COMP417
Introduction to Robotics and Intelligent Systems

Visual SLAM and Visual Odometry

McGill

MRL Mobile Robotics Lab at McGill University

Recap & admin stuff

ADMIN

- We have one more assignment. Two more quizzes.
- For the next assignment, the due date is slightly flexible.
- Quiz dates: Nov 21, Nov 30?
- Recall: Classes end Thursday, December 7

RECAP

- We are talking about computer vision.
- We have considered camera geometry, the ill-posed nature or vision, and nature of shape-from-X
- Many methods depend on computing properties of specific points, maybe in multiple images.
- Today we "zoom in" on these problems and consider specific methods, but not individually in great detail.

- Visual SLAM
  - Localization and mapping with measurements usually coming from tracking **image features**:
    - keypoints/corners
    - edges
    - image intensity patches
  - Can use one or more cameras

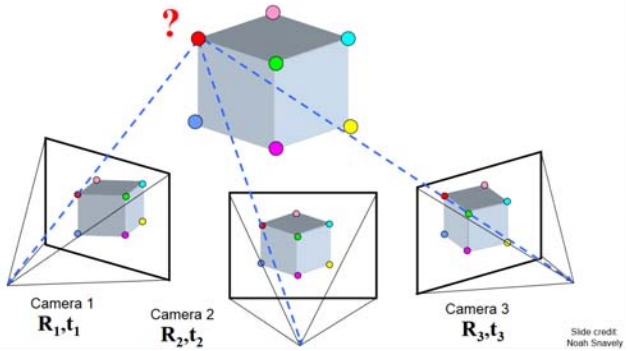
- Visual Odometry (VO)
  - Real-time localization with measurements usually coming from tracking **image features**:
    - keypoints/corners
    - edges
    - image intensity patches
  - Can use one or more cameras

Multi-view geometry problems

A.K.A. mapping

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point

3D point coordinates are unknown and to be estimated

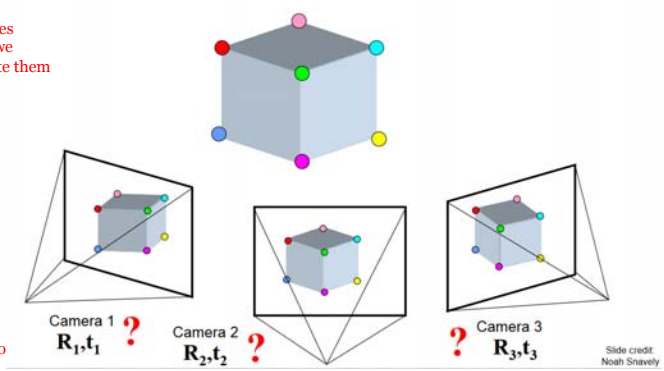


Camera frame transformations are known

Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters

3D point coordinates are unknown, but we won't try to estimate them

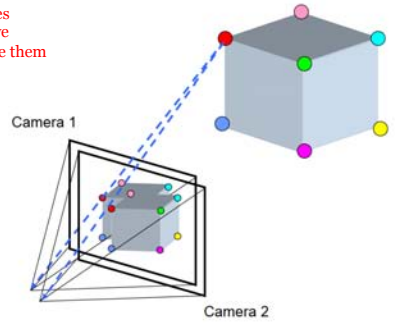


Camera frame transformations are unknown and to be estimated

Multi-view geometry problems

- **Optical flow:** Given two images, find the location of a world point in a second close-by image with no camera info.

3D point coordinates are unknown, but we won't try to estimate them



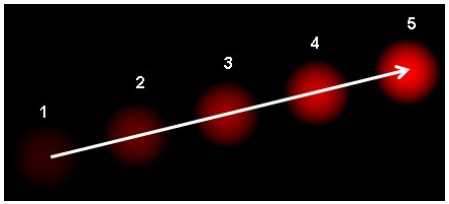
Many variants with two OR MORE images

We are estimating pixel displacement from one image to the next

Camera frame transformations are unknown, but we won't try to estimate them

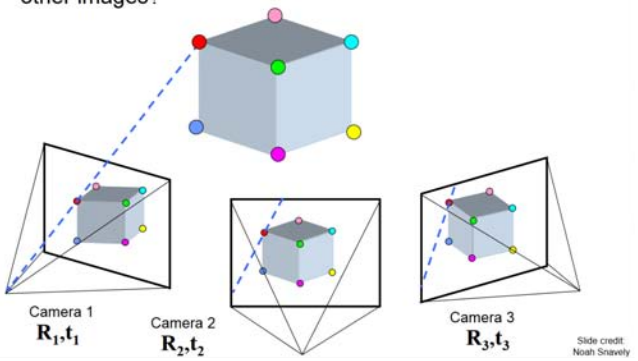
Optical flow

- Optical flow is a 2D problem: it's about the image (image domain).
- Motion is (unless otherwise specified) a 3D problem: it's about world coordinates (maybe with a scaling error).



Multi-view geometry problems

- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?

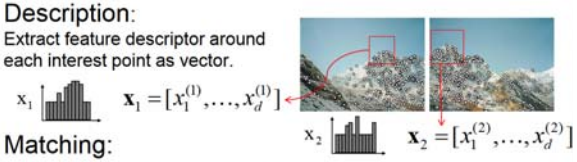


Basic underlying component in many of these problems:
keypoint detection and
matching across images

Local features: main components

1) Detection:
Find a set of distinctive key points.

2) Description:
Extract feature descriptor around each interest point as vector.



3) Matching:
Compute distance between feature vectors to find correspondence.

$$d(\mathbf{x}_1, \mathbf{x}_2) < T$$

K. Grauman, B. Leibe

Ideally, we want the descriptor to be invariant when there are

- viewpoint changes (small rotation or translation of the camera)

- scale-changes

- illumination changes

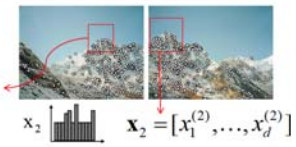
Invariant means it does **not change when these things happen**. In practice, we rarely do better than "pseudo-invariant" (changes little).

The point of the **descriptor** is to allow a point to be re-recognized to permit the matching.

Local features: main components

1) Detection:
Find a set of distinctive key points.

