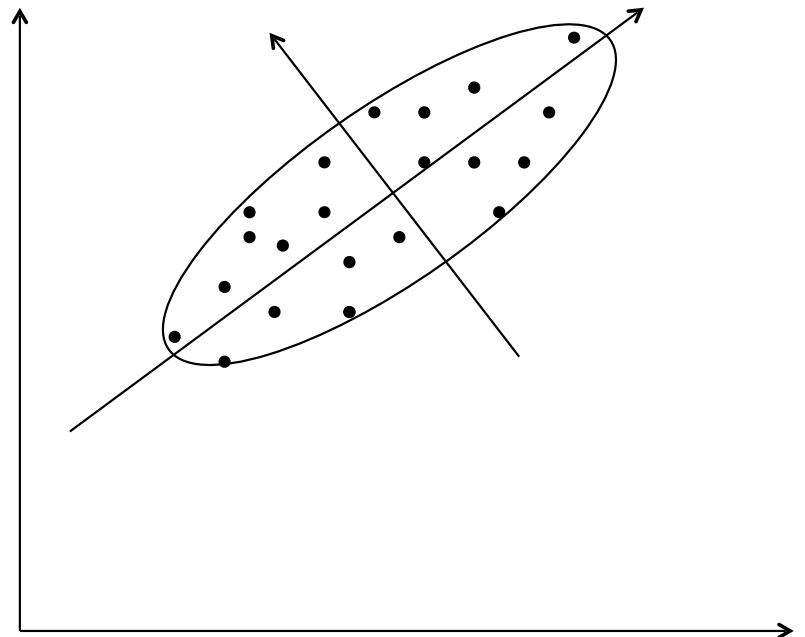


Principal Component Analysis

- Eigen Vectors show the direction of axes of a fitted ellipsoid
- Eigen Values show the significance of the corresponding axis
- The larger the Eigen value, the more separation between mapped data
- For high dimensional data, only few of Eigen values are significant





What is PCA?

- Finding Eigen Values and Eigen Vectors
- Deciding on which are significant
- Forming a new coordinate system defined by the significant Eigen vectors
(→ lower dimensions for new coordinates)
- Mapping data to the new space

à Compressed Data

How is PCA used in Recognition?

- A training set is used for LEARNING phase
 - Applying PCA to training data to form a new coordinate system defined by significant Eigen vectors
 - Representing each data in PCA coordinate system (weights of Eigen vectors)
- A test set is used for TESTING phase
 - Same PCA coordinate system is used
 - Each new data is represented in PCA coordinates
 - New data is recognized as the closest training data (Euclidean distance)

PCA for images

- Each image is represented as a 1-D data Γ_i
- Finding Eigen values/vectors is expensive
- Turk/Pentland Trick:

$$\text{average picture : } \Psi = \frac{1}{M} \sum_{i=1}^M \Gamma_i$$

$$\Phi_i = \Gamma_i - \Psi$$

$$\text{Covariance Matrix : } C = \frac{1}{M} \sum_{i=1}^M \Phi_i \Phi_i^T = AA^T$$

$$\text{image : } N \times N \rightarrow C : N^2 \times N^2$$

$$\text{Trick : } AA^T : N^2 \times N^2 \text{ but } A^T A : M \times M$$

$$A^T A v_i = \lambda_i v_i \rightarrow AA^T A v_i = \lambda_i A v_i$$

$$v_i : \text{eigenvector of } A^T A \rightarrow A v_i : \text{eigenvector of } AA^T$$



What are Eigenfaces?

- Turk and Pentland used PCA method for face images
- All faces are about the same size
- Each face image is a data vector.
- Each Eigen vector is actually an image called an Eigenface.



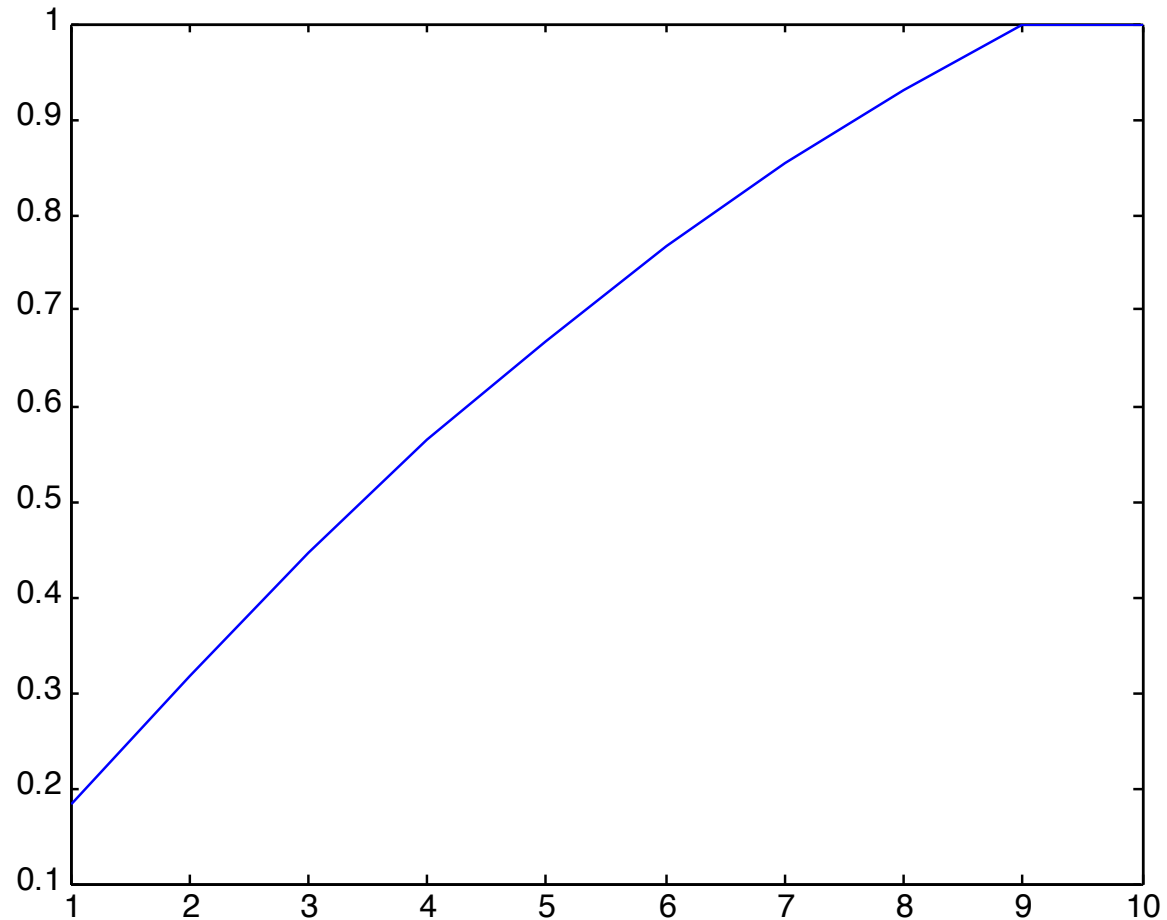
Eigenfaces

Average image



Significant Components

$$g(m) = \sum_{i=1}^m \lambda_i$$



References

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- 4) ‘Principal components analysis’, http://en.wikipedia.org/wiki/Principal_component_analysis.
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