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THESIS OF BACHELOR

论文题目：节点移动对随机传感网络栅栏覆盖的影响

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Impact of Mobility towards Asymptotic General Barrier Coverage with Heterogeneous Wireless Sensor Networks

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ABSTRACT

Barrier coverage, which is of significant value in intruder detection within battlefields and homeland security, has attracted much attention since the concept appeared. Numbers of excellent works have been done on the critical density or critical number of sensors to ensure barrier coverage, while few work focused on critical sensing range under a given number of sensors, specific kind of intruders and famous sensor mobility models, which is quite the ubiquitous analyzing approach in wireless sensor networks (WSNs). In this paper, we successfully derived the critical sensing radius (CSR) under the general barrier coverage condition with curved-path intruders and uniformly deployed sensors, with i.i.d and random walk mobility model, respectively. We firstly analyze the preliminary barrier coverage, deriving the CSR under intruders' fast-motion detection model, in which intruders come basically as line-based path. Then we use different techniques to achieve the critical conditions for general barrier coverage, whose intruders can move freely in any curved path, under the two sensing mobility models. Finally, we provide extensive simulations for our analytical results and an informative comparison with related works. The theoretical conclusion within this paper can be utilized into the practical situation of deploying barrier sensor networks.

Keywords: wireless sensor network, barrier coverage, heterogeneity, mobility, circuitous intruding

摘要

栅栏覆盖在被提出以来，因其应用于战场和国土安全的入侵者监测上的重要价值，引起了学者们广泛的关注。对于临界的传感器密度，或者说是达到栅栏覆盖所需要的临界传感器数量，学者们做了许多优秀的研究工作。但是对于在一定的传感器数目，特定形式的入侵者以及应用常用的节点移动模型的条件下的临界的传感半斤的推导以及分析却几乎无人问津。但这却是无线传感器网络（WSNs）领域中的常见的重要的科研问题。在本文中，我们成功地导出了在广义栅栏覆盖（允许入侵者曲线入侵）、传感器一致分布、节点移动的条件下临界传感半径（CSR），并且临界半径的的导出是对I.I.D移动模型和随机游动移动模型两种情况分别进行的。在一种情况中，我们首先分析初步栅栏覆盖，在快速入侵的移动模型，也就是入侵者直线入侵的情况下导出临界传感半径。然后我们用不同的手段，分别对于I.I.D模型和随机游动模型，来导出广义栅栏覆盖的临界条件，也就是曲线入侵栅栏覆盖的临界传感半径。最后，针对我们的理论推导结果，我们做了大量的仿真分析来阐述它的合理性。同时，我们也和其他著名学者的相关工作做了详尽的对比分析。我们相信，本文的理论结果可以被应用于实际的栅栏传感网络的部署以及设置。

关键词: 传感器网络，栅栏覆盖，不均匀性，节点移动，曲线入侵

Contents

Chapter 1 Introduction

1.1 Wireless Sensor Network

We first come to basic concepts and theories related to wireless sensor network, which is necessary to understand the topic and issue in our paper of WSNs' barrier coverage.

1.1.1 Introduction on WSNs

Wireless sensor networkYick et al. (2008), whose appearance has induced a new technical revolution and its applications are all around us Mainwaring et al. (2002) Werner-Allen et al. (2006). Among all the issues related to WSNs, the most heated topic is remote environment surveillance and target tracking. This trend of applications over WSNs has been enabled by the availability, especially in recent years, of sensors which are smaller, cheaper, more delicate and sensitive.

Sensors, is one kind of monitoring devices who can sense the targeted information and transact the sensed signals to electrical impulses and other forms of output, in order to satisfy the demands on information gathering, processing, storing and displaying. Recently, the sensor producers has been able to provide new type of sensors who is based on wireless environment and are small, with limited processing and computing resources, and they are not that costive compared to prototype sensors. The sensor nodes can sense, measure, and gather information from the environment monitored and, based on some local decision process or algorithm, they can transmit the sensed data to the user or concerning people.

Current WSNs are mostly deployed on land, underground, and underwater. Depending on the environment varies, a sensor network faces different challenges and constraints. There are five major types of WSNs: terrestrial WSN, underground WSN, underwater WSN, multi-media WSN, and mobile WSN, which is a major concern in our paper.

1.1.2 Fundamentals of Wireless Sensor Network

This trend of wireless sensor networks (WSNs) increasing usage inspires research for WSNs. We introduce several important properties and fundamentals, which is mainly introduced in Freris et al. (2010) , of WSNs on which researchers has did numerous excellent works and convincing results on.

Coverage Ghosh and Das (2008), is one of the most basic and practical issues. There are many kinds of coverage, within which full coverage is the most basic concern. Full coverage is mean to make the whole monitored area covered by sensing range of deployed sensors. Recent years, researchers has casted many results over full coverage problems. In Kumar et al. (2004), Kumar finished a theory concerning k-coverage of a sleeping wireless sensor network. And in Wang et al. (2011), Wang has solved the full k-coverage problems in mobile WSNs with prevail mobility models. Coverage is always a fundamental and significant requirement of a successful sensor network, especially for target monitoring and tracking applications.

Connectivity, is a concept related to coverage. Wireless network that is connected means that every randomly selected two sensor nodes in the network has a connected path between them. In other words, the whole wireless sensor network constitutes a connected graph. This is particularly meaningful in sensor networks, where achieving a common application aims requirement of communication among all the nodes. In Freris et al. (2010), Freris introduced two important random graphic models to evaluate and achieve connectivity, which is Erdos-Renyi Graphs and Random Geometric Graphs.

Clock synchronization is a problem generated by the fact that distributed clocks don't generally agree precisely. Yet, clock synchronization is required for several applications in wireless sensor networks. Applications include coordinating events in a distributed system, tracking targets, monitoring areas, target localization and closed loop control over networks. Scheduled operations like power-efficient duty-cycling which can result in significant and evident energy savings are grounded in accurate clock synchronization.

In network computation is evoked by the sensors' strong computing capacities from which technology guarantees. Sensors are now able to not only gathering and transacting data, but also do quite a measurement of calculation. Sensor networks are mostly interested to gather results over functions of distributed data, and sensor nodes are sufficient to run the function computations. There are two basic type of computing functions. One is type threshold function means that only the environmental data surpass a given threshold can the sensor be activated to calculation and data gathering. The other type is type sensitive function, meaning every delicate variation in the environment monitored can cause the function result change. The computation over the two types of functions almost cover all kinds of real world data processing within sensor networks. And because that only the processed data is ready for user to distinguish and understand, in network computation is thriving in recent research topics in WSNs theories.

1.2 Barrier Coverage

Barrier coverage Gage (1992) can be utilized into many significant events, e.g., national border, critical resource and infrastructure protection, security surveillance and intruder detection, etc. In a wireless sensor network, the barrier is formed by a group of sensors whose sensing ranges are heterogeneous and span across the monitored area. Every intruder can be detected when it enter the circles covered by the sensors' sensing radius. Compared with full coverage, the number of sensors which barrier coverage demands is much smaller. In this way, barrier coverage is considered to be more applicable and desirable for large scale deployment in practical concern.

1.2.1 Developments and Practical Concerns

The deployment of barrier sensors is always a heated topic. There has been strong barrier coverage which means there is no gaps between sensor's sensing span and sensors are placed side by side regularly along straight lines across the monitored region Kumar et al. (2005). This is an ideal sensor deployment for its simplicity and efficiency. Liu and Dousse have studied critical conditions of achieving this strong barrier coverage Liu et al. (2008). Recently, Chen and Li raised a new concept of strong barrier coverage, which is one-way barrier coverage Chen et al. (2011). In their novel protocols, intruders coming from outside is illegal while exiting from inside is ignorable. Both the two works have given excellent insights into strong barrier condition analysis and deployment strategy. However, this kind of coverage is quite difficult to implement in realistic situations considering complex terrains, other hard-to-reach areas and the huge number of sensors, which all make this specific deployment infeasible. The general condition of sensors' deployment, which is also known as random uniform deployment, may include dropping a large number of sensors from vehicles such as aircrafts along predetermined routes Pister (2001)Saipulla et al. (2008). In Saipulla et al. (2008), they give this deployment a high random property and make it much more practical and feasible than the previous one.

In our paper, we firstly define a belt region where its length is up to kilometers and width up to several meters, which is exactly the same as those described in S. Kumar's paper Kumar et al. (2005). And our region meets well with the sensor deployment scheme as enlightened in Saipulla et al. (2008). In Kumar et al. (2005), S. Kumar gives a definition of weak coverage where intruders only come in identical straight line-path. However, S. Kumar and his collaborates failed to consider the path of inclined lines, whether this angle can make a difference is investigated in our paper. After the basic sensor region and deployment are approved by researchers, many of them give their efforts to the research into sensor collaboration schemes, dimensionality on barrier coverage, and the construction and monitoring of barrier coverage Chen et al. (2008)Yang and Qiao (2010)Barr et al. (2009)Yang and Qiao (2009).

1.2.2 Critical Sensing and Mobile Networking

Recently, researchers have paid attention to the critical sensor density under different sensor deployments or models. For the strong barrier coverage, Liu has given their insights into the critical conditions of implementing strong barrier coverage. And due to the narrowed application of the strong barrier, Saipulla and Westphal study over line-based deployed sensor barrier Saipulla et al. (2009). They work out the influence of sensor density and deployment scheme on the barrier coverage. Another work of

Saipulla and Liu has reflected the impact of limited mobility towards barrier coverage Saipulla et al. (2010). In addition, Wang and Cao pay attention to camera barrier coverage, whose detection will be much more informative than scalar barrier coverage Wang and Cao (2011). They analyze the problem of constructing camera barrier, camera sensor deployment and the critical number of cameras for barrier coverage. The above enlightening papers and some other famous works focus on a given sensor's sensing range and try to find a critical sensor density for achieving solid barrier coverage. However, none of these works gives an explicit formula defines the critical sensing range under the mentioned sensing deployment scheme.