2) Description:
Extract feature descriptor around each interest point as vector.



3) Matching:
Compute distance between feature vectors to find correspondence.

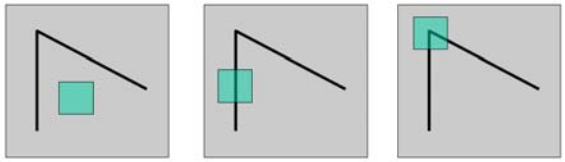
$$d(\mathbf{x}_1, \mathbf{x}_2) < T$$

K. Grauman, B. Leibe

Local measures of uniqueness

Suppose we only consider a small window of pixels

- What defines whether a feature is a good or bad candidate?

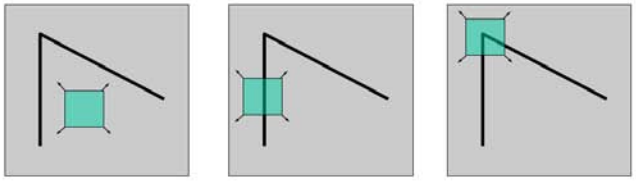


Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

Feature detection

Local measure of feature uniqueness

- How does the window change when you shift by a *small amount*?



"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

Slide adapted from Darya Frolova, Denis Simakov, Weizmann Institute.

"Corner" detectors

Corners are a class of feature, also known as **keypoints**.

In this context, corners are **not actual corners** of 3D (or 2D) objects, but places where the image varies in certain ways.

- Harris (1988)
- FAST (features from accelerated segment test)
- Laplacian of Gaussian detector (LoG)
- SUSAN*
- Forstner

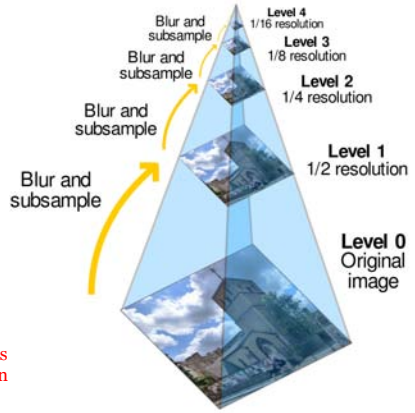
**Don't bother to memorize that this is "smallest univalue segment assimilating nucleus"*

Superimposed Harris keypoints

500 strongest keypoints

Digital Image Processing: Bernd Girod, © 2013 Stanford University -- Keypoint Detection 23

Scale-space representation



Feature detection:

search for "corners"/keypoints across many scales, and return a list of (x, y, scale) keypoints

Characteristics of good features

- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Kristen Grauman

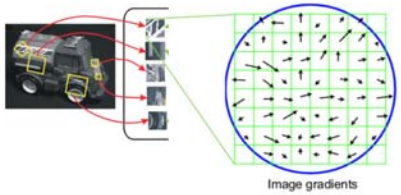
Additional modern feature detectors

- SIFT
- SURF (fast approximation of SIFT)

- (more listed on a later slide)

SIFT descriptor formation

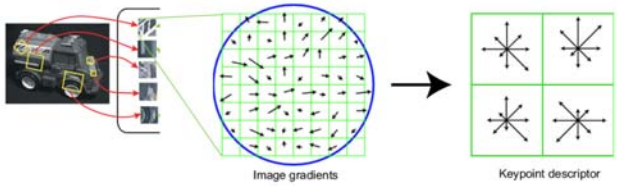
- Compute on local 16 x 16 window around detection.
- Rotate and scale window according to discovered orientation Θ and scale σ (gain invariance).
- Compute gradients weighted by a Gaussian of variance half the window (for smooth falloff).



Actually 16x16, only showing 8x8

SIFT vector formation

- 4x4 array of gradient orientation histograms weighted by gradient magnitude.
- Bin into 8 orientations x 4x4 array = 128 dimensions.

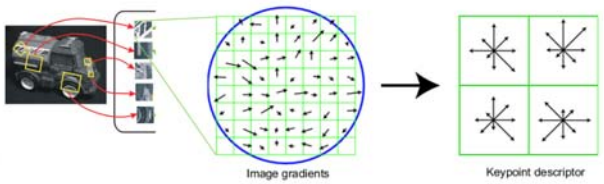


Showing only 2x2 here but is 4x4

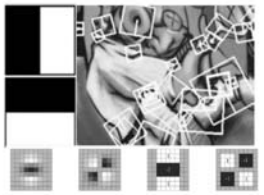
James Hays

Reduce effect of illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
  - After normalization, clamp gradients > 0.2
  - Renormalize



Local Descriptors: SURF



Fast approximation of SIFT idea

Efficient computation by 2D box filters & integral images
⇒ 6 times faster than SIFT
Equivalent quality for object identification

GPU implementation available

Feature extraction @ 200Hz
(detector + descriptor, 640×480 img)
http://www.vision.ee.ethz.ch/~surf

[Bay, ECCV'06], [Cornelis, CVGPU'08]

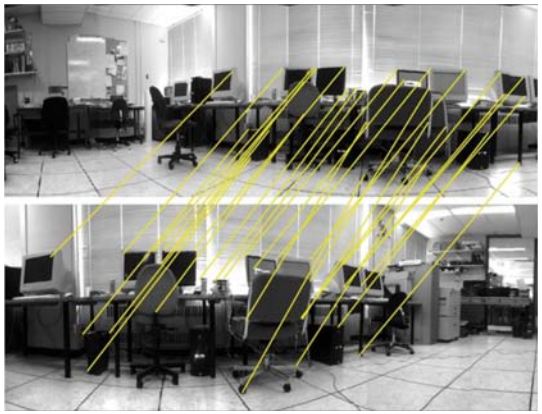
K. Grauman, B. Leibe

- ORB
- BRIEF
- FREAK
- RootSIFT-PCA

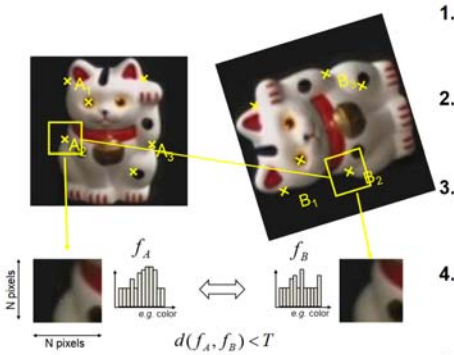
ORB, for example is quite popular and we use it in my lab.
It is FASTER that SURF, but probably not quite as robust.

