightarrow \quad f_\gamma (x, y, \gamma) = 0
\]
Triangulation based on circle intersection

- The angle $\alpha_{ij} (= \theta_j - \theta_i)$ between the lines connecting $P$ and any two landmarks $R_i$ and $R_j$, constrains the robot position to be on a circular arc.
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\[ \gamma = \theta_3 - \theta_1 \quad \Rightarrow \quad f_\gamma(x, y, \gamma) = 0 \]

**Calculation of position:**

\[
\begin{align*}
    f_\alpha(x, y, \alpha) &= 0 \\
    \cap \\
    f_\beta(x, y, \beta) &= 0
\end{align*}
\]

$\Rightarrow \quad p = \begin{bmatrix} x \\ y \end{bmatrix}$
**Triangulation based on circle intersection**

- The angle $\alpha_{ij} (= \theta_j - \theta_i)$ between the lines connecting P and any two landmarks $R_i$ and $R_j$, constrains the robot position to be on a circular arc.
- The use of **three landmarks** ($R_1, R_2, R_3$) allows to determine the position of P from the intersection of the arcs associated with each pair of landmarks.

**Calculation of orientation:**

$$\psi (p, \theta_i) = \arctan \frac{y_i - y}{x_i - x} - \theta_i$$

$\forall i = 1, 2, 3$

**Calculation of position:**

$$\left\{ \begin{array}{l}
f_\alpha (x, y, \alpha) = 0 \\
f_\beta (x, y, \beta) = 0 \\
\end{array} \right\} \Rightarrow p = \begin{bmatrix} x \\ y \end{bmatrix}$$
Singularities of the method

- Singularities arise when point P lies on the critical circumference that define the three landmarks used ($R_1, R_2, R_3$).
- The position of P is **undetermined** since the three circumferences are the same and they cannot be intersected.
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Geometric Localization Using Triangulation

Error Analysis for the Robot Position

Dynamic Localization From Discontinuous Measurements

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Conclusions

Design of an Automatic Omnidirectional Wheelchair
Error analysis. Bounded error in the measurements

- **Bounded error** in the measurements: \( \theta_i = \theta_i^r + \delta \theta_i \in \left[ \theta_i^r - \Delta \theta_m, \theta_i^r + \Delta \theta_m \right] \).

- Each circumference has an **uncertainty area** associated: \( Z_\alpha, Z_\beta, Z_\gamma \).

\[
\alpha \in \left[ \alpha^r - 2\Delta \theta_m, \alpha^r + 2\Delta \theta_m \right] \rightarrow Z_\alpha
\]
Error analysis. Bounded error in the measurements

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\alpha & \in [\alpha^r - 2\Delta \theta_m, \alpha^r + 2\Delta \theta_m] \rightarrow Z_\alpha \\
\beta & \in [\beta^r - 2\Delta \theta_m, \beta^r + 2\Delta \theta_m] \rightarrow Z_\beta \\
\gamma & \in [\gamma^r - 2\Delta \theta_m, \gamma^r + 2\Delta \theta_m] \rightarrow Z_\gamma
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Error analysis. Bounded error in the measurements

- **Bounded error** in the measurements: \( \theta_i = \theta_i^r + \delta \theta_i \in \left[ \theta_i^r - \Delta \theta_m, \theta_i^r + \Delta \theta_m \right] \).
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\begin{align*}
\alpha & \in \left[ \alpha^r - 2\Delta \theta_m, \alpha^r + 2\Delta \theta_m \right] \rightarrow Z_\alpha \\
\beta & \in \left[ \beta^r - 2\Delta \theta_m, \beta^r + 2\Delta \theta_m \right] \rightarrow Z_\beta \\
\gamma & \in \left[ \gamma^r - 2\Delta \theta_m, \gamma^r + 2\Delta \theta_m \right] \rightarrow Z_\gamma
\end{align*}
\]

\[
Z_P = \bigcap_{i,j=1}^{3} Z_{\alpha_{ij}} = Z_\alpha \cap Z_\beta \cap Z_\gamma
\]
Error analysis. Bounded error in the measurements

- **Bounded error** in the measurements: \( \theta_i = \theta_i^r + \delta \theta_i \in \left[ \theta_i^r - \Delta \theta_m, \theta_i^r + \Delta \theta_m \right] \).