Meanwhile, many studies has been casted towards mobility in wireless networks, which is quite a heated topic and we hope to employ in our analysis. Mobile WSN applications include but are not constrained to environment monitoring, target tracking, rescuing survivals, and real-time monitoring of vital material. For environmental monitoring in disaster areas, manual deployment might not be possible. With mobile sensors, they can move to areas of events after original deployment to provide the required coverage. In military surveillance, area protecting and target tracking, mobile sensor nodes can collaborate and make decisions based on the wanted. Mobile sensor nodes can achieve a higher degree of coverage and connectivity compared to static sensor nodes, as is indicated in Liu et al. (2005). In the existance of obstacles in the field, mobile sensor nodes can plan ahead and move appropriately to blocked regions to increase possibility of target's exposure.

The mobile networks' overall performance can be better than the stationary networks. For instance, in Wang et al. (2011) Wang proved that mobility can increase coverage performance by reducing demanded critical radius, as well as reducing energy consumption. Grossglauser and Tse has argued in Grossglauser and Tse (2002) that mobility of the nodes in an ad-hoc wireless network can increase the its throughput. On the other hand, mobility has negative influences to the networking performance in some specific situations. The coverage at sea surface is always disturbed due to the currency and wind forces. Luo and Wang has studied the mobility pattern in sea surface Luo et al. (2009) and design sensing patterns to achieve full coverage in underwater sensing environment.

1.3 Highlights on Our Research

In our work, we utilize a different analyzing approach comparing with our formers, which considers the critical sensing range when the number of sensors grows to infinity. We separate our analysis to two main parts for i.i.d model and random walk model respectively.

In i.i.d model, due to the fact that belt shaped barrier whose width is much more smaller than its length, as well as the intruders' fast speed when they are crossing a monitored area, we suppose that the intruders approximately goes a straight line path when intruding the barrier. This intruder mobility model is also raised in Kumar et al. (2005) by S. Kumar. We define the intruding path either identical or inclined, then the preliminary barrier coverage condition is analyzed and our result on it is proved. In addition, we may prove that if the identical line condition is guaranteed, the inclined one is guaranteed anyway. Finally, we move on to the general barrier coverage whose intruders may come with a curved path. We partition the barrier into numbers of sub-barriers and utilize the preliminary barrier coverage analysis into every sub-barrier. Admitting that every intruding path in a sub-barrier is straight line and path between neighboring sub-barriers is connected, we are able to guarantee a curved path in the integral barrier which is the combination and integrity of the sub-barriers. We may also find out that the more sub-barriers we partition, the more approximately the path presents as a curve. In our analysis, partition times of the integral barrier goes towards infinity as $n \to \infty$, thus we may almost definitely have a curved intruding path in the integral barrier. Afterwards, we made amends and estimations to the intruders' speed, and achieved the speed resolution to make it clear how fast an intruder can be for deployment guidance.

In the random walk mobility model, which is mostly 1-dimensional here for barrier coverage, we have firstly give similar definitions to intruder path and speed as the i.i.d model. Then we derive proof on the preliminary barrier coverage, in which any given value of radius can serve as CSR. This is because the line based movement in random walk model already guarantees barrier coverage which is a basically line coverage problem. Then we give several vital reasons that the general barrier condition is not applicable and move forward to full barrier coverage with its convincing advantages. With techniques alike in previous part, we successfully derived the CSR in full barrier coverage under 1-dimensional random walk mobility model.

Last but not least, we also consider other attributes in WSNs to make our work solid and dedicated. Heterogeneity is an important consideration in coverage problems Wang et al. (2011). In large scale wireless sensor networks, the numerous sensors maybe come from different manufacturers. As is known to all, the manufactures have different standards, thus this discrepancy causes in heterogeneous sensor performances, like sensing radius, transmission/receiving power and antenna designs, etc. Meanwhile, when the sensors exceed the expiring date or are damaged by natural or human factor, heterogeneity of the WSNs is displayed as well. Therefore, a model with heterogeneity is more realistic and practical, in our model, we incorporate this property and we hope to derive more pragmatic results.

Our results demonstrate that as the number of sensors within a barrier grows, both preliminarily and generally, the critical sensing range of each sensor decreases fast. Within our result, if the number of sensors in a given barrier region is known to us, we are able to determine the critical sensing range (CSR) of each sensor. In this way, we can have the sensors' ranges not lower than CSR, so that every intruder can be detected. We can also make sure that the sensing range is not much higher than CSR, so that we can promise a smarter energy saving strategy and cost efficiency achievement.

1.4 Chapter Preview

The rest of the paper is organized as follows. We give the definition of our model and describe the coverage problem in chapter 2. We derived the CSR for preliminary and general barrier coverage in i.i.d mobility model in chapter 3 . Afterwards, in chapter 4 we give a definition of speed resolution to categorize the intruder speed in a given CSR and barrier width. Additionally, we made full analysis towards the barrier coverage in random walk mobility model, both preliminarily and generally at chapter 5. According to the results we get, evaluation is made in chapter 6, and finally we conclude the work we've done and give inspiration for future work in chapter 7 .

1.5 Main Results

Our main accomplishments are presented as below, each condition is based on random uniform deployment:

- Under the I.I.D sensing mobility model, preliminary barrier coverage condition whose intruders move in line-paths have the critical sensing radius $\Theta(\frac{\log n}{2n})$. And in the general condition with intruders in curved-path, we use results in sections of preliminary condition and achieve a meticulous proof. We demonstrate that in the general barrier coverage, the critical sensing radius should be $\Theta(\frac{\log n(\log n - \log\log n)}{2n})$.

- Under I.I.D mobility model, to achieve successful barrier coverage, the intruders' speed should not pass below a certain threshold which we define as the speed resolution. The scaling law expression of speed resolution is $\Theta(\frac{r(w_0-2r)^2-\frac{4}{3}\pi r^2}{4r(w_0-2r)})$, in which $w_0 = \frac{w}{\log n}$. w represents the width of the barrier region while the length is unified to 1. And r is the critical sensing radius in the given barrier condition.

- Under 1-dimensional random walk mobility model, the preliminary barrier coverage is achieved naturally by the line based mobile pattern which means any positive value whatever tiny can serve as the CSR. And the general barrier coverage condition is achieved by employing full barrier coverage to the belt region. The CSR in full barrier coverage is $\Theta\frac{3w(\log n+\log\log n)}{4n}$ and w represents the width of barrier region while length is defined as 1.

Chapter 2 Notations and Models

In this section, we present sensing and deployment model we use in our work, describe the definition and meaning of barrier coverage under the fast-motion detection model, and define critical sensing range to evaluate the conditions for achieving barrier coverage.

2.1 Sensing and Deployment Model

In heterogeneous wireless sensor networks, we assume that n sensors are initially deployed in a belt area with the unit length and the width of w. A sensor S_i has its own sensing range r_i, which correspond to a sensing area A_i (i.e., a circle centered at S_i with the radius of r_i). Any intruder moving pass the area A_i will be detected by the sensor S_i.

Definition 2.1. *In fast-motion detection model, the intruder's velocity is fast enough such that when the intruder goes through the belt region, the velocity remains the same (i.e., the magnitude and the orientation of the velocity keep unchanged).*

Definition 2.2. *In fast-motion detection model, the crossing path is actually a straight*

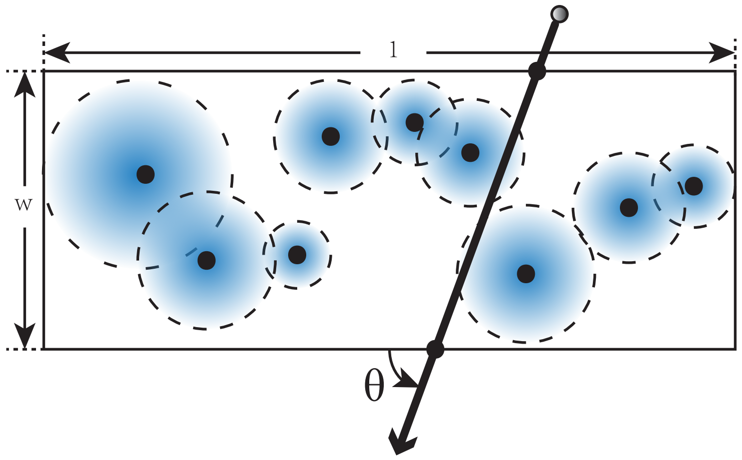


Figure 2.1 The crossing path and the incoming angle.

line. As you can see in the figure, we define the angle between the edge of the belt region and the crossing path as the incoming angle.

We consider heterogeneous sensors similar to Wang et al. (2011). Sensors are divided into u different groups $G_1, G_2, \cdots, G_u$, where u is a positive constant. For $y = 1, 2, \cdots, u$, group G_y consists of $n_y = c_y n$ sensors, where n is the total number of sensors in the network and $c_y (y = 1, 2, \cdots, u)$ is called the *grouping index*, which is a positive constant invariant to n and $\sum_{y=1}^{u} c_y = 1$. All sensors in group G_y own identical sensing radius r_y. We mainly study the asymptotic coverage here, implying that n is a variable approaching to infinity, whereas r_y is dependent variables of n, sometimes denoted by $r_y(n)$. When the total number of sensors n changes, the requirements for $r_y(n)$ should change along with n.

In our work, sensors are deployed according to uniform deployment, which means that n sensors are randomly and uniformly deployed in the operational region, independent of each other. The operational region is a belt region, which is supposed to be a torus so that we can ignore the boundary effect.