- This was a research "industry". There is little new work on algorithms for feature detection because:
- (a) it's close to **solved** in the general case, and
- (b) **data-driven (learning methods)** are quite efficient in specific domains (and maybe even the almost-general case)

Feature matching



Overview of Keypoint Matching



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

K. Grauman, B. Leibe

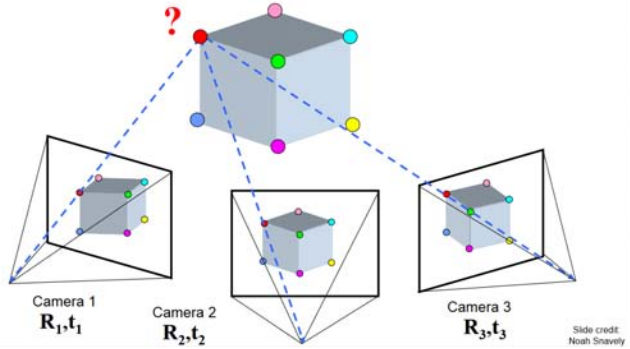
Problem #1: landmark triangulation

Multi-view geometry problems

A.k.a. mapping

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point

3D point coordinates are unknown and to be estimated



Camera frame transformations are known

Slide credit: Noah Snavely

Stereo revisited

1. Use 2 cameras
2. **Match points** between left and right cameras (i.e your eyes)
3. Observe how much the projected **point moves** (shifts position) between the two images.
4. **Compute the true 3D distance** based on the size of the shift using the **geometry** of projection.

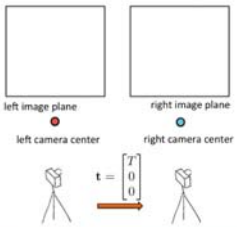
- You need to know these steps. The following slides provide further detail.

Stereo

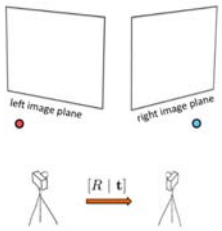
Epipolar geometry

- Case with two cameras with parallel optical axes ← **First this**
- General case

Parallel stereo cameras:

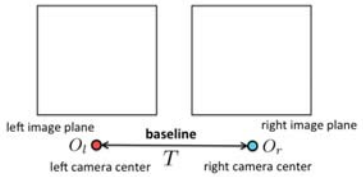


General stereo cameras:



Stereo: Parallel Calibrated Cameras

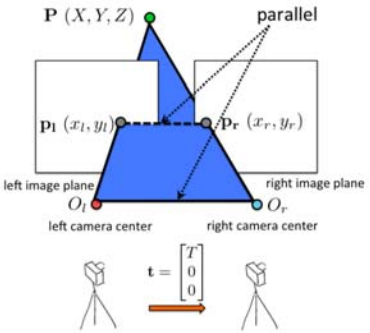
- We assume that the two calibrated cameras (we know intrinsics and extrinsics) are parallel, i.e. the right camera is just some distance to the right of left camera. We assume we know this distance. We call it the **baseline**.



The right camera is shifted to the right in X direction

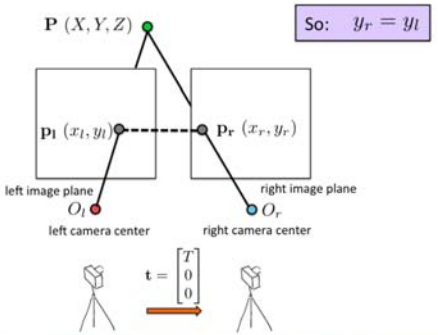
Stereo: Parallel Calibrated Cameras

- Points $\mathbf{O_l}$, $\mathbf{O_r}$ and $\mathbf{P}$ (and $\mathbf{p_l}$ and $\mathbf{p_r}$) lie on a plane. Since two image planes lie on the same plane (distance f from each camera), the lines $\mathbf{O_l O_r}$ and $\mathbf{p_l p_r}$ are parallel.



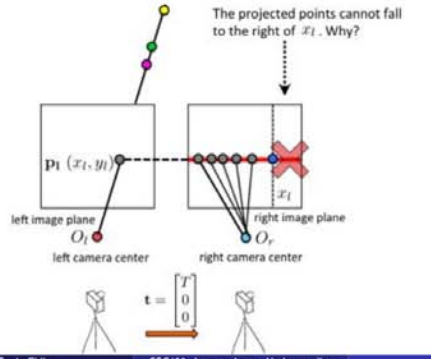
Stereo: Parallel Calibrated Cameras

- Since lines $\mathbf{O_l O_r}$ and $\mathbf{p_l p_r}$ are parallel, and $\mathbf{O_l}$ and $\mathbf{O_r}$ have the same y, then also $\mathbf{p_l}$ and $\mathbf{p_r}$ have the same y: $y_r = y_l$!



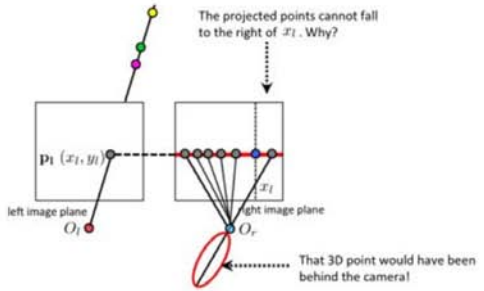
Stereo: Parallel Calibrated Cameras

- Another observation: No point from $\mathbf{O_l p_l}$ can project to the right of x_l in the right image. **Why?**



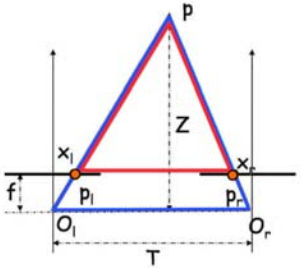
Stereo: Parallel Calibrated Cameras

- Because that would mean our image can see behind the camera...