- Each circumference has an **uncertainty area** associated: \( Z_\alpha, Z_\beta, Z_\gamma \).

**Monte Carlo Simulation**
Error analysis. Bounded error in the measurements

- Mapping of the **Maximum Error** (for $\Delta \theta_m = 0.2$ mrad)
Error analysis. Gaussian error in the measurements

- **Gaussian error** in the measurements: \( \theta_i = \theta_i^r + \delta \theta_i; \quad \delta \theta_i \sim N\left(0, \sigma^2_{\theta_i}\right) \)

- **Position error:**

\[
\delta p = p - p^r = \sum_{i=1}^{3} \frac{\partial p}{\partial \theta_i} \delta \theta_i + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial^2 p}{\partial \theta_i \partial \theta_j} \delta \theta_i \delta \theta_j + O^3
\]
Error analysis. Gaussian error in the measurements

- **Gaussian error** in the measurements: \( \theta_i = \theta_i^r + \delta \theta_i ; \quad \delta \theta_i \sim N(0, \sigma^2_\theta) \)

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  \]

- **Expected value** of the position error:
  \[
  E\{\delta p\} = \frac{\sigma^2_\theta}{2} \left( \frac{\partial^2 p}{\partial \theta_1^2} + \frac{\partial^2 p}{\partial \theta_2^2} + \frac{\partial^2 p}{\partial \theta_3^2} \right)
  \]
Error analysis. Gaussian error in the measurements

- Mapping of the **Expected Value of the Position Error** (for $\sigma_\theta = 1$ mrad)
Error analysis. Gaussian error in the measurements

- **Gaussian error** in the measurements: \( \theta_i = \theta_i^r + \delta \theta_i; \quad \delta \theta_i \sim N(0, \sigma_\theta^2) \)

- **Position error**: \( \delta p = p - p^r = \sum_{i=1}^{3} \frac{\partial p}{\partial \theta_i} \delta \theta_i + \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial^2 p}{\partial \theta_i \partial \theta_j} \delta \theta_i \delta \theta_j + O^3 \)

- **Expected value** of the position error: 
  \[
  E\{\delta p\} = \frac{\sigma_\theta^2}{2} \left( \frac{\partial^2 p}{\partial \theta_i^2} + \frac{\partial^2 p}{\partial \theta_j^2} + \frac{\partial^2 p}{\partial \theta_s^2} \right)
  \]

- **Covariance** of the position error:
  \[
  C = E\{\delta p \delta p^T\} = \sigma_\theta^2 J J^T; \quad J = \left[ \frac{\partial p}{\partial \theta} \right]
  \]

**Uncertainty Ellipsoid**: 
\[
S = \left\{ p \mid (p - p^r)^T C^{-1} (p - p^r) \leq K \right\}
\]
Uncertainty ellipsoid

- The dispersion in the robot position is represented by an **Uncertainty Ellipsoid**.
Uncertainty ellipsoid

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\[ S \equiv \left\{ p \mid \left( p - p^r \right)^T C^{-1} \left( p - p^r \right) \leq K \right\} \]

- The ellipsoid represents the set of points in the x-y plane that have a certain probability to contain the measured position of P (K defines this probability).
Uncertainty ellipsoid

- The dispersion in the robot position is represented by an **Uncertainty Ellipsoid**.

\[ S \equiv \left\{ p \mid (p - p^r)^T C^{-1} (p - p^r) \leq K \right\} \]

- The ellipsoid represents the set of points in the x-y plane that have a certain probability to contain the measured position of P (K defines this probability).

- Ellipsoids for different probabilities (K)
Uncertainty ellipsoid (for $\kappa$ defining 95% probability)

$\sigma_\theta = 0.1 \text{ mrad}$

$\sigma_\theta = 1 \text{ mrad}$
Uncertainty ellipsoid (for $\kappa$ defining 95% probability)

$\sigma_\theta = 0.1 \text{ mrad}$

$\sigma_\theta = 1 \text{ mrad}$

Error Analysis for the Robot Position
Novel triangulation method based on straight-line intersection

- If the orientation angle $\psi$ is known, the position of $P$ can be determined using straight-line intersection.
- The method has no singularities if three unaligned landmarks are used.

