Sonar effects

(a) Sonar providing an accurate range measurement
(b-c) Lateral resolution is not very precise; the closest object in the beam’s cone provides the response
(d) Specular reflections cause walls to disappear
(e) Open corners produce a weak spherical wavefront
(f) Closed corners measure to the corner itself because of multiple reflections \(\rightarrow\) sonar ray tracing

Sonar modeling

(resolution: time / space)

Sonar Modeling

response model \((Kuc)\)

\[
K_{\text{r}}(c, z, \alpha, a) = \frac{2c \cos \alpha}{\pi a} \sqrt{z} \left[ 1 - \frac{c^2 z^2}{4a^2c^2} \right]
\]

• Models the response, \(K_{\text{r}}\), with
  - \(c\) = speed of sound
  - \(a\) = diameter of sonar element
  - \(t\) = time
  - \(z\) = orthogonal distance
  - \(\alpha\) = angle of environment surface

• Then, add noise to the model to obtain a probability:

\[
p(\mathbf{S} | \mathbf{o})
\]

chance that the sonar reading \(\mathbf{S}\), given an obstacle at location \(\mathbf{o}\)

Using sonar to create maps

What should we conclude if this sonar reads 10 feet?

What would a local map look like?
Using sonar to create maps
What should we conclude if this sonar reads 10 feet?

Local Map
- unoccupied
- occupied

There isn’t something here
There is something somewhere around here

10 feet

or...

Using sonar to create maps
What should we conclude if this sonar reads 10 feet?

Local Map
- unoccupied
- no information
- occupied

There isn’t something here
There is something somewhere around here

10 feet

Using sonar to create maps
What should we conclude if this sonar reads 10 feet?

Local Map
- unoccupied
- occupied

There isn’t something here
There is something somewhere around here

10 feet

and how do we add the information that the next sonar reading (as the robot moves) reads 10 feet, too?

Combining sensor readings
- The key to making accurate maps is combining lots of data.
- But combining these numbers means we have to know what they are!

What should our map contain?

- small cells
- each represents a bit of the robot’s environment
- larger values => obstacle
- smaller values => free

What is in each cell of this sonar model/map?
What is it a map of?

Several answers to this question have been tried:

- It’s a map of occupied cells.
- It’s a map of probabilities:
  \[ p(\text{occupied} | S_1, S_2, \ldots, S_i) \]
  \[ p(\text{unoccupied} | S_1, S_2, \ldots, S_i) \]

Each cell is either occupied or unoccupied -- this was the approach taken by the Stanford Cart.

What information should this map contain, given that it is created with sonar?

- maintaining related values separately?
- initialize all certainty values to zero
- contradictory information will lead to both values near 1
- combining them takes some work...

Combining probabilities

How to combine two sets of probabilities into a single map?

What is it a map of?

Several answers to this question have been tried:

- It’s a map of occupied cells.
- It’s a map of probabilities:
  \[ p(\text{occupied} | S_1, S_2, \ldots, S_i) \]
  \[ p(\text{unoccupied} | S_1, S_2, \ldots, S_i) \]

The odds that a cell is occupied, given the sensor readings \( S_1, S_2, \ldots, S_i \)

\[ \text{odds}(\text{occupied} | S_1, S_2, \ldots, S_i) = \frac{p(\text{occupied} | S_1, S_2, \ldots, S_i)}{p(\text{unoccupied} | S_1, S_2, \ldots, S_i)} \]
An example map

Evidence grid of a tree-lined outdoor path

- lighter areas: lower odds of obstacles being present
- darker areas: higher odds of obstacles being present

Conditional probability

Some intuition...

- $p(\text{o} | \text{S})$ = The probability of event $\text{o}$, given event $\text{S}$.
  - The probability that a certain cell $\text{o}$ is occupied, given that the robot sees the sensor reading $\text{S}$.

- $p(\text{S} | \text{o})$ = The probability of event $\text{S}$, given event $\text{o}$.
  - The probability that the robot sees the sensor reading $\text{S}$, given that a certain cell $\text{o}$ is occupied.

- What is really meant by conditional probability?
- How are these two probabilities related?