Stereo: Parallel Calibrated Cameras

- We can then use similar triangles to compute the depth of the point P



Similar triangles:

$$\frac{T}{Z} = \frac{T + x_r - x_l}{Z - f}$$

$$Z = \frac{f \cdot T}{x_l - x_r}$$

baseline

focal length

disparity

In metric, not in pixel coordinates. To convert to pixel coordinates need to use elements of the camera calibration matrix.

[Adopted from: J. Hays]

Synopsis: if you have a **well-calibrated and rectified** (parallel) stereo camera you do not need to do least squares triangulation.

You can estimate depth via the disparity map.

Stereo: Parallel Calibrated Cameras

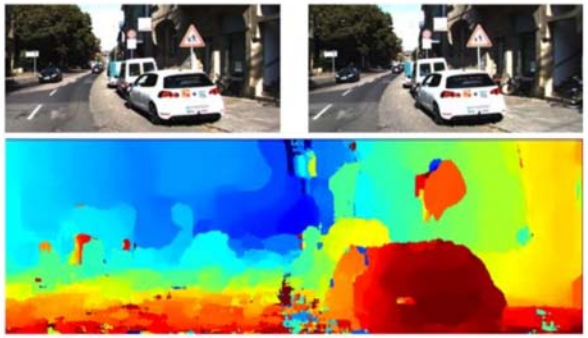
- For each point $\mathbf{p_l} = (x_l, y_l)$, how do I get $\mathbf{p_r} = (x_r, y_r)$? By matching. Patch around (x_r, y_r)) should look similar to the patch around (x_l, y_l).

left image

Do this for all the points in the left image!

Stereo: Parallel Calibrated Cameras

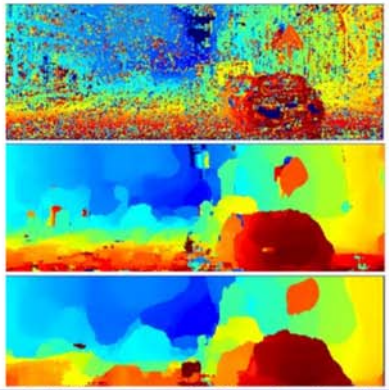
- We get a disparity map as a result



Result: **Disparity map**
(red values large disp., blue small disp.)

Stereo: Parallel Calibrated Cameras

- Smaller patches: more detail, but noisy. Bigger: less detail, but smooth



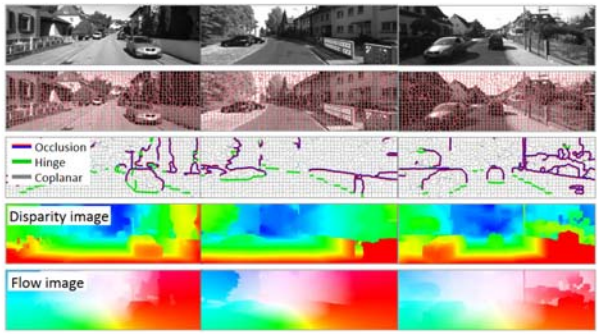
patch size = 5

patch size = 35

patch size = 85

You Can Do It Much Better...

[K. Yamaguchi, D. McAllester and R. Urtasun, ECCV 2014]



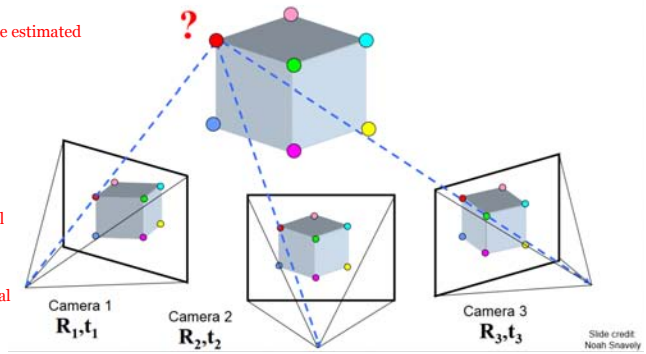
Multi-view geometry problems

- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point

3 x 7 = 21 variables to be estimated

7 pixel observations in each camera, so 21 pixel observations across all cameras

→ 42 constraints in total



Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

Actual pixel observation of a keypoint by camera frame k

Expected pixel observation of 3D point ${}^{w}p$ by camera frame k

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{fm_x X}{Z} + c_x \\ \frac{fm_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

Enumerate all cameras that observed the keypoint.

The only term to be optimized. The rest are known.

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{fm_x X}{Z} + c_x \\ \frac{fm_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ {}^{w}X^*, {}^{w}Y^*, {}^{w}Z^* = \underset{{}^{w}p=[{}^{w}X, {}^{w}Y, {}^{w}Z]}{\operatorname{argmin}} \sum_{k=1}^{K} ||\bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw})\,]\, ||^2 $$

3D point expressed in the frame of camera k

${}^{k}p = {}^{k}_{w}R\, {}^{w}p + {}^{k}t_{kw}$

$$ h_{\text{pinhole}}(X, Y, Z) = \begin{bmatrix} \frac{fm_x X}{Z} + c_x \\ \frac{fm_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ]}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k_w R\, {}^w p + {}^k t_{kw}) \,] ||^2 $$

Reprojection error of point ${}^kp = {}^k_w R\, {}^w p + {}^k t_{kw}$ into camera k's frame

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ \begin{aligned} {}^wX^*, {}^wY^*, {}^wZ^* &= \underset{{}^wp=[{}^wX,{}^wY,{}^wZ]}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k_w R\, {}^w p + {}^k t_{kw}) \,] ||^2 \\ &= \underset{{}^wp=[{}^wX,{}^wY,{}^wZ]}{\operatorname{argmin}} \sum_{k=1,\, {}^wp \to {}^kp}^{K} || \bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k p) \,] ||^2 \end{aligned} $$

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

$$ \begin{aligned} {}^wX^*, {}^wY^*, {}^wZ^* &= \underset{{}^wp=[{}^wX,{}^wY,{}^wZ]}{\operatorname{argmin}} \sum_{k=1}^{K} || \bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k_w R\, {}^w p + {}^k t_{kw}) \,] ||^2 \\ &= \underset{{}^wp=[{}^wX,{}^wY,{}^wZ]}{\operatorname{argmin}} \sum_{k=1,\, {}^wp \to {}^kp}^{K} || \bar{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k p) \,] ||^2 \\ &= \underset{{}^wp=[{}^wX,{}^wY,{}^wZ]}{\operatorname{argmin}} \sum_{k=1,\, {}^wp \to {}^kp}^{K} || \begin{bmatrix} \bar{u}^{(k)} \\ \bar{v}^{(k)} \end{bmatrix} - \begin{bmatrix} \frac{f m_x {}^kX}{{}^kZ} + c_x \\ \frac{f m_y {}^kY}{{}^kZ} + c_y \end{bmatrix} ||^2 \end{aligned} $$