P not aligned with $R_i$ and $R_j$  

P aligned with $R_i$ and $R_j \rightarrow$ a third unaligned landmark is needed ($R_k$)
Before triangulating the orientation $\psi$ is not known, but an approximation of it can be used: $\psi_e \equiv \psi + \epsilon_\psi$ → Orientation Error

If $\psi_e \neq \psi$, then the lines intersect in three points $O_{12}, O_{13}, O_{23}$ (these define the error triangle).

Triangulation based on straight lines. Error triangle

Approximate Orientation

$\psi_e \equiv \psi + \epsilon_\psi$

Iterative Method

[Cohen and Koss 1992]
Triangulation based on straight lines. Error triangle

- Is there any geometric relationship between the error triangle and $\varepsilon_\psi$?
- Error triangles for different values of $\varepsilon_\psi$: 

![Diagram showing error triangles for different values of $\varepsilon_\psi$.]
Triangulation based on straight lines. Error triangle

- Is there any geometric relationship between the error triangle and $\varepsilon_{\psi}$?
- Error triangles for different values of $\varepsilon_{\psi}$:

Novel Triangulation Method Based on Straight Lines
Triangulation based on straight lines. Error and centres triangles.
Similarity ratio between the triangles:

\[ r \equiv \frac{\rho_o}{\rho_c} = 2 \sin \varepsilon \psi \]

\[ |\varepsilon \psi| = \arcsin \left( \frac{|O_{ij}O_{ik}|}{2|C_{ij}C_{ik}|} \right) \]

Geometric relationships

- Similarity ratio between the triangles:
  \[ r = \frac{\rho_O}{\rho_C} = 2 \sin \varepsilon_\psi \]
  \[ |\varepsilon_\psi| = \arcsin \left( \frac{|O_{ij}O_{ik}|}{2|C_{ij}C_{ik}|} \right) \]

- The angle between corresponding sides of the triangles is \( \pi/2 - \varepsilon_\psi \) rad.

- The sign of \( \varepsilon_\psi \) can be determined using the following expression:
  \[ \text{sign}(\varepsilon_\psi) = \text{sign} \left( \overrightarrow{O_{ij}O_{ik}} \wedge \overrightarrow{C_{ij}C_{ik}} \right)_z \]

Introduction

Geometric Localization Using Triangulation

Error Analysis for the Robot Position

**Dynamic Localization From Discontinuous Measurements**

Experimental Results

Conclusions

Design of an Automatic Omnidirectional Wheelchair
**The Dynamic Localization Problem**

Landmarks are detected from different configurations of the robot.

Triangulation is **inconsistent** if the sensor measurements are directly used.
Suggested solution: *Dynamic Angular Estimation*

Real-time simulation of the landmark angles based on odometric information

Guarantees the **consistent** use of the triangulation methods using the simulated angles

**Dynamic Localization From Discontinuous Measurements**
Angular odometry

- Between actual sensor measurements, angle $\theta_i$ can be predicted integrating the following equation that governs its time rate of change:

\[
\frac{\partial}{\partial t}(\psi + \theta_i) = \frac{v_L \sin \theta_i - v_T \cos \theta_i}{\rho_i}
\]

\[
\theta_i(t) = \theta_i(t_0) + \int_{t_0}^{t} \frac{v_L \sin \theta_i - v_T \cos \theta_i}{\rho_i} - \psi \, dt
\]
Angular odometry

- Between actual sensor measurements, angle $\theta_i$ can be predicted integrating the following equation that governs its time rate of change:

$$\frac{\partial}{\partial t} (\psi + \theta_i) = \frac{v_L \sin \theta_i - v_T \cos \theta_i}{\rho_i}$$

$$\theta_i(t) = \theta_i(t_0) + \int_{t_0}^{t} \frac{v_L \sin \theta_i - v_T \cos \theta_i}{\rho_i} \, \psi \, dt$$

- Cumulative error in the odometric prediction
- Error in the discontinuous angular observations

Variables $v_L, v_T, \psi$ are obtained from the odometric measurements.