2.2 Description of Preliminary Barrier Coverage

Preliminary barrier coverage was first proposed by Kumar in Kumar et al. (2005), in which he called it weak barrier coverage for its limitations of detecting only straight-identical-path intruders. We modify the definition under the fast-motion detection model, and extend its concept to accept inclined-straight-path intruders. We define this kind of barrier coverage the preliminary barrier coverage

Definition 2.3. *Given a belt region with n sensor nodes, the preliminary barrier coverage can be achieved if and only if for any possible crossing path (i.e., straight lines or inclined straight line) of the intruder, it can be detected by at least one sensor.*

From the definition, we know that the incoming angle θ of the crossing path $\in [0, \pi]$. We mainly focus on the condition that $\theta = \pi/2$, in this case, the crossing path is parallel to the width of the belt region. We derive the critical result in this case and further we prove that this result is still sufficient for other incoming angles, thus we consider this

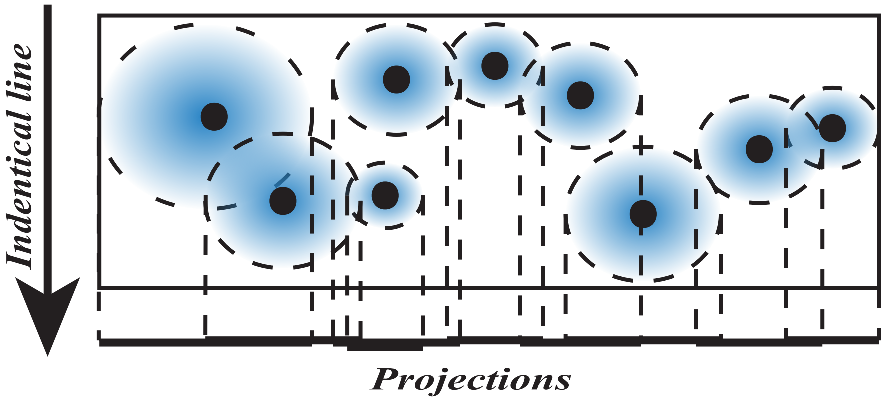


Figure 2.2 The intruders and the projections of the sensing area.

result is the critical one under the assumption that we don't know what actually the intruder's crossing path is.

2.3 Description of General Barrier Coverage

Few researchers has payed attention to the barrier coverage's properties whether their intruders goes straightly or circuitously. This is because that previous works are mostly done on the critical density to achieve strong barrier coverage, in which sensors are bounded one by one and no intervals exists as is introduced in Liu et al. (2008)Chen et al. (2011).

As the previous section presents, S. Kumar proposed the weak barrier coverage and derived the critical conditions of achieving it. In Kumar et al. (2005), Kumar holds that it is quite difficult to derive the CSR when intruders are coming in a curved path, while this kind of intruders are the conditions meet the real world for on one while move strictly in a line. Based on its difficulty and its practical utilities, we propose the definition of general barrier coverage and give insightful analysis to it in our paper.

Definition 2.4. *Given a belt region with n sensor nodes, the general barrier coverage* *can be achieved if and only if for any possible crossing path, whether it is circuitous or smooth, of the intruder, it can be detected by at least one sensor.*

From the definition, we know that this kind of barrier coverage is the most strong and practical model in this research area. Compared with weak and preliminary barrier coverage, the intruder's trace has no limitations for detection, and a lot more significance is added. Concerning the strong barrier coverage Liu et al. (2008), their low stability when any sensor is failed and the difficulty of deployment makes its less pragmatic than our general barrier coverage.

2.4 Definition of Critical Sensing Range

In heterogeneous wireless sensor networks, sensors in different group G_y have different sensing range r_y. We denote the weighted summation of all sensors' sensing ranges $r_\star = \sum_{y=1}^{u} c_y r_y$. Let $\mathcal{H}$ denote the event that the barrier coverage is achieved. The value of critical sensing range (CSR) must suffice the following inequalities.

Definition 2.5. *$R_\star$ is the critical sensing range (CSR) for event $\mathcal{H}$ if*

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{H}) = 1, \textit{ if } r_\star \geq cR_\star \textit{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{H}) < 1, \textit{ if } r_\star \leq cR_\star \textit{ for any } 0 < c < 1.$$

Here $R_\star$ is the CSR under the i.i.d model. Similarly, the corresponding CSR for 1-dimensional random work model $R_\diamond$ is achievable. According to the definition, no matter what value of r_y is for a special group G_y, when the order of the weighted sum $r_\star$ exceeds the order of CSR, event $\mathcal{H}$ is sure to happen asymptotically. On the other hand, $\mathcal{H}$ may not happen if the order of $r_\star$ is lower than that of CSR. Then CSR is a centralized parameter to judge whether barrier coverage will be achieved in heterogeneous sensor networks as n approaches to infinity. It provides a unified standard and an overall judgment for sensors of different sensing radii.

2.5 Coverage Type

Barrier coverage has two different types due to the kinds of mobility models.

- *[Coverage At An Instinct]* A point in the operational region is said to be covered at an instinct $t(t \geq 0)$ if it is sensed by one sensor in the detected region. Let $\eta(t)$ be the fraction of the whole operational region that is covered at instinct t.

- *[Coverage Over A Time Interval]* A point in the operational region is said to be covered during a time interval $\tau = [0, t)$ if it has been sensed by one sensor at the end of the interval. Let $\eta(\tau)$ be the fraction of the whole operational region that is covered during τ.

2.6 Mobility Pattern

Sensors move according to certain mobility patterns.

- *[I.I.D. Mobility Model]* The sensing process is partitioned into time slots with identical unit length. At the starting of each time slot, each sensor will randomly and uniformly choose a position within the operational region and stay stationary in the rest of the time slot.

- *[1-Dimensional Random Walk Mobility Model]* Sensors in each group are classified into two types of equal quantity, H-nodes and V-nodes. And sensors of each type move horizontally and vertically, respectively. The sensing process is also divided into time slots with unit length. At the very beginning of each time slot, each sensor will randomly and uniformly choose a direction along its moving dimension and travel in the selected direction a certain distance D which is a random variable uniformly distributed from 0 to 1. We do not set requirements on the velocity of sensor during its movement, but sensor must reach destination within the time slot.

The i.i.d. mobility pattern is widely used since it can provide kind of intuitions and characterize the upper bound or lower bound of a concerning variable. We will present the main approach to asymptotic coverage problems under this model. And to be straight forward, this mobility pattern is easy to accomplish by the most sleeping strategy mentioned in Kumar et al. (2004). In this strategy, the sensors are randomly awaken with a proposed possibility p at the beginning of a given time slot, and be on guard during this time slot. On concern of simplification, we omit the utilization of sleeping strategy in our analysis, for which we can simply replace our n to np to get the results under this sleeping model.

The 1-dimensional mobility model is motivated by certain networks whose nodes move along determined traces such as networks employed in streets, systems consisted of satellites moving in fixed orbits and so forth. And to be more applicable in our specific barrier region, we do not employ the 2-dimensional mobility pattern and we also omit V-nodes who move identically to the barrier boundaries due to the narrow width of our monitored area, in which mostly influencing movements are horizontal to the barrier boundaries, say the H-nodes' movements.

2.7 Chapter Review

In this chapter, we give important notations on definitions and models we will refer to later in our paper. In section 2.1 we give definitions on sensing and deployment model which is widely equipped in most IEEE papers of coverage problems. In section 2.2 we give brief but exact introduce of weak barrier coverage and developed our preliminary barrier coverage. In section 2.3 we provide the current flaws in definitions of barrier coverage, and we give a more applicable model of general barrier coverage. Then in section 2.4 we give definition to a vital concept in our paper, the CSR. It is the very method we evaluate barrier coverage's critical condition. And to achieve the critical condition, we make formulations of CSR by sensor number n, which can easily controlled by the network deploying engineer. Our main efforts in this paper is to derive the equation of CSR in terms of n. Meanwhile, we also considered mobility in wireless sensor networks. In section 2.6 we introduced two prevail mobility models which are i.i.d and random walk. Due to the two kinds of background models we utilize in barrier coverage, we need to include two types of coverage problems in section 2.5, which are coverage over an instinct and coverage over a interval. Based on the above concepts and models, we will start our research of analysis over barrier coverage's CSR in next several chapters.

Chapter 3 Barrier Coverage Under I.I.D Model

3.1 Preliminary Barrier Coverage

In this section, we give our result and analysis over the preliminary barrier coverage condition, whose intruders comes only with the straight-line paths.

3.1.1 Geometric Analysis

We first focus on the normal incidence condition, (i.e., crossing paths are parallel to the width of the belt Kumar et al. (2005)). Since the speed of the intruders is high, so the crossing path is almost a straight line, we draw a line segment $\mathbb{L}$ parallel to the bottom boundary with length 1, then we can get projections of all sensing areas corresponding to the sensor location and the sensor sensing range. For a sensor S_i with sensing range r_i, the projection will be a line segment centered at the horizontal coordination of S_i with the length of $2r_i$. Intuitively, every crossing path parallel to the width has a projection–a point locates at the horizontal coordination of the entrance at the top boundary.

Preliminary barrier coverage can be achieved if and only if that all projections of sensors suffice to cover $\mathbb{L}$. The preliminary barrier coverage now is actually a line coverage problem.