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Triangulation as a least squares problem

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Note: unconstrained optimization does not guarantee that the solution will be in the camera's field of view. For example, it could happen that it returns ${}^kZ < 0$ which is an invalid solution (i.e. behind the camera)

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Potential pitfalls with triangulation: near parallel rays

"point at infinity"

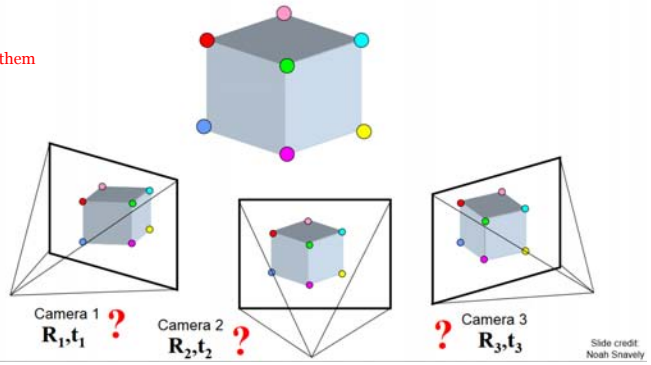
Intersection point is too far away, dominated by noise and insufficient image resolution.
Triangulating these points is typically impossible without sufficient baseline between camera frames.

Problem #2: camera localization/visual odometry

Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters

3D point coordinates are unknown, but we won't try to estimate them



Camera frame transformations are unknown and to be estimated

Camera localization as a least squares problem?

$$ {}^k_w R^*, {}^k t^*_{kw} = \underset{{}^k_w R, {}^k t_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \tilde{z}^{(k)} - \mathbb{E}[\, h_{\text{pinhole}}({}^k_w R\, {}^w p + {}^k t_{kw}) \,] ||^2 $$

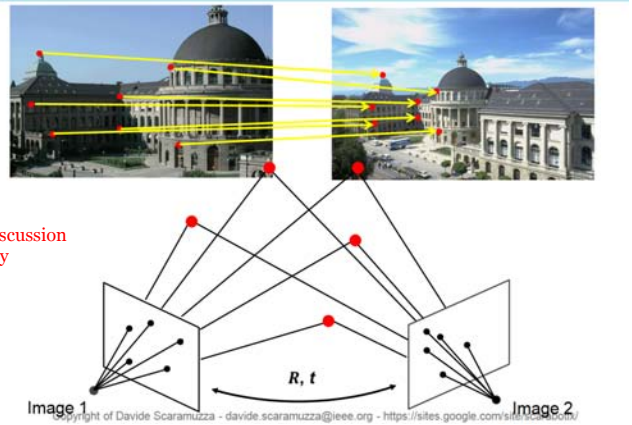
The only terms to be optimized.

But, 3D position is unknown!

So, we cannot solve the problem using the reprojection error unless we know the 3D position corresponding to the keypoint.

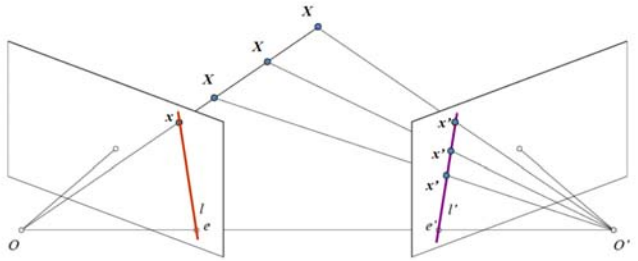
$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{f m_x X}{Z} + c_x \\ \frac{f m_y Y}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0, R) \quad \text{noise (in pixels)} $$

Working Principle



Let's restrict the discussion to two cameras only

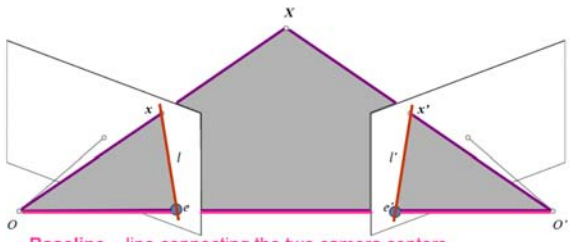
Key idea: Epipolar constraint



Potential matches for x' have to lie on the corresponding line l.

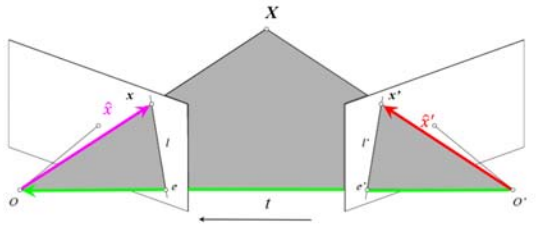
Potential matches for x have to lie on the corresponding line l'.

Epipolar geometry: notation



- **Baseline** – line connecting the two camera centers
- **Epipoles**
  = intersections of baseline with image planes
  = projections of the other camera center
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

Epipolar constraint: Calibrated case



$$\hat{x} = K^{-1}x = X \qquad \hat{x}' = K'^{-1}x' = X'$$

Homogeneous 2d point (3D ray towards X)

2D pixel coordinate (homogeneous)

3D scene point

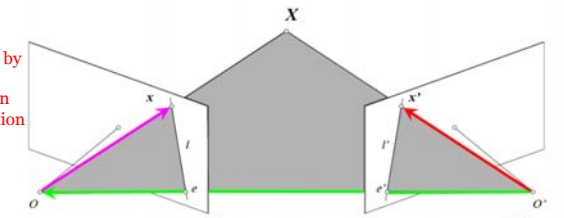
3D scene point in 2nd camera's 3D coordinates

$$\hat{x} \cdot [t \times (R\hat{x}')] = 0$$

(because $\hat{x}, R\hat{x}'$, and t are co-planar)

Essential matrix

The "5-point algorithm" by David Nister computes essential matrix and then decomposes it into rotation and translation.