Kalman Filtering (EKF)
Angular odometry. Kinematics of the robot SPHERIK-3x3

- **3-DOF** mobile robot with **3 omnidirectional wheels** consisting of two spherical rollers. Each wheel is actuated by a single motor.

Motorized motion
- Controlled by the motor

Free motion
- It adapts to the kinematics imposed by the set of motors
Angular odometry. Kinematics of the robot SPHERIK-3x3

- 3-DOF mobile robot with 3 omnidirectional wheels consisting of two spherical rollers. Each wheel is actuated by a single motor.

\[
\begin{align*}
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 
\end{bmatrix} &= 
\frac{1}{r} 
\begin{bmatrix}
0 & -1 & -L \\
\cos \alpha & \sin \alpha & -s \\
-\cos \alpha & \sin \alpha & -s 
\end{bmatrix} 
\begin{bmatrix}
v_L \\
v_T \\
\dot{\psi}
\end{bmatrix} \\
J_\omega &= 
\begin{bmatrix}
0 & -1 & -L \\
\cos \alpha & \sin \alpha & -s \\
-\cos \alpha & \sin \alpha & -s 
\end{bmatrix}
\end{align*}
\]

\[
\begin{bmatrix}
v_L \\
v_T \\
\dot{\psi}
\end{bmatrix} = J^{-1}\omega 
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 
\end{bmatrix}
\]

motor encoders
LONGITUDINAL DOF

Dynamic Localization From Discontinuous Measurements
Dynamic Localization From Discontinuous Measurements
ROTATIONAL DOF
Angular-State Extended Kalman Filter (EKF)

- Angular state-vector: \( x_k = \{\theta_{1,k}, \theta_{2,k}, \theta_{3,k}\}^T \)

**Prediction Phase**

(odometric evolution of the state-vector)

\[
x_{k+1} = f(x_k, u_k, w_k)
\]

- State transition equation

\[
\theta_{i,k+1} = \theta_{i,k} + \Delta t \left( \frac{v_{L,k} \sin \theta_{i,k} - v_{T,k} \cos \theta_{i,k}}{\rho_{i,k}} - \dot{\psi}_k \right)
\]

- Prediction of the angular state-vector from odometric measurements:

\[
\bar{x}_{k+1} = f(\tilde{x}_k, u_k, 0)
\]

- Prediction of the error covariance:

\[
\bar{P}_{x,k+1} = \nabla f_x \bar{P}_{x,k} \nabla f_x^T + \nabla f_w Q_k \nabla f_w^T
\]
Angular-State Extended Kalman Filter (EKF)

**Correction Phase**

(angle $\theta_{i,k}$ is measured by the sensor)

- Kalman gain matrix is determined:
  \[
  K_k = \frac{1}{\sigma_{\theta_i,k}^2 + R} \bar{P}_{x,k} \nabla h_x^i \nabla h_x^i^T
  \]
  
  minimizes squared error

- Correction of the odometrically predicted state-vector:
  \[
  \tilde{x}_k = \bar{x}_k + K_k \left( \theta_{i,k} - \bar{\theta}_{i,k} \right)
  \]
  \[
  \tilde{\theta}_i = \bar{\theta}_i + \frac{\sigma_{\theta_i}^2}{\sigma_{\theta_i}^2 + R} (\theta_i - \bar{\theta}_i)
  \]
  \[
  \tilde{\theta}_j = \bar{\theta}_j + \frac{\sigma_{\theta_i,\theta_j}}{\sigma_{\theta_i}^2 + R} (\theta_i - \bar{\theta}_i); \ i \neq j
  \]

- Estimation of the error covariance:
  \[
  \bar{P}_{x,k} = \left( I - K_k \nabla h_x^i \nabla h_x^i \right) \bar{P}_{x,k}
  \]
Angular-State Extended Kalman Filter (EKF)

**Correction Phase**

(angle $\theta_{i,k}$ is measured by the sensor)