3.1.2 Coverage Problem Transformation

Lemma 3.1. *If the line could be seen as a $1 \times \varphi(n)$ virtual grid $\mathbb{M}$, (i.e., all points segmented from the line at a distance of $1/\varphi(n)$), then the line coverage problem is equivalent to the grid point coverage problem: if all the virtual grid points on the line is covered by sensing radius $r^{'}$, then the whole line is covered by the same network but with sensing radius r. Satisfying $r^{'} = r - \frac{1}{2l}$, $l = \frac{1}{\varphi(n)}$*

Proof. We let v be an arbitrary point in the barrier line (length 1). Without loss of generality, we may assume it is between two virtual grid point a, b as shown in Figure 6. Also, towards no loss of generality, we may assume that it is closest to point b. By assumption, there exists at least one active sensors that covers point b. Let one of the sensors' projections on the barrier line located at u as is shown in Figure 3.1. Then we

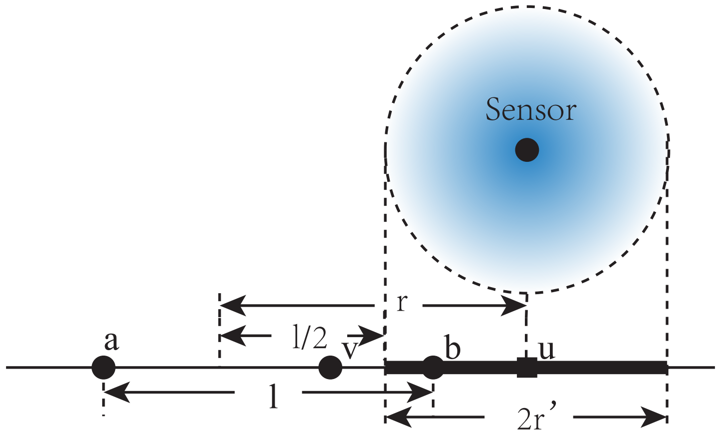


Figure 3.1 Relationship between grid coverage and line coverage

have $d(u, b) < r'$. From the relationship shown in the figure, we have

$$d(u, v) \leq d(u, b) + d(v, b) < r' + \frac{1}{2l} = r$$

The same holds for other sensors' projections covering point b. Therefore, we conclude that every point on the line is covered by using a sensing radius of r, if all the points on the virtual grid are covered using a sensing radius of r'. □

Lemma 3.2. *We define $\varphi(n) = n$ for our one dimensional condition, name the barrier line be partitioned by n. And the partition is sufficient of guaranteeing preliminary barrier coverage. l is the grid length and r' is the sensing range for grid coverage.*

Proof. S. Kumar proved in that in binary sensing model if $m = \varphi(n) \geq n \log n$, then the condition realizing the coverage of the points in a $\sqrt{m} \times \sqrt{m}$ dense grid is sufficient to guarantee the coverage of the whole unit square. That is to say $m = n \log n$ is large enough to ensure the asymptotic coverage in 2-dimensional condition.

And we can utilize the result in Wang's work Wang et al. (2011). Their result is based on Kumar et al. (2004) and we may have a critical sensing radius under partition $\sqrt{m} \times \sqrt{m} = m$ of a unit square from their work.

Similarly, we partition our barrier line into $1 \times \varphi(n) = n$ and apply our result in Theorem 1, thus we can give a brief but warranted comparison between 2-dimensional and 1-dimensional conditions.

Towards the rightness of our $\varphi(n) = n$ partition, we use the ratio between the *coverage radius* $r^{'}$ and *the grid length* l to evaluate the feasibility of this partition.

Thus, we have $\frac{1}{l} = o(r^{'})$, then $r = r^{'} + \frac{1}{2l}$ and $r^{'}$ are in the same order. If not, the result of $r^{'}$ is meaningless, because we will always have $r = \Omega(\frac{1}{2l})$.

We begin our comparison :

For $\sqrt{m} \times \sqrt{m}$ dense 2-dimensional grid area, the grid length is $1/\sqrt{m} = 1/\sqrt{n \log n}$, and the sensing radius is equal to $\sqrt{\frac{\log n + \log\log n}{\pi n}}$, so the ratio $r^{'}/l = \Theta(\log n)$, and this is the critical condition for successful coverage.

For $\varphi(n) = n$ dense 1-dimensional grid barrier line, the length of the grid is $l = 1/\varphi(n) = 1/n$, and the sensing range under this segmentation is $\Theta(\frac{\log n}{n})$, so the ratio $r^{'}/l = \Theta(\log n)$, this scenario suffices the Kumar's and Wang's critical condition for coverage problem conversion.

Therefore, we may understand that our $r^{'}$ is in a properly higher level over l just the same as Kumar and Wang's work. We conclude that our partition is warranted and reasonable. □

Namely, conditions to achieve coverage of $\mathbb{M}$ will also ensure preliminary barrier coverage of the belt. On the other hand, the coverage of $\mathbb{M}$ is obviously necessary for preliminary barrier coverage of the whole area, so the coverage of the belt is equivalent to the coverage of $\mathbb{M}$. We can focus our attention on the latter in the following analysis.

3.1.3 Necessary Condition Analysis

Let $\mathcal{H}$ denote the event that all points (density grid $\mathbb{M}$) on the line are covered. And we derive the critical sensing range to guarantee asymptotic full coverage on the line.

Definition 3.1. *$R_\star$ is the critical sensing range (CSR) for event $\mathcal{H}$ if*

$$\lim_{n\to\infty} \mathbb{P}(\mathcal{H}) = 1, \textit{if } r_\star \geq cR_\star \textit{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}(\mathcal{H}) < 1, \textit{if } r_\star \leq cR_\star \textit{ for any } 0 < c < 1.$$

We have several supporting lemmas to achieve our proof of necessary condition.

Lemma 3.3. *Given x and y are both positive functions of n. If x, x^2y approach to zero as $n \to +\infty$, then $(1-x)^y \sim e^{-xy}$*

Proof. The proof is given by Wang et al. (2011). □

Lemma 3.4. *If $R_\star(n) = \frac{\log n + \omega(n)}{2n}$, for any fixed $\beta < 1$ and $m = n$,*

$$m\prod_{y=1}^{u}(1-2r_y)^{c_y n} \geq \beta e^{-\omega}, \tag{3-1}$$

Proof. Taking logarithm of the left hand side of (3-1), we obtain

$$\begin{aligned}
&\log(m\prod_{y=1}^{u}(1-2r_y)^{c_y n})\\
=\;& \log n + \sum_{y=1}^{u}((c_y n\log(1-2r_y)))\\
=\;& \log n - \sum_{y=1}^{u}((c_y n)\sum_{i=1}^{+\infty}\frac{(2r_y)^i}{i})\\
=\;& \log n - \sum_{y=1}^{u}(c_y n(\sum_{i=1}^{2}\frac{(2r_y)^i}{i} + \delta_y))
\end{aligned} \tag{3-2}$$

where, we can have the upper bound of δ_y.

$$0 < \delta(x) = \sum_{i=3}^{+\infty}\frac{(x)^i}{i} < \sum_{i=3}^{+\infty}\frac{x^i}{3} = \frac{1}{3}\frac{x^3}{1-x} < \frac{x^2}{3} \tag{3-3}$$

Substituting $x = 2r_y$ into (3-3), we have

$$\delta_y = \sum_{i=3}^{+\infty}\frac{(2r_y)^i}{i} < \frac{4}{3}(r_y)^2 \tag{3-4}$$

Combine (3-2) and (3-4), we can obtain

$$\begin{aligned}
&\log(m\prod_{y=1}^{u}(1-2r_y)^{c_y n}) \\
&\geq \log n - \sum_{y=1}^{u}((c_y n)(2r_y + \frac{10}{3}(r_y)^2)) \\
&\geq \log n - 2nr_\star - \frac{10}{3}n\sum_{y=1}^{u}(c_y(r_y)^2) \\
&= -\omega - \frac{10}{3}n\sum_{y=1}^{u}(c_y(s_y)^2). \qquad (3\text{-}5)
\end{aligned}$$

Note that n we considered here is sufficient large, this ensures that $r_y < c_y$ for $i, j = 1, 2, \cdots, u$. We can simplify the equation

$$\sum_{y=1}^{u}(c_y(r_y)^2) \leq (\sum_{y=1}^{u}(c_y r_y))^{3/2} = (r_\star)^{3/2} \qquad (3\text{-}6)$$

Substituting (3-6) into (3-5)

$$\frac{10}{3}n\sum_{y=1}^{u}(c_y(s_y)^2) \leq \frac{10}{3}n(r_\star)^{3/2} \rightarrow 0, \text{as } n \rightarrow +\infty. \qquad (3\text{-}7)$$

For any $\epsilon > 0$, and all $n > N_\epsilon$

$$\log(m\prod_{y=1}^{u}(1-2r_y)^{c_y n}) \geq -\omega - \epsilon \qquad (3\text{-}8)$$

Let $\beta = e^{-\epsilon}$ and taking the exponent of both sides, the result follows. □

Proposition 3.1. *In the hetergeneous WSNs deployed in a belt region, if* $R_\star = \frac{\log n + \omega(n)}{2n}$, *then*

$$\liminf_{n\rightarrow+\infty} \mathbb{P}(\overline{\mathcal{H}}) \geq e^{-\omega} - e^{-2\omega}, \qquad (3\text{-}9)$$

where $\omega = \lim_{n\rightarrow+\infty}\omega(n)$.

Proof. We first study the case where $R_\star = \frac{\log n + \omega}{2n}$ for a fixed ω. We derive the following inequalities from Bonferroni inequality.

$$\begin{aligned}\mathbb{P}(\overline{\mathcal{H}}) &\geq \sum_{P_i \in \mathbb{M}} \mathbb{P}(\{\text{some point } P_i \text{ is not covered}\}) \\ &\geq \sum_{P_i \in \mathbb{M}} \mathbb{P}(\{P_i \text{ is the only uncovered point}\}) \\ &\geq \sum_{p_i \in \mathbb{M}} \mathbb{P}(\{P_i \text{ is not covered}\}) \\ &- \sum_{P_i, P_j \in \mathbb{M}}^{P_i \neq P_j} \mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}).\end{aligned} \tag{3-10}$$

Respectively, we can evaluate the two terms on the right hand size of (3-10). For the first term, we have

$$\begin{aligned}&\mathbb{P}(\{P_i \text{ is not covered}\}) \\ &= \prod_{y=1}^{u} \mathbb{P}(\{P_i \text{ is not covered by sensors in } G_y\}) \\ &= \prod_{y=1}^{u} (1 - 2r_y)^{c_y n}.\end{aligned} \tag{3-11}$$

Using lemma 3.4, we bound the first term for any const $\beta < 1$,

$$\sum_{P_i \in \mathbb{M}} \mathbb{P}(\{P_i \text{ is not covered}\}) \geq \beta e^{-\omega}, \tag{3-12}$$

for all $n > N_\omega$.

