A Basis Function Model of Parietal Cortex

(Pouget, A. and Sejnowski, T.J., "Spatial transformations in the Parietal cortex using basis functions", *Journal of Cognitive Neuroscience*, Vol. 9, No. 2, pp 222-237, 1997)

Functions that map from one coordinate system to another in 3 dimensions are generally nonlinear.

As the parietal cortex is involved in performing these coordinate transformations, it must be capable of approximating nonlinear functions.

One way to approximate nonlinear functions is to use linear combinations of nonlinear *basis functions*. For example, one can approximate a function by a sum of sinusoids (Fourier Series).

Two types of (basis) functions have been often used as models of neuron response:

Gaussian

Sigmoid

Pouget and Sejnowski propose that the basis function framework can be used to interpret the reponse of the "gain-field" neurons in the parietal cortex.

For example, if a motor command, M, is anonlinear function of its inputs (such as the location of a visual stimulus on the retina and the current eye position), then it could be generated by a linear combination of basis functions of the input signals:

$$M = \sum_i c_i B_i(\vec{S}, \vec{P})$$

where $\vec{S}$ is a vector of sensory inputs, and $\vec{P}$ is a vector of postural inputs (e.g. eye, head, or arm positions).

B_i are the basis functions and c_i are the weights associated with the basis functions.

The weights c_i would depend on the particular motor command that was being computed.

The advantages of using the basis function approach are:

- Once the basis functions have been computed, only a linear summation is required to generate the motor command
- Multiple systems can share the basis functions
- Development or learning of the basis functions can proceed in an unsupervised manner since their choice is independent of the output functions being computed.

Posterior Parietal neuron responses are consistent with the basis function hypothesis. These neurons have receptive fields fixed on the retina, but their amplitudes are modulated by eye position.

The response of one of these cells can be modeled as a product of a gaussian function of retinal location with a sigmoid function of eye position. The products of a set of basis functions also form a set of basis functions.

A Neural Network Model

Pouget and Sejnowski simulated a neural network model of Posterior parietal using the basis function approach.

This model was similar to the Zipser and Andersen model, except that it was shown to allow many different coordinate mappings to be computed with the same network.

Their neural network was a 3-layer network, with the input layer consisting of a retinotopic map of visual stimuli with gaussian receptive fields and an eye position map with sigmoidal receptive fields.

The sigmoidal eye location receptive fields were different than in the Zipser and Andersen model, which used a piecewise linear response pattern, which did not saturate at a maximum level. A sigmoidal response pattern is more realistic.

The gaussian visual receptive fields were uniformly distributed about a ± 60 degree area in increments of 12 degrees, with a width (standard deviation) of 18 degrees.

The inflection points of the sigmoidal eye position receptive fields were also uniformly spread between -40 and $+40$ degrees in steps of 8 degrees. The slope factor T was set to 8 degrees.

Unlike the hidden units in the Zipser and Andersen model, the hidden units in the Pouget and Sejnowski model were not learned, but were set to compute the product of the gaussian visual receptive fields and the sigmoidal eye position fields.

That is, the hidden layers implement the following basis functions:

$$h_i = \frac{e^{-\frac{(r_x - r_{x_i})^2}{2\sigma^2}}}{1 + e^{-(e_x - e_{x_i})T}}$$

A delta rule supervised learning procedure was used to determine the weights applied to the inputs to the output layer, coming from the hidden layer.

It was shown that, using this learning procedure, the neural network could learn many different types of coordinate transformations:

Retinotopic-to-Head

Retinotopic-Retinotopic

By adding in more input signals, such as head or body position, more coordinate systems can be represented.

All of the possible coordinate transformations, even the trivial ones such as retinotopic to retinotopic, are computed in exactly the same way.

Complexity and Modularity

The basis function approach shows how many different frames of reference can be encoded by a single set of basis function neurons.

A single basis function representation, however, demands a large number of neurons, as the number of basis functions required increases exponentially with the number of stimulus dimensions

(location of the stimulus on the retina, eye position, head position, arm position, auditory localization, vestibular inputs, somatosensory inputs...)

The number of neurons could be reduced by selectively sampling the input space. That is, some parts of the input space may be represented more accurately than others.

One could impose correlations between certain input variables. For example, for many parietal neurons, it appears that the dependence of the neurons response to head and eye position is correlated along a particular direction in space specific to each neuron.

A compromise between extreme modularity and a signle multi-purpose basis function is possible. A sensori-motor transformation might be decomposed into a number of steps, where each step may involve a multi-purpose basis function modules.

Each of these modules would be used to represent 2 or 3 different frames of reference and would be used in several types of behaviour, facilitating crosstalk and coordination between the behaviours.