$\hat{x} \cdot [t \times (R\hat{x}')] = 0$ ⇒ $\hat{x}^T E \hat{x}' = 0$ with $E = [t]_\times R$

E is a 3x3 matrix which relates corresponding pairs of normalized homogeneous image points across pairs of images – for *K* calibrated cameras.

Essential Matrix
(Longuet-Higgins, 1981)

Estimates relative position/orientation.

Note: $[t]_\times$ is matrix representation of cross product $[t]_\times = \begin{bmatrix} 0 & -t_2 & t_1 \\ t_2 & 0 & -t_0 \\ -t_1 & t_0 & 0 \end{bmatrix}$

After estimating the essential matrix, we extract t, R.

However, the translation t, is only estimated up to a multiplicative scale.

- Translation is not fully observable with a single camera.

- To make it observable we need stereo

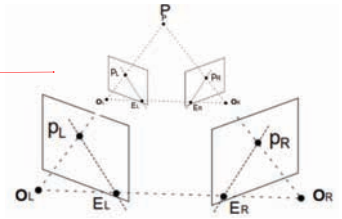
Visual odometry with a single camera: translation is recovered only up to a scale

- Scale = relationship between real-world metric distance units and estimated map distance units

Camera placements (1) and (2) generate the same observation of P. In fact, infinitely many possible placements of the two camera frames along their projection rays could have generated the same measurement.

(1)

(2)

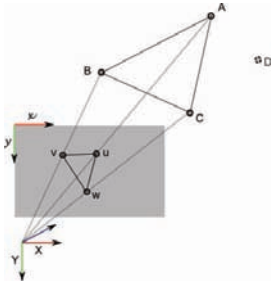


Visual odometry with a single camera: translation is recovered only up to a scale

- Scale = relationship between real-world metric distance units and estimated map distance units

Q: Is there a way to obtain true metric distances only with a single camera?

A: The only way is to have an object of known metric dimensions in the observed scene. For example if you know distances AB, BC, CA then you can recover true translation. This is commonly referred to as the Perspective-3-Point (P3P), or in General, the Perspective-n-Point (PnP) problem.

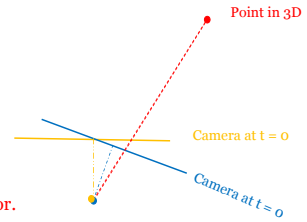


Visual odometry (VO) with a single camera: translation is recovered only up to a scale

- Scale = relationship between real-world metric distance units and estimated-map distance units

Q: Does scale remain constant throughout the trajectory of a single camera?

A: No, there is **scale drift,** which is most apparent during in-place rotations (i.e. pure rotation, no translation), because depth estimation for 3D points is unconstrained, so it is easily misestimated.
This is a form of error accumulation, just like the dead reckoning error.



2D-to-2D Algorithm

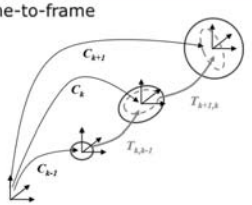
Motion estimation		
2D-2D	3D-3D	3D-2D

Algorithm 1: VO from 2D-to-2D correspondeces

1 Capture new frame I_k
2 Extract and match features between I_{k-1} and I_k.
3 Compute essential matrix for image pair I_{k-1}, I_k
4 Decompose essential matrix into R_k and t_k, and form T_k
5 Compute relative scale and rescale t_k accordingly
6 Concatenate transformation by computing $C_k = C_{k-1}T_k$
7 Repeat from 1

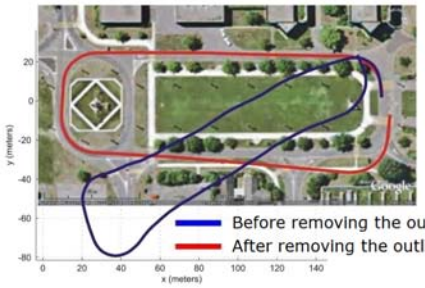
VO Drift

- The errors introduced by each new frame-to-frame motion accumulate over time
- This generates a drift of the estimated trajectory from the real path



The uncertainty of the camera pose at C_k is a combination of the uncertainty at C_{k-1} (black solid ellipse) and the uncertainty of the transformation $T_{k,k-1}$ (gray dashed ellipse)

Influence of Outliers on Motion Estimation



- Error at the loop closure: 6.5 m
- Error in orientation: 5 deg
- Trajectory length: 400 m

Are points-at-infinity useful for localization?

Points-at-infinity can help estimate the camera's rotation, similarly to how we use stars for navigation, without estimating how far they are.

For estimating translation, most likely no. For estimating rotation, yes.
Look up "Inverse Depth Parameterization for Monocular SLAM" for more info.

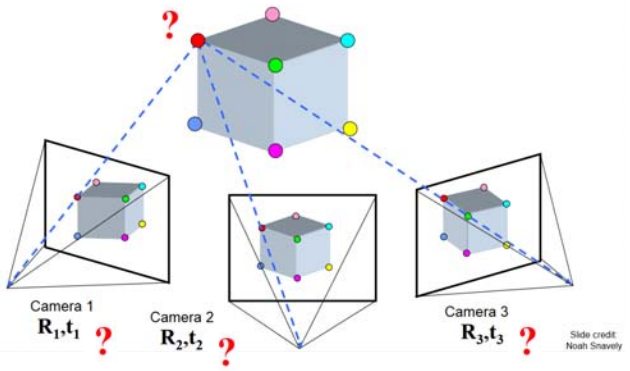
Problem #3: Visual SLAM

Structure from Motion

How can we estimate both 3D point positions and the relative camera transformations?

Sometimes also called bundle adjustment

Q: Why is it different than SLAM?



