Techniques for Minimization

The Tikhonov approach for regularizing ill-posed problems requires that a functional be minimized with respect to the space of candidate solutions.

In computer vision algorithms this minimization is usually performed using one of the following methods:

- **Calculus of Variations:** Derive a (set of) partial differential equation(s) (Euler equations) whose solution is the quantity that minimizes the functional. This equation is usually discretized and solved numerically.
- **Gradient Descent:** If the space, Z, of solutions admits a gradient on it, then one can select a point in the solution space and move in the direction of the negative of the gradient of the functional. That is:

  $$\frac{d\vec{z}}{dt} = -\nabla_Z \vec{E}(\vec{z})$$

  One follows the negative of the gradient until the gradient becomes zero, corresponding to a minimum of the functional. This does not guarantee reaching the global minimum, however.

- **Exhaustive Search:** Discretize the solution space and examine the value of the functional at every point.
- **Stochastic Search:** Randomly sample the solution space, evaluating the functional at each of these samples.
  - **Monte-Carlo methods:** Samples the space randomly, but uniformly (doesn't use any prior knowledge of the solution). An example is the *Metropolis Algorithm*. In the Metropolis algorithm a the candidate solution is perturbed by adding a random offset. The functional is evaluated at this new candidate solution. If the functional value has decreased then the perturbation is retained, otherwise the original solution is used. The magnitude of the perturba-

tions can be varied over time, being large at the beginning and then reducing (see annealing). TO avoid getting trapped in local minima a small percentage of perturbations that increase the functional value are retained.

– **Simulated Annealing:** Uses a model of the physical process by which solids form as they are cooled from a liquid form. In simulated annealing the global minimum of some optimization problem is associated with the ground state of some imaginary physical system. The temperature of the system is held fixed and the minimum energy configuration of the system is computed, then the temperature is lowered, and a new configuration found. This process continues until the temperature reaches absolute zero. If the temperature reduction is done slowly enough (annealing) then the final solution will be the global minimum.

- **Deterministic Annealing (Mean-Field):** This uses statistical averages to determine the minimum value.
- **(Nonlinear) Resistive Networks:** Electrical networks naturally minimize the power (or more generally, the "co-content") dissipated in the network. Networks can be constructed to solve various minimization problems.
- **Dynamic Programming:** Some optimization problems can be recursively decomposed into simpler subproblems. Solutions to the subproblems can be linked to provide a solution to the more general problem.