$R_\star(n) = \sum_{y=1}^{u} c_y r_y(n) = \frac{\log n + \omega}{2n}$. Hence for all $y = 1, 2, \cdots, u, r_y(n) = \Theta(\frac{\log n + \omega}{2n})$, this suffices that $r_y(n)$ and $r_y^2(n) c_y n$ approach 0 as $n \to \infty$. From lemma 3.3, we derive for any arbitrary positive constant α

$$(1 - \alpha r_y(n))^{c_y n} \sim e^{-\alpha r_y(n) c_y n} \tag{3-13}$$

Thus, for two points P_i and P_j in $\mathbb{M}$, we obtain that

$$\begin{aligned}&\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\\ \geq\ & \prod_{y=1}^{u}(1-4r_y)(1-4r_y)^{c_y n}\\ \sim\ & e^{-4n\sum_{y=1}^{u} r_y c_y n}. \end{aligned} \tag{3-14}$$

We can get the upper bound in a similar way.

$$\begin{aligned}&\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\\ \leq\ & \prod_{y=1}^{u} 2r_y(1-2r_y)^{c_y n} + (1-4r_y)^{c_y n}\\ \sim\ & e^{-4n\sum_{y=1}^{u} r_y c_y n}. \end{aligned} \tag{3-15}$$

From (3-14) and (3-15), we can close the gap between the upper bound and the lower bound, thus,

$$\mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\}) \sim e^{-4n\sum_{y=1}^{u} r_y c_y n}. \tag{3-16}$$

Note that the (P_i,P_j) pair is quite arbitrary, so the following results hold

$$\begin{aligned}&\sum_{P_i,P_j\in\mathbb{M}}^{P_i\neq P_j} \mathbb{P}(\{P_i \text{ and } P_j \text{ are not covered}\})\\ \sim\ & m^2 e^{-4n\sum_{y=1}^{u} r_y c_y n}\\ =\ & n^2 e^{-4nr_\star}\\ =\ & e^{-2\omega}. \end{aligned} \tag{3-17}$$

By substituting (3-12) and (3-17) to (3-10), we have

$$\mathbb{P}(\overline{\mathcal{H}}) \geq \beta e^{\omega} - e^{-2\omega}. \tag{3-18}$$

for any positive constant β.

As for the case that ω is a function of n with $\omega = \lim_{n\to+\infty}\omega(n)$, we know $\omega(n) \leq \omega+\delta$ for any $\delta > 0$ all $n > N_\delta$. Since $\mathbb{P}(\overline{\mathcal{H}})$ is monotonously decreasing in $r_\star$. and thus in ω, we have

$$\mathbb{P}(\overline{\mathcal{H}}) \geq \beta e^{\omega+\delta} - e^{-2(\omega+\delta)}. \tag{3-19}$$

The result follows. □

From the result of the proposition 3.1, we find that when $r_\star = \frac{\log n+\omega}{2n}$, the probability of event $\mathcal{H}$ has a upper bound lower than one, thus $r_\star \geq \frac{\log n}{2n}$ is necessary for achieving preliminary barrier coverage.

3.1.4 Sufficient Condition Analysis

Proposition 3.2. *In the hetergeneous WSNs deployed in a belt region, if* $R_\star = c\frac{\log n}{2n}$, $c > 1$ *is a constant, then*

$$\liminf_{n\to+\infty}\mathbb{P}(\overline{\mathcal{H}}) = 0, \tag{3-20}$$

Proof. Let $\mathcal{F}_i$ denote the event that grid point P_i in $\mathbb{M}$ is not covered, if $r_\star = cR_\star(n)$ where $c > 1$, then

$$\begin{aligned}\mathbb{P}(\bigcup_{i=1}^{m}\mathcal{F}_i) &\leq \sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i)\\ &= n\prod_{y=1}^{u}(1-2r_y)^{c_y n_0}\\ &= ne^{-2nr_\star}\\ &= \frac{1}{n^{c-1}}.\end{aligned} \tag{3-21}$$

For any positive constant c, we have the following result for $n\to\infty$.

$$\lim_{n\to\infty}\mathbb{P}(\bigcup_{i=1}^{m}\mathcal{F}_i) = 0 \tag{3-22}$$

Which can be rewritten as

$$\liminf_{n\to+\infty} \mathbb{P}(\overline{\mathcal{H}}) = 0 \tag{3-23}$$

Thus, we finish the proof. □

This result shows that $r_\star \geq \frac{\log n}{2n}$ is sufficient to guarantee the preliminary barrier coverage of the belt region.

3.1.5 CSR for Preliminary Barrier Coverage

From both the sufficient and necessary analysis, we have the following theorem concerning the CSR under preliminary barrier condition in i.i.d mobility model.

Theorem 3.1. *Under i.i.d mobility model, the CSR for mobile heterogeneous WSNs to achieve asymptotic preliminary barrier coverage is*

$$r_\star(n) = \frac{\log n}{2n} \tag{3-24}$$

3.2 CSR in Different Incoming Angle Condition

In our model, the intruder's crossing path can have multiple incoming angles, not just the one parallel to the width of the belt, and we show that the CSR in normal incidence still holds for different incoming angles vary from $[0, 2\pi)$. Intuitively, when the incoming angle is $\theta \in [0, 2\pi)$, the projections will be derived along the crossing path (line) to the line $\mathbb{L}$. The length of the projection of sensor S_i should be modified to $\frac{2r_i}{\sin\theta}$, and the center of the projection should come to the point with a different coordination according to sensor S_i's location as well as its distance to the bottom. Obviously, in this case the centers of the projections are still distributed with i.i.d. scheme, but the projections have a larger length ($\frac{2r_i}{\sin\theta} > 2r_i$). So if the $R_\star$ suffices to reach line coverage with length $2r_i(i = 1, 2, 3, \cdots, n)$, the result still holds for length $\frac{2r_i}{\sin\theta}$. Thus, the CSR ($\frac{\log n}{2n}$) is sufficient to ensure the preliminary barrier coverage.
On the other hand, when $r_\star < \frac{\log n}{2n}$, then the incoming angle is $\pi/2$, from the above

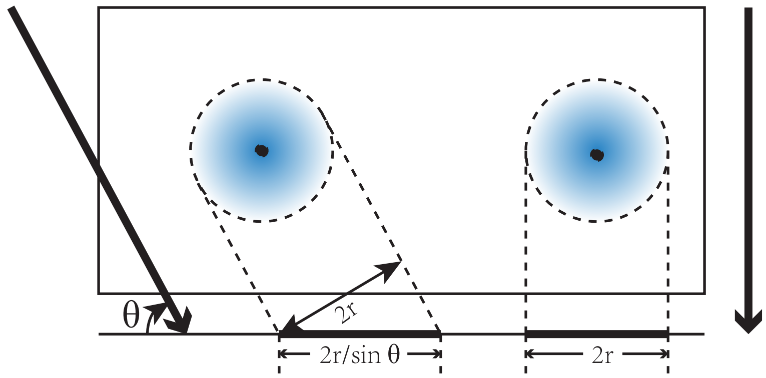


Figure 3.2 Intruders with the incoming angle of θ.

analysis, we know that the probability of event $\mathcal{H}$ is bounded by a constant which is lower than 1, this means that $r_\star \geq \frac{\log n}{2n}$ is necessary to guarantee preliminary barrier coverage.

Combine the above two parts together, we can conclude that when we take multiple incoming angles into consideration, the CSR in the normal incidence still holds, and this CSR becomes the CSR for all kinds of intruder orientations.

3.3 General Barrier Coverage

3.3.1 Geometric Analysis

In this section, we utilize our basic condition of angled straight-line intruders to analysis a much more general and applicable scenario that the intruders can turn their directions while they are passing the barrier region. Thus their paths of intruding are like a curve, and we will develop our insight into this general condition by a differential idea. We divide the width of the barrier into $w(n)$ partitions and every sub-barrier of the original barrier will be a new individual target of our analysis. The value of $w(n)$ will be given at the definition below and is warranted from Liu et al. (2008).

We first derive the new sensing radius of a partitioned barrier (sub-barrier) with our previous result of an integral barrier. And we will present a proof of our result with both necessary perspective and sufficient perspective to guarantee that our sensing radius is warranted and reasonable.

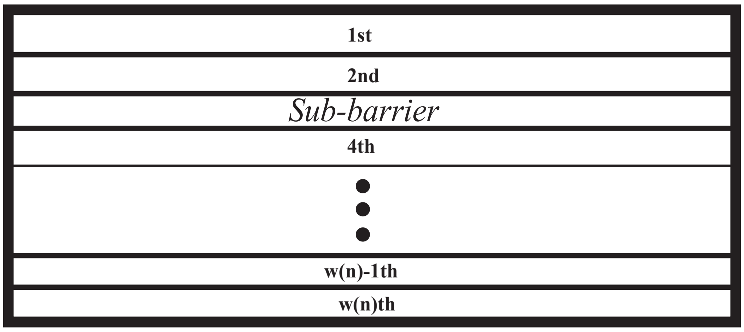


Figure 3.3 Partitioned barrier

Definition 3.2. *Differential barrier: Partition the barrier width into $w(n) = log(n)$ parts and make $w(n)$ sub-barriers. In a given sub-barrier, the intruder can always select its path in an inclined-straight-line pattern. Between different barriers, the intruders can change their coming angles, however their path in two sub-barriers must connects with each other. Thus all the $w(n)$ barrier combines together to make the intruders' path a curve line, which is a general condition.*

Let event $\mathcal{H}$ denote that the sub-barrier coverage is achieved. And event $\mathcal{G}$ denote that the integral barrier coverage is achieved. Here the definition 5.1 still holds for this section's analysis.