- Kalman gain matrix is determined:
  \[
  K_k = \frac{1}{\sigma_{\theta_i,k}^2 + R} \bar{P}_{x,k} \nabla h_x^T
  \]

  minimizes squared error

- Correction of the odometrically predicted state-vector:
  \[
  \tilde{x}_k = \bar{x}_k + K_k \left( \theta_{i,k} - \bar{\theta}_{i,k} \right)
  \]

  \[
  \begin{align*}
  \tilde{\theta}_i &= \bar{\theta}_i + \frac{\sigma_{\theta_i}}{\sigma_{\theta_i}^2 + R} \left( \theta_i - \bar{\theta}_i \right) \\
  \tilde{\theta}_j &= \bar{\theta}_j + \frac{\sigma_{\theta_i,\theta_j}}{\sigma_{\theta_i}^2 + R} \left( \theta_i - \bar{\theta}_i \right); \ i \neq j
  \end{align*}
  \]

- Estimation of the error covariance:
  \[
  \tilde{P}_{x,k} = \left( I - K_k \nabla h_x^T \right) \bar{P}_{x,k}
  \]
### Comparison with the standard Pose-State EKF

- The usual method to dynamically estimate the robot pose from discontinuous angular measurements is the **Pose-State EKF**.

- **Pose state-vector:** \( y_k = \{x_k, y_k, \psi_k\}^T \)

#### State-transition equation:

\[
\begin{align*}
\begin{bmatrix}
    x_{k+1} \\
    y_{k+1} \\
    \psi_{k+1}
\end{bmatrix}
&= \begin{bmatrix}
    x_k \\
    y_k \\
    \psi_k
\end{bmatrix} + \Delta t \begin{bmatrix}
    v_L \cos \psi_k - v_T \sin \psi_k \\
    v_L \sin \psi_k + v_T \cos \psi_k \\
    \dot{\psi}
\end{bmatrix}
\end{align*}
\]

#### Measurement equation:

\[
\theta_{i,k} = \arctan \left( \frac{y_i - y_k}{x_i - x_k} \right) - \psi_k + \delta \theta_m \quad \text{non-linear}
\]

---

**POSE-STATE EKF**

- [Wiklund *et al.* 1988]
- [Durrant-Whyte 1994]
- [Hu i Gu 2000]
- [Sgorbissa 2000]
- [Piaggio *et al.* 2001]
Comparison with the standard Pose-State EKF

POSE STATE SPACE

Prediction → Correction → pose estimation

odometric measurements

laser measurement

Change of the State Space of the Kalman Filter

ANGULAR STATE SPACE

Prediction → Correction → Triangulation → pose estimation

odometric measurements

laser measurement

angular estimation
Comparison with the standard Pose-State EKF

**Pose-State EKF**
- The algorithm fuses the *pose odometry* with the discontinuous angular measurements.

**Angular-State EKF**
- The algorithm fuses the *angular odometry* with the discontinuous angular measurements.
**Comparison with the standard Pose-State EKF**

<table>
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<td>• The measurement equation is <strong>non-linear</strong> → gives rise to important errors when the state prediction is uncertain.</td>
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## Comparison with the standard Pose-State EKF

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<td>• Each angular measurement corrects the odometric pose estimation only in the <strong>perpendicular direction</strong> to the viewed landmark.</td>
<td>• Once the <strong>angular state-vector</strong> is estimated, the robot pose is geometrically determined using <strong>triangulation</strong>.</td>
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Presentation Contents

Introduction

Geometric Localization Using Triangulation

Error Analysis for the Robot Position

Dynamic Localization From Discontinuous Measurements

Experimental Results

Conclusions

Design of an Automatic Omnidirectional Wheelchair
Experimental Setup

- We need a system to measure the trajectory of the centre of the laser sensor and the robot orientation with an accuracy below \( mm \) (in position) and \( mrad \) (in orientation).
- We designed a metrological system consisting of two rotary encoders and a linear potentiometer.