Structure from Motion

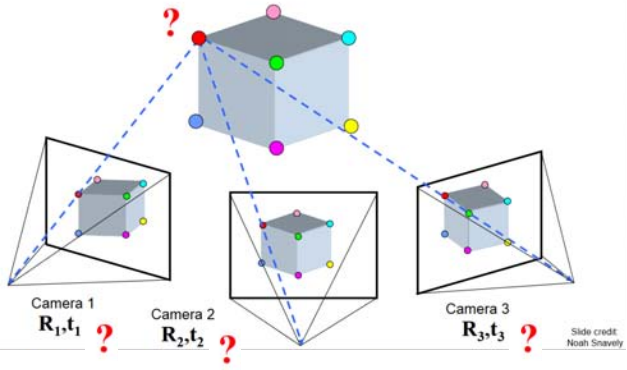
How can we estimate both 3D point positions and the relative camera transformations?

Sometimes also called bundle adjustment

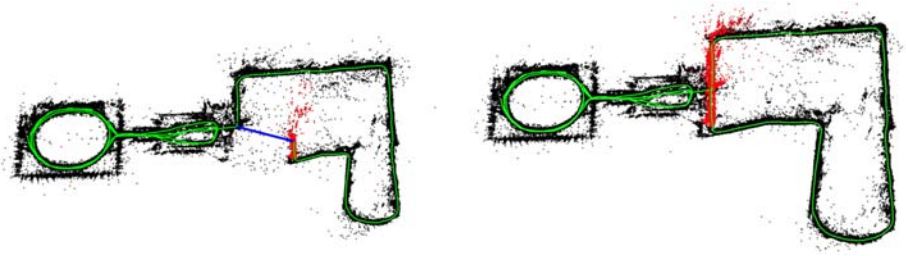
Q: Why is it different than SLAM?

A: SLAM potentially includes

- loop closure
- dynamics constraints
- velocities, accelerations



Loop Closure in Visual SLAM



ORB-SLAM, Mur-Artal, Tardos, Montiel, Galvez-Lopez

Bundler (bundle adjustment/structure from motion)

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \tilde{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ] ||^2 $$

Indicates the frame of the k-th camera.

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{fm_xX}{Z} + c_x \\ \frac{fm_yY}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \tilde{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ] ||^2 $$

Actual pixel measurement of 3D point ${}^wp$ from camera k

Expected pixel projection of 3D point ${}^wp$ onto camera k

$$ h_{\text{pinhole}}(X,Y,Z) = \begin{bmatrix} \frac{fm_xX}{Z} + c_x \\ \frac{fm_yY}{Z} + c_y \end{bmatrix} + n, \quad n \sim \mathcal{N}(0,R) \quad \text{noise (in pixels)} $$

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} || \tilde{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ] ||^2 $$

Q: Is the scale of these two estimates accurate/unambiguous when measurements are done from a monocular (single) camera in motion? I.e. is it observable?

Note: **scale** = relationship between real-world metric distances and estimated map distances. I.e. relationship between distance units.

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} ||\tilde{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ]\ ||^2 $$

Q: Is the scale of these two estimates accurate/unambiguous when measurements are done from a monocular (single) camera in motion? I.e. is it observable?

A: No, regardless of how many common keypoints are matched in between camera frames. Without external reference distance, e.g. stereo baseline, or real size of observed object, the scale is ambiguous and unobservable, just as it was in Visual Odometry.

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} ||\tilde{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ]\ ||^2 $$

Q: Is the scale of these two estimates constant during the entire experiment, if we use a monocular camera?

Structure from Motion as Least Squares

$$ {}^k_wR^*, {}^kt^*_{kw}, {}^wX^*, {}^wY^*, {}^wZ^* = \underset{{}^wp=[{}^wX,{}^wY,{}^wZ],\ {}^k_wR,\ {}^kt_{kw}}{\operatorname{argmin}} \sum_{k=1}^{K} ||\tilde{z}^{(k)} - \mathbb{E}[\ h_{\text{pinhole}}({}^k_wR\,{}^wp + {}^kt_{kw})\ ]\ ||^2 $$

Q: Is the scale of these two estimates constant during the entire experiment, if we use a monocular camera?

A: No. During in-place rotations there is not enough baseline between camera frames to triangulate new points. So, error in structure and in motion accumulates → **scale drift**

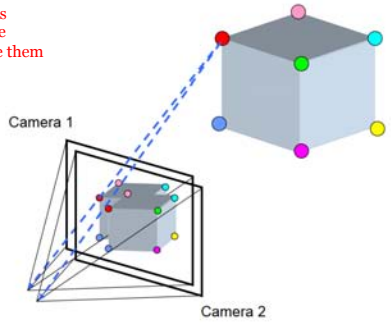
Similarly to visual odometry with a single camera.

Problem #4: optical flow

Multi-view geometry problems

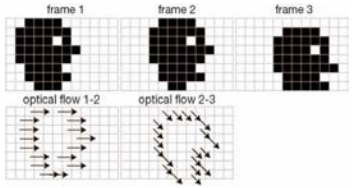
- **Optical flow:** Given two images, find the location of a world point in a second close-by image with no camera info.

3D point coordinates are unknown, but we won't try to estimate them



Camera frame transformations are unknown, but we won't try to estimate them

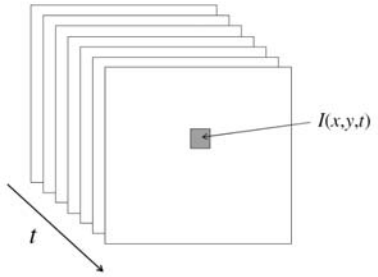
We are estimating pixel displacement from one image to the next



http://tcr.amegroups.com/article/viewFile/3200

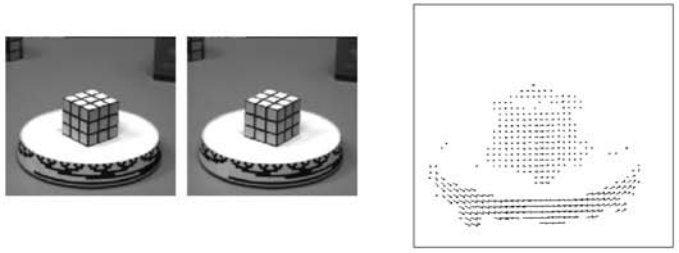
Video

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



Motion estimation: Optical flow

Optic flow is the **apparent** motion of objects or surfaces



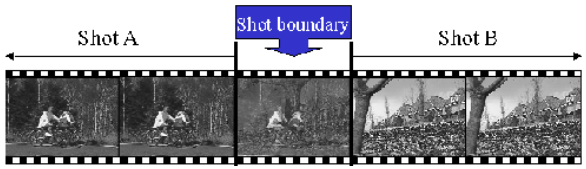
Will start by estimating motion of each pixel separately
Then will consider motion of entire image