Theorem 3.2. *If n sensors are randomly and uniformly deployed in the integral barrier region, then for every differential sub-barrier region, the critical sensing range (CSR) to reach the necessary condition and sufficient condition for sub-barrier coverage is*

$$R_{\star}(n) = \frac{\log n(\log n - \log\log n)}{2n}. \tag{3-25}$$

Proof. In every sub-barrier,there are

$$n_0 = \frac{n}{\log n} \tag{3-26}$$

sensors deployed in it. Thus we investigate into this very sub-barrier, we have n_0 sensors in a belt region. Thus according to Theorem 3.1, we substitute n by our new sensor amount n_0. And we have our CSR as below:

$$R_\star(n) = \frac{\log n_0}{2n_0}. \tag{3-27}$$

And apply (3-26) in (3-27), we have

$$R_\star(n) = \frac{\log n(\log n - \log\log n)}{2n}. \tag{3-28}$$

□

Then we will proof that this CSR for sub-barrier coverage is the very CSR for integral barrier coverage. Also we will give our proof from two perspectives, both necessary condition and sufficient condition.

3.3.2 Necessary Condition for General Barrier Coverage

In this section, we aim at the achievement of analysis towards necessary condition for general barrier coverage under i.i.d mobility model. To prove our proposed CSR is the very critical value that guarantees barrier coverage, we first need several lemmas and propositions as indicated below.

Lemmas and propositions

From previous section 3.1, we can utilize some revised lemmas and propositions in it for our analysis here. We may find that lemma 3.4 still holds for our general condition's CSR, while the lower bound of the formula is not a constant but related to n.

Lemma 3.5. *For general barrier coverage: If* $R_\star(n) = \frac{\log n(\log n - \log\log n) + \omega(n)}{2n}$, *for any fixed* $\beta < 1$ *and* $m = \frac{n}{\log n}$,

$$m\prod_{y=1}^{u}(1-2r_y)^{c_y m} \geq \beta^{-\frac{1}{\log n}} e^{-\frac{\omega}{\log n}}, \tag{3-29}$$

And the proof is just similar to the original lemma 3.4.

Proof. Concerning the formulation of the left side on the greater equation symbol of (3-29), we achieve

$$\begin{aligned}
&\log(m\prod_{y=1}^{u}(1-2r_y)^{c_y m}) \\
&= \log m + \sum_{y=1}^{u}((c_y m \log(1-2r_y))) \\
&= \log m - \sum_{y=1}^{u}((c_y m)\sum_{i=1}^{+\infty}\frac{(2r_y)^i}{i}) \\
&= \log m - \sum_{y=1}^{u}(c_y m(\sum_{i=1}^{2}\frac{(2r_y)^i}{i}+\delta_y))
\end{aligned} \tag{3-30}$$

where, we can have the upper bound of δ_y.

$$0<\delta(x)=\sum_{i=3}^{+\infty}\frac{(x)^i}{i}<\sum_{i=3}^{+\infty}\frac{x^i}{3}=\frac{1}{3}\frac{x^3}{1-x}<\frac{x^2}{3} \tag{3-31}$$

Substituting $x=2r_y$ into (3-31), we have

$$\delta_y=\sum_{i=3}^{+\infty}\frac{(2r_y)^i}{i}<\frac{4}{3}(r_y)^2 \tag{3-32}$$

Combine (3-32) and (3-30), we can obtain

$$\begin{aligned}
&\log(m\prod_{y=1}^{u}(1-2r_y)^{c_y m}) \\
&\geq \log m - \sum_{y=1}^{u}((c_y m)(2r_y+\frac{10}{3}(r_y)^2)) \\
&\geq \log m - 2mr_\star - \frac{10}{3}m\sum_{y=1}^{u}(c_y(r_y)^2) \\
&= -\frac{\omega}{\log n}-\frac{10}{3}\frac{n}{\log n}\sum_{y=1}^{u}(c_y(s_y)^2).
\end{aligned} \tag{3-33}$$

Note that n we considered here is sufficient large, this ensures that $r_y < c_y$ for $i, j =$

$1, 2, \cdots, u$. We can simplify the equation

$$\sum_{y=1}^{u}(c_y(r_y)^2) \leq (\sum_{y=1}^{u}(c_y r_y))^{3/2} = (r_\star)^{3/2} \tag{3-34}$$

Substituting (3-34) into (3-33)

$$\frac{10}{3}\frac{n}{\log n}\sum_{y=1}^{u}(c_y(s_y)^2) \leq \frac{1}{\log n}\frac{10}{3}n(r_\star)^{3/2} \tag{3-35}$$

And we have

$$\frac{10}{3}n(r_\star)^{3/2} \to 0, \text{as } n \to +\infty. \tag{3-36}$$

For any $\epsilon > 0$, and all $n > N_\epsilon$

$$\log(m\prod_{y=1}^{u}(1-2r_y)^{c_y n}) \geq -\frac{\omega+\epsilon}{\log n} \tag{3-37}$$

Let $\beta = e^{-\epsilon}$ and taking the exponent of both sides, the result follows. □

We also have proposition 3.1 holds for our general barrier coverage's CSR, only with some tiny modifications. We may understand that in a sub-barrier, every properties stay the same as the previous line-intruders condition. Thus we have our new proposition which can be derived in the same way as proposition 3.1.

Proposition 3.3. *In the heterogeneous WSNs deployed in a sub-barrier region, if* $R_\star = \frac{\log n(\log n - \log\log n)+\omega(n)}{2n}$*, then*

$$\liminf_{n\to+\infty} \mathbb{P}(\overline{\mathcal{H}}) \geq \beta^{-\frac{1}{\log n}} e^{-\frac{\omega}{\log n}}, \tag{3-38}$$

where $\omega = \lim_{n\to+\infty}\omega(n)$*. Here event* $\mathcal{H}$ *denote that the sub-barrier coverage is achieved.*

The proof will be achieved similarly by applying lemma 3.3 and lemma 3.5 as the previous parts indicates. We give it to illustrate the difference between preliminary and general one.

Proof. We first study the case where $R_\star = \frac{\log n+\omega}{2n}$ for a fixed ω. We derive the following inequalities from Bonferroni inequality.

$$\mathbb{P}(\overline{\mathcal{H}}) \geq \sum_{P_i \in \mathbb{M}} \mathbb{P}(\{\text{some point } P_i \text{ is not covered}\}) \tag{3-39}$$

From lemma 3.5, we can evaluate the term on the right hand size of (3-39). We have

$$\begin{aligned}
&\mathbb{P}(\{P_i \text{ is not covered}\}) \\
&\quad = \prod_{y=1}^{u} \mathbb{P}(\{P_i \text{ is not covered by sensors in } G_y\}) \\
&\quad = \prod_{y=1}^{u} (1-2r_y)^{c_y n}.
\end{aligned} \tag{3-40}$$

Using lemma 3.5, we bound the first term for any const $\beta < 1$,

$$\sum_{P_i \in \mathbb{M}} \mathbb{P}(\{P_i \text{ is not covered}\}) = \mathbf{C}_m^1 \prod_{y=1}^{u} (1-2r_y)^{c_y n} \geq \beta^{-\frac{1}{\log n}} e^{-\frac{\omega}{\log n}}, \tag{3-41}$$

for all $n > N_\omega$.

By substituting (3-41) to (3-39), we have

$$\mathbb{P}(\overline{\mathcal{H}}) \geq \beta^{-\frac{1}{\log n}} e^{-\frac{\omega}{\log n}}. \tag{3-42}$$

for any positive constant β.

As for the case that ω is a function of n with $\omega = \lim_{n\to+\infty} \omega(n)$, we know $\omega(n) \leq \omega+\delta$ for any $\delta > 0$ all $n > N_\delta$. Since $\mathbb{P}(\overline{\mathcal{H}})$ is monotonously decreasing in $r_\star$. and thus in ω, we have

$$\mathbb{P}(\overline{\mathcal{H}}) \geq \beta^{-\frac{1}{\log n}} e^{-\frac{\omega+\delta}{\log n}}. \tag{3-43}$$

The result follows. □

From proposition 3.3, we understand that when $R_\star = \frac{\log n(\log n - \log\log n)+\omega(n)}{2n}$, meaning the integral barrier coverage's CSR is slightly added up a positive constant, the failure possibility of every sub-barrier is a constant powered by $\frac{1}{\log n}$. We may present in the afterwards sections that the power can be eliminated by our partition strategy, thus the integral barrier coverage's failure possibility can have a positive lower bound which is the vital point of our necessary condition analysis.

Proof of Necessary condition

In this section we use propositions and lemmas in the previous part to achieve a dedicated proof of the necessary condition. Thus to simplify our equation, we have

Definition 3.3. *As $n \to +\infty$, we define*

$$\overline{P}(n) = \limsup_{n\to+\infty} \mathbb{P}(\overline{\mathcal{H}}). \tag{3-44}$$

Here we always considers the probability of event $\overline{\mathcal{H}}$ equals $\overline{P}(n)$.

From proposition 3.3, we have

$$\overline{P}(n) = \liminf_{n\to+\infty}(\mathbb{P}(\overline{\mathcal{H}})) \geq \beta^{-\frac{1}{\log n}} e^{-\frac{\omega+\delta}{\log n}} \tag{3-45}$$

We define

$$\gamma = \beta^{-\frac{1}{\log n}} e^{-\frac{\omega+\delta}{\log n}}. \tag{3-46}$$

Thus we have an on-going proposition.

Proposition 3.4. *In the heterogeneous WSNs deployed in a sub-barrier region, if $R_\star = \frac{\log n(\log n - \log\log n)+\omega(n)}{2n}$, then*

$$\overline{P}(n) \geq \gamma \tag{3-47}$$

And as our barrier region contains approximately sensors of infinity amount, we may have the lower bound of $\overline{\mathbb{P}}(\mathcal{H})$, which is $\limsup_{n\to+\infty} \mathbb{P}(\overline{\mathcal{H}})$. It represents the function of probability concerning the event $\overline{\mathcal{H}}$ when $n \to +\infty$.