**Position of P:**
\[
\begin{align*}
\{x_k^m\} &= \begin{bmatrix} l_k \cos \varphi_{1,k} \\ l_k \sin \varphi_{1,k} \end{bmatrix} \\
\{y_k^m\} &= \begin{bmatrix} \end{bmatrix}
\end{align*}
\]

**Robot orientation:**
\[
\psi_k^m = \varphi_{1,k} + \varphi_{2,k}
\]
Experimental Setup

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Position of $P$:

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\end{align*}
\]

Robot orientation:

\[
\psi^m_k = \varphi_{1,k} + \varphi_{2,k}
\]
Methodology

- **Dynamic estimation of the robot pose** from odometric and angular measurements.
- **Pose measurement** by means of the high-accuracy metrological system.
Control system and data acquisition module

- Layout of the electronic components on the robot.
Experimental environment and trajectories

- Three trajectories were tested.

Trajectory 1
Experimental environment and trajectories

- Three trajectories were tested.
Experimental environment and trajectories

- Three trajectories were tested.
Results. Lateral error (trajectory 1)

3rd detection

<table>
<thead>
<tr>
<th></th>
<th>( e_{lat} )</th>
<th>RMS (( e_{lat} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular-State EKF</td>
<td>-4,24 mm</td>
<td>Angular-State EKF</td>
</tr>
<tr>
<td>Pose-State EKF</td>
<td>-84,31 mm</td>
<td>Pose-State EKF</td>
</tr>
</tbody>
</table>
Results. Orientation error (trajectory 1)

3rd detection

<table>
<thead>
<tr>
<th></th>
<th>$e_\psi$</th>
<th>RMS ($e_\psi$)</th>
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</thead>
<tbody>
<tr>
<td>Angular-State EKF</td>
<td>$-1.74 \text{ mrad}$</td>
<td>$1.06 \text{ mrad}$</td>
</tr>
<tr>
<td>Pose-State EKF</td>
<td>$23.28 \text{ mrad}$</td>
<td>$3.26 \text{ mrad}$</td>
</tr>
</tbody>
</table>
## Table of results

- RMS value of the lateral error and the orientation error.

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Angular-State EKF + Triangulation</th>
<th>Pose-State EKF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS ($e_{lat}$)</td>
<td>RMS ($e_{\psi}$)</td>
</tr>
<tr>
<td></td>
<td>(mm)</td>
<td>(mrad)</td>
</tr>
<tr>
<td>(1)</td>
<td>1.30</td>
<td>1.06</td>
</tr>
<tr>
<td>(2)</td>
<td>2.43</td>
<td>4.89</td>
</tr>
<tr>
<td>(3)</td>
<td>3.20</td>
<td>5.13</td>
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**1.30 3.20 mm 3.37 17.24 mm**

Table of results

RMS value of the lateral error and the orientation error.

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<tr>
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<td>RMS ($e_{lat}$) (mm)</td>
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</tr>
<tr>
<td>(1)</td>
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1,06 ÷ 5,13 mrad  3,26 ÷ 5,37 mrad

Introduction

Geometric Localization Using Triangulation

Error Analysis for the Robot Position

Dynamic Localization From Discontinuous Measurements

Experimental Results

Conclusions

Design of an Automatic Omnidirectional Wheelchair
Conclusions

- A Dynamic Angular-State Estimator based on Kalman Filtering has been developed. It uses odometric data to track the evolution of the angular measurements in between discontinuous landmark detections.

- It guarantees the **consistent and continuous** use of the triangulation methods.

- Together with triangulation, it **improves the accuracy** of the widespread Pose-State Extended Kalman Filter.

- The method is **robust**: It is easy to notice **erroneous measurements** (due to bad reflections) or **non-detected landmarks**.
Design of an Automatic Omnidirectional Wheelchair
Tri-Spherical Wheelchair

- Same concept as SPHERIK-3x3
- 3 motors $\rightarrow$ 3 DOF
The 3 motion modes of the wheelchair

- Transverse mode
- Longitudinal mode
- Minimum space rotation mode
Tri-Spherical Wheelchair

- Longitudinal motion
- Transverse motion
- Rotation
- General motion
Advantages of the omnidirectional wheelchair
Advantages of the omnidirectional wheelchair

Possibility of a lateral motion

Lateral motion is impossible (no slipping)
Advantages of the omnidirectional wheelchair

Minimum space rotation