Cards: got 2 pair, what are odds for getting full house?

- $p(\text{FH} | 2\text{pr}) = \frac{1}{12}$
- $p(2\text{pr} | \text{FH}) = \frac{3}{5}$

Bayes Rule

- Conditional probabilities
  - $p(\text{o} \wedge \text{S}) = p(\text{o} | \text{S}) p(\text{S})$

- Bayes rule relates conditional probabilities
  - $p(\text{o} | \text{S}) = \frac{p(\text{S} | \text{o}) p(\text{o})}{p(\text{S})}$
Bayes Rule

- Conditional probabilities
  \[ p(o \times S) = p(o | S) p(S) \]

- Bayes rule relates conditional probabilities
  \[ p(o | S) = \frac{p(S | o) p(o)}{p(S)} \]

- So, what does this say about \( \text{odds}(o \times S_2 \cap S_1) \)? Can we update easily?

Combining evidence

So, how do we combine evidence to create a map?

What we want --
\[ \text{odds}(o \times S_2 \cap S_1) \]
the new value of a cell in the map after the sonar reading \( S_2 \)

What we know --
\[ \text{odds}(o \times S_1) \]
the old value of a cell in the map (before sonar reading \( S_2 \))
\[ p(S_i | o) \text{ & } p(S_i | \sigma) \]
the probabilities that a certain obstacle causes the sonar reading \( S_i \)

Combining evidence

\[ \text{odds}(o \times S_2 \cap S_1) = \frac{p(o \times S_2 \cap S_1)}{p(\sigma \times S_2 \cap S_1)} \]
def \( \text{odds} \)

\[ \text{odds}(o \times S_2 \cap S_1) = \frac{p(o \times S_2 \cap S_1)}{p(\sigma \times S_2 \cap S_1)} \\ \]
\[ = \frac{p(S_2 \times S_1 | o) p(\sigma)}{p(S_2 \times S_1 | \sigma) p(o)} \]
Combining evidence

\[
\text{odds}(o \mid S_2 \land S_1) = \frac{p(o \mid S_2 \land S_1)}{p(\overline{o} \mid S_2 \land S_1)} \quad \text{def' of odds}
\]

\[
- = \frac{p(S_2 \land S_1 \mid o) p(o)}{p(S_2 \land S_1 \mid \overline{o}) p(\overline{o})} \quad \text{Bayes' rule (+)}
\]

\[
- = \frac{p(S_2 \mid o) p(S_1 \mid o) p(o)}{p(S_2 \mid \overline{o}) p(S_1 \mid \overline{o}) p(\overline{o})} \quad \text{Bayes' rule (+)}
\]

\[\text{Bayes' rule (+) conditional independence of } S_1 \text{ and } S_2\]

\[\text{precomputed values}\]

Update step = multiplying the previous odds by a precomputed weight.

Evidence grids

- hallway with some open doors
- lab space

known map and estimated evidence grid

CMU – Hans Moravec
Path Planning in Evidence Grids

Reduces to a search problem within the graph of cells, and the graph is created on the fly.

Search for a minimum-cost path, depending on:
- length
- probability of collision

\[ \text{cost} = K \sum_{(i,j) \in \text{path}} \text{step}_{ij} - \sum_{(i,j) \in \text{path}} \log(1-p) \]

Suppose \( p \) is the chance of colliding with an obstacle in cell \((i,j)\).

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Learning the Sensor Model

The sonar model depends dramatically on the environment; we’d like to learn an appropriate sensor model...

\[ \text{cost} = \sum_{(i,j) \in \text{path}} \text{step}_{ij} + \sum_{(i,j) \in \text{path}} \text{collision penalty} \]

Suppose \( p \) is the chance of colliding with an obstacle in cell \((i,j)\).
Learning the Sensor Model

The sonar model depends dramatically on the environment -- we'd like to learn an appropriate sensor model rather than hire someone to develop another one...

Sensor fusion

Incorporating data from other sensors -- e.g., IR rangefinders and stereo vision...

1. create another sensor model
2. update along with the sonar

3d evidence grids

the idealized model

the mapping results of a model that had an even better match score (against the ideal map)