Motion Applications:
Segmentation of video

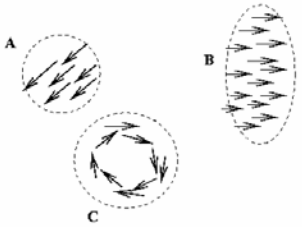
- Background subtraction
  - A static camera is observing a scene
  - Goal: separate the static *background* from the moving *foreground*

Motion Applications: Segmentation of video

- Shot boundary detection in edited video
  - Edited video is usually composed of *shots* or sequences showing the same objects or scene
  - Goal: segment video into shots for summarization and browsing (each shot can be represented by a single keyframe in a user interface)
  - Difference from background subtraction: the camera is not necessarily stationary

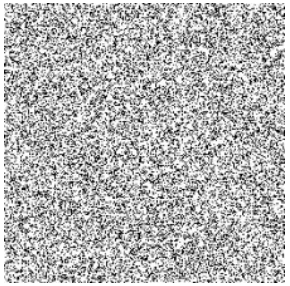


- Background subtraction
- Shot boundary detection
- Motion segmentation
  - Segment the video into multiple coherently moving objects



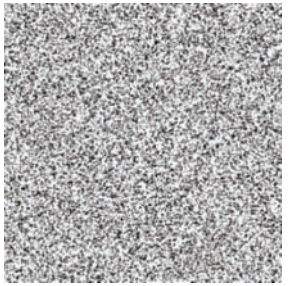
Motion and perceptual organization

- Sometimes, motion is the only cue



Motion and perceptual organization

- Sometimes, motion is the only cue

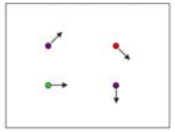


Motion and perceptual organization

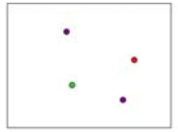
https://www.youtube.com/watch?v=n9TWwG4SFWQ

Experimental study of apparent behavior.
Fritz Heider & Marianne Simmel. 1944

Problem definition: optical flow



$I(x,y,t)$



$I(x,y,t+1)$

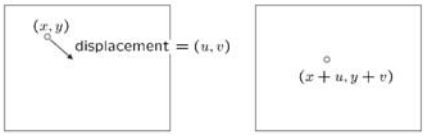
How to estimate pixel motion from image $I(x,y,t)$ to $I(x,y,t+1)$?

- Solve pixel correspondence problem
  - Given a pixel in I(x,y,t), look for nearby pixels of the same color in $I(x,y,t+1)$

Key assumptions

- Small motion: Points do not move very far
- Color constancy: A point in $I(x,y,t)$ looks the same in $I(x,y,t+1)$
  - For grayscale images, this is brightness constancy

Optical flow constraints (grayscale images)



$I(x,y,t)$

$I(x,y,t+1)$

- Let's look at these constraints more closely
  - Brightness constancy constraint (equation)

$$I(x,y,t) = I(x+u, y+v, t+1)$$

  - Small motion: (u and v are less than 1 pixel, or smooth)

From Taylor expansion of I

$$I(x+u,y+v,t+1) = I(x,y,t+1) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

Optical flow equation

- Combining these two equations

$$0 = I(x+u,y+v,t+1) - I(x,y,t)$$
$$\approx I(x,y,t+1) + I_x u + I_y v - I(x,y,t)$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$ for t **or** $t+1$)

Optical flow equation

- Combining these two equations

$$\begin{aligned} 0 &= I(x+u, y+v, t+1) - I(x,y,t) \\ &\approx I(x,y,t+1) + I_x u + I_y v - I(x,y,t) \\ &\approx [I(x,y,t+1) - I(x,y,t)] + I_x u + I_y v \\ &\approx I_t + I_x u + I_y v \end{aligned}$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$ for t **or** $t+1$)

Optical flow equation

- Combining these two equations

$$\begin{aligned} 0 &= I(x+u, y+v, t+1) - I(x,y,t) \\ &\approx I(x,y,t+1) + I_x u + I_y v - I(x,y,t) \\ &\approx [I(x,y,t+1) - I(x,y,t)] + I_x u + I_y v \\ &\approx I_t + I_x u + I_y v \end{aligned}$$

(Short hand: $I_x = \frac{\partial I}{\partial x}$ for t **or** $t+1$)

In the limit as u and v go to zero, this becomes exact

Brightness constancy constraint equation

$$I_x\, u + I_y\, v + I_t = 0$$

The brightness constancy constraint

Can we use this equation to recover image motion (u,v) at each pixel?

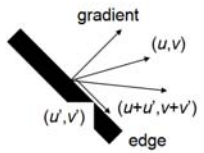
$$0 = I_t + \nabla I \cdot \langle u, v \rangle \quad \text{or} \quad I_x u + I_y v + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation (this is a scalar equation!), two unknowns (u,v)

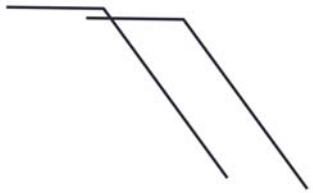
The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured

If (u, v) satisfies the equation, so does $(u+u', v+v')$ if

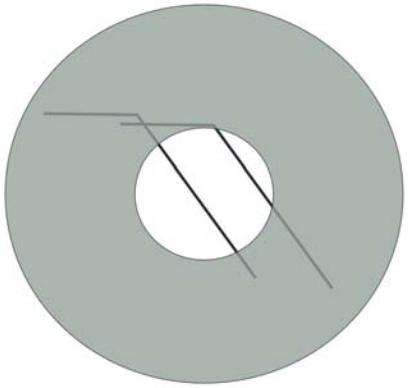
$$\nabla I \cdot [u' \; v']^T = 0$$



Aperture problem



Aperture problem



Solving the ambiguity…

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

- How to get more equations for a pixel?
- **Spatial coherence constraint**
- Assume the pixel's neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u\ v]$$

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}$$

Solving the ambiguity…

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix} \quad \underset{25\text{x}2}{A}\ \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$