After having analyzed a given sub-barrier's achieving probability, we here have

Definition 3.4. *The probability of failing to achieve general barrier coverage before the kth sub-barrier is P_k*

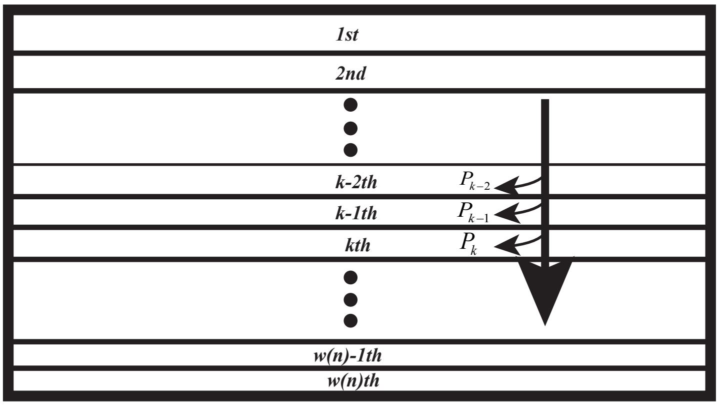


Figure 3.4 The relationship between $P_{k-2}, P_{k-1} and P_k$

And to derive the formulation of P_k, we have to use a method of recursion. The following lemma is constracted

Lemma 3.6.

$$P_k = \beta^{-\frac{k}{\log n}} e^{-\frac{k\omega}{\log n}} \tag{3-48}$$

Proof. This is because the general barrier coverage achieved at kth sub-barrier should satisfy both the below condition:

- At every sub-barrier include and before k the intruder should not be detected.
- The possibility of intruder detection between different sub-barriers has no correlation or interference to each other, meaning the intruding detection is independent.

And we derive

$$\begin{aligned} & P_k \\ = \ & \overline{P}(n)P_{k-1} \\ = \ & \overline{P}(n)^2 P_{k-2} \\ = \ & \overline{P}(n)^k \\ = \ & \beta^{-\frac{k}{\log n}} e^{-\frac{k\omega}{\log n}}. \end{aligned} \tag{3-49}$$

Therefore the proof is achieved. □

We can use the above lemma to obtain the lower bound of P_k and thus get the result of $\mathbb{P}(\mathcal{G})$, where $\mathcal{G}$ denotes that the integral barrier coverage is achieved.

Proposition 3.5. *In the heterogeneous WSNs deployed in an integral barrier region, and intruders can come with a curve path, if* $R_\star(n) = \frac{\log n(\log n - \log\log n) + w(n)}{2n}$, *then*

$$\limsup_{n\to+\infty} \mathbb{P}(\mathcal{G}) = 1 - \beta e^{-\omega} < 1 \tag{3-50}$$

Proof. We have

$$\begin{aligned} &\mathbb{P}(\mathcal{G}) \\ &= 1 - P_{w(n)} \\ &= 1 - \overline{P}(n)^{\log n} \\ &= 1 - \beta e^{-\omega}. \end{aligned} \tag{3-51}$$

And thus we get

$$\limsup_{n\to+\infty} \mathbb{P}(\mathcal{G}) = 1 - \beta e^{-\omega} < 1 \tag{3-52}$$

Finally the proof is achieved. □

From the result of the proposition 3.5, and based on the general barrier coverage condition, we find that when $r_\star = \frac{\log n(\log n - \log\log n) + w(n)}{2n}$, the probability of event $\mathcal{G}$ has a upper bound lower than one, thus $r_\star \geq \frac{\log n(\log n - \log\log n)}{2n}$ is necessary for achieving barrier coverage.

3.3.3 Sufficient condition for general barrier coverage

To achieve the sufficiency of general barrier coverage's CSR, we first propose a proposition concerning this issue.

Proposition 3.6. *In he heterogeneous WSNs deployed in a sub-barrier region, if* $R_\star =$

$c\frac{\log n(\log n - \log\log n)}{2n}$,$c > 1$ *is a constant,then*

$$\overline{P} = \liminf_{n\to+\infty} \mathbb{P}(\overline{\mathcal{H}}) = 0 \tag{3-53}$$

Proof. Let $\mathcal{F}_i$ denote the event that grid point P_i in $\mathbb{M}$ is not covered, if $r_\star = cR_\star(n)$ where $c > 1$, then

$$\begin{aligned}
\mathbb{P}(\bigcup_{i=1}^{m}\mathcal{F}_i) &\le \sum_{i=1}^{m}\mathbb{P}(\mathcal{F}_i) \\
&= n_0\prod_{y=1}^{u}(1-2r_y)^{c_y n_0} \\
&= n_0 e^{-2n_0 r_\star} \\
&= \frac{1}{n_0^{c-1}} && (3\text{-}54) \\
&= (\frac{\log n}{n})^{c-1}. && (3\text{-}55)
\end{aligned}$$

Where $n_0 = \frac{n}{\log n}$. For any positive constant c, we have the following result for $n \to \infty$.

$$\lim_{n\to\infty} \mathbb{P}(\bigcup_{i=1}^{m}\mathcal{F}_i) = 0 \tag{3-56}$$

Which can be rewritten as

$$\liminf_{n\to+\infty} \mathbb{P}(\overline{\mathcal{H}}) = 0 \tag{3-57}$$

Thus, we finish the proof. □

From proposition 3.6, we know that it is impossible ($\overline{P} = 0$) for a intruder to pass a single sub-barrier as $n \to +\infty$, let alone can it pass through the $\log n$ sub-barriers. This can be represented as $\overline{\mathbb{P}}(\mathcal{G}) = P^{\log n} \to 0$.

Thus the result shows that $r_\star \ge \frac{\log n(\log n - \log\log n)}{2n}$ is sufficient to guarantee the general barrier coverage of the integral belt region. And we have the below theorem.

Theorem 3.3. *If n sensors are randomly and uniformly deployed in a barrier region,* *sensing radius*

$$r_\star = \frac{\log n(\log n - \log\log n)}{2n} \tag{3-58}$$

is large enough to guarantee the general barrier coverage under i.i.d mobility model of the whole belt region.

3.3.4 CSR for General Barrier Coverage under I.I.D mobility model

From the previous sections analyzing the sufficiency and necessity of the CSR for general barrier coverage under i.i.d model, we have the following theoretical highlights on it.

Theorem 3.4. *If n sensors are randomly and uniformly deployed in a barrier region, the critical sensing radius to reach the necessary condition for general barrier coverage (curve path intruders) under i.i.d mobility model is*

$$R_\star(n) \geq \frac{\log n(\log n - \log\log n)}{2n}. \tag{3-59}$$

3.4 Chapter Review

In this chapter, we focused on the CSR derivation under i.i.d mobility model, for both preliminary barrier coverage and general barrier coverage. Firstly, we dig into the relative simple problem, the preliminary barrier coverage, as is in section 3.1. In this section, we assume intruders only intrude in identical straight lines. Our work is major based on previous works about full coverage and some rational modify to their approach is demanded. Afterwards, we analyzed the impact of intruding angle, which is the intersection angle between the barrier bound and the intruding path, to the preliminary barrier coverage. And we prove that if the identical intruding detection is achieved, any inclined intruding is guaranteed either. However, due to the limitation of line intruding path's pragmatic utilities, we need to move on to general barrier coverage. In the access to general barrier coverage under i.i.d mobility model, we need to firstly handle on curved path. We all understand that a curved line is combined by numerous numbers of straight line intervals. In this point of view, we can partition the barrier region into proper numbers of sub-barrier region and in each sub-barrier region, we apply the preliminary barrier coverage. The critical partition is $w(n) = \log n$ which is supported in Liu et al. (2008). And based on this partition strategy, we successfully derived the general barrier coverage's CSR under i.i.d mobility model.

Chapter 4 Speed Resolution

4.1 Models and Definitions

In this section, we discuss the impact of intruder speed to the barrier detection under I.I.D mobility model. As we indicate in 2.1, the intruders in our model should be in a fast motion speed. However, how much this fast motion could be and should be is not analyzed,which is quite an important issue in intruder detection problem. Therefore, to explain the measure of defining fast motion, we include a new parameter of speed resolution to characterize how much an intruders' motion should be.

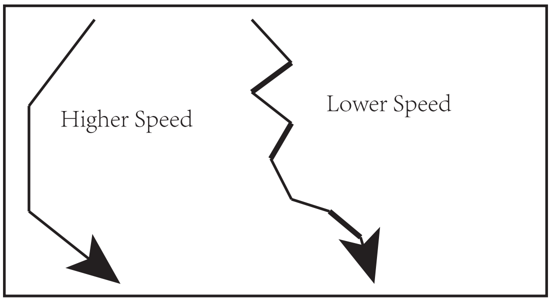
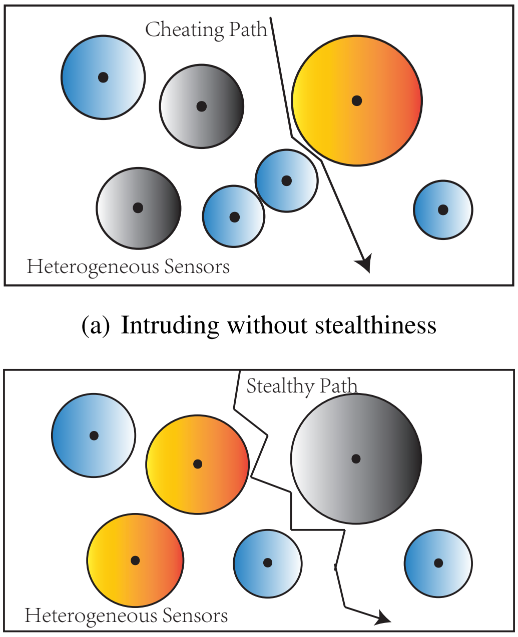


Figure 4.1 Intruding Path

Definition 4.1 (Swerving reaction time)**.** *In this background, we define the time costed by intruder changing direction be 1 second. In this motion model, the trace of intruders should be folding line divided by 1 second time slot. The path should be as the above figure.*

Definition 4.2 (Stealthiness)**.** *The sensor network is said to satisfy the stealthiness assumption if no intruder is aware of the locations of the sensors. In this condition, the swerving pattern of a given intruder shall not turn suddenly between two hidden sensors, meaning the moving pattern in figure 4.2(b) is reasonable while the figure 4.2(a) make no sense.*



(a) Intruding without stealthiness

(b) Intruding with stealthiness

Figure 4.2 Stealthiness

Definition 4.3 (Interval distance). *The smallest distance between two neighbor sensor's coverage area, which is denoted by* $D(i, j)$*. Here* i, j *represents two neighbor sensors.*

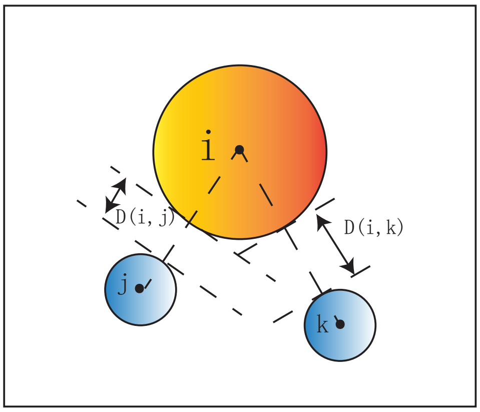


Figure 4.3 Speed Resolution : Interval Distance

Definition 4.4 (Speed resolution). *The speed threshold that determine the lowest speed a intruder obsess in order to be detected. In a direct opinion, the threshold is determined by the interval distance of two neighbor sensors.*

4.2 Analysis towards resolution expectation

To derive the formulation of speed resolution, we shall pay attention to the value of $D(i, j)$, where i, j represents randomly selected neighbor sensors. With stealthiness background, the intruders can only pass the intervals when their speed value is no larger than the interval distance. Therefore, we analyze $D(i, j)$ to determine the resolution.

The upper bound of speed resolution is the width of a sub-barrier, which is $w_0 = \frac{w}{log(n)}$. The lower bound of resolution is 0, in which case the width of sub-barrier is almost same as critical sensing radius, also called strong barrier. Our target is deriving the expectation of resolution Res, which shall be $Res = \mathbb{E}[D(i, j)]$.

Lemma 4.1. *For a rectangle region $ABCD$, and suppose the bottom edge's middle point is the origin of coordinates. We can have the integral equation as below. The details of this lemma can be seen in below figure.*

$$\iint_{ABCD} \sqrt{x^2 + y^2}\, dx\, dy. = \int_0^{w_0 - 2r} S_{\Delta EOF}\, dy. \tag{4-1}$$

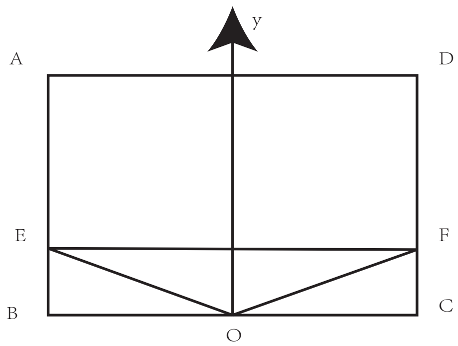


Figure 4.4 Integral equation

Lemma 4.2. *Given a randomly selected two horizontal lines, whose interval is the sub-barrier width $w_0 = \frac{w}{\log n}$, in the whole barrier region, a sub barrier is formed between the two arbitrary bound lines.*

The above two lemmas' proof is quite straight forward. Since lemma 7 is an equation on area of rectangle $ABCD$ and lemma 8 is due to the sensors in barrier region is random uniformly deployed. Then now we are ready to derive the expectation of the resolution. We will have the below proposition.

Proposition 4.1. *Randomly select a deployed sensor O, and make a sub barrier region as lemma 4.2 indicated. The expectation of interval space between O and its neighbor sensor i is shown as the below formula, in which r represents the sensing radius and $w_0 = \frac{w}{log(n)}$ denotes the width of a sub barrier.*

$$\mathbb{E}[D(O,i)] = \frac{r(w_0 - 2r)^2 - \frac{4}{3}\pi r^2}{4r(w_0 - 2r)} \tag{4-2}$$

Proof. The situation which this proposition suppose can be shown in the below figure.

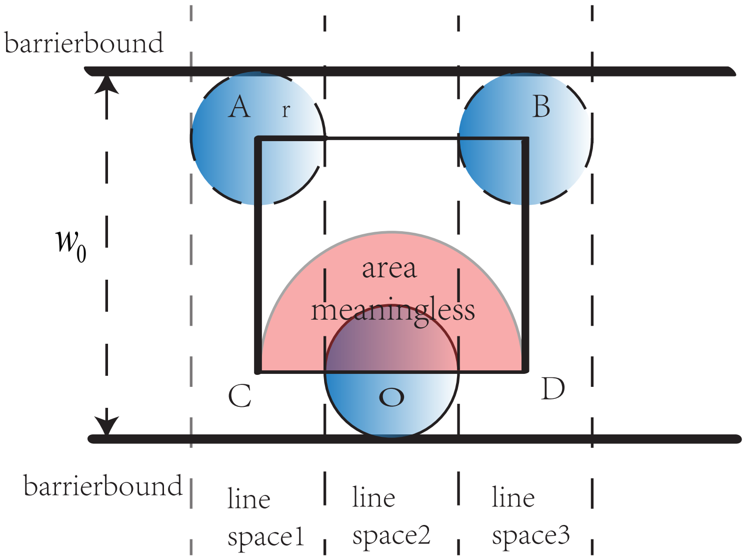


Figure 4.5 Resolution expectation analysis

Firstly we need to explain what is a sensor's neighborhood. As the figure presents, there is a line spaces on either side of the target sensor O. To satisfy barrier coverage, there must be sensors in the line space to make its projection cover the line spaces besides target sensor. Meanwhile, in the target sensor's line space region, it is free for random sensors to be located. In this way , the neighbor sensor's central location must lies in the rectangle region $ABCD$. If not, there shall be a interval space in the bound line without coverage of sensors' projection.

In order to investigate into the speed resolution, we pay our attention to the interval distance between target O and the randomized sensor i in any locator within rectangle $ABCD$. As shown in figure 4.5, the half circle area dyed in red because any sensor center located in it has zero interval distance with target O. And the vital area of our analysis shall be the difference of $ABCD$ and the half circle, which denote as S^* here.

$$\begin{aligned}
\mathbb{E}[D(O,i)] &= \frac{\int\int_{S^*} \sqrt{x^2+y^2}\, dx\, dy}{S_{ABCD}} \\
&= \frac{\int\int_{S_{ABCD}} \sqrt{x^2+y^2}\, dx\, dy - \int\int_{S_{Semicircle}} \sqrt{x^2+y^2}\, dx\, dy}{4r(w_0-2r)} \\
&= \frac{\int_0^{w_0-2r} S_{\Delta EOF}\, dy - \int_0^{2r} \frac{1}{2}\pi r^2\, dr}{4r(w_0-2r)} \\
&= \frac{\int_0^{w_0-2r} 2ry\, dy - \frac{4}{3}\pi r^3}{4r(w_0-2r)} \\
&= \frac{r(w_0-2r)^2 - \frac{4}{3}\pi r^3}{4r(w_0-2r)}
\end{aligned} \tag{4-3}$$

The proof is achieved. □

4.3 Practical Analysis

Afterwards we have derived the expectation of speed resolution, a numerical simulation is desired to make it more appealing and convincing whose detail is provided in the coming chapter 6.

In Kumar et al. (2005), Kumar points out that the width of barrier region is normally 1 percent of its length, and here we assign $w \leq 0.01l$ for simplifying our simulation. Be aware of the length is uniformed to 1 as well as sub barrier width being $w_0 = \frac{w}{log(n)}$, we can start our simulation with independent variable n (the total sensor number).

Firstly, we handle on the boundary conditions when $w_0 = 2r$ and $w_0 = 4r$. Keep in mind that r decreases as n increases, and before the first boundary, the interval distance of neighbor sensors (O, i) is zero. Meanwhile, after the second boundary, the interval distance is achievable as the equation 4-2. However, between the two boundaries, the speed resolution is unavailable and we define it not applicable due to our approach of derivation. The boundary effect $g(n)$ can be described as below.

Lemma 4.3. *Concerning the practical expression of the resolution, the original* $\mathbb{E}[D(O,i)]$ *should be shaped by a window function* $g(n)$*, which is*

$$g(n) = \begin{cases} 0 & 0 < w_0 < 2r \\ -1 & 2r < w_0 < 4r \\ 1 & w_0 \geq 4r \end{cases} \tag{4-4}$$

More over, we have the practical function who properly indicate the properties of speed resolution towards the speed resolution. The result is as below.

Definition 4.5. *The practical expression of speed resolution shall be*

$$Res(n) = g(n)\mathbb{E}[D(O,i)] = g(n)\frac{r(w_0 - 2r)^2 - \frac{4}{3}\pi r^3}{4r(w_0 - 2r)} \tag{4-5}$$

where $g(n)$ *is provided in above lemma.*

4.4 Properties of Resolution

In this section, we pay attention to properties on speed resolution. We give optimal directions on how to achieve high or low resolution due to various situation demands. Finally, we present the resolutions' utilities and significance.

Proposition 4.2. *As* $n \to \infty$*, we have* $Res(n) = \frac{w}{4\log n}$*, where* w *is the barrier width(length unified to* 1*).*

Proof. We derive the limitation of resolution. Firstly, we have that $r = \Theta(\frac{(\log n)^2}{n})$, $\frac{r^2}{w_0} = \Theta(\frac{(\log n)^3}{n^2})$, $\frac{r}{w_0} = \Theta(\frac{(\log n)^2}{n})$ are all infinitesimal of higher order to $w_0 = \Theta(\frac{1}{\log n})$, and the infinitesimal with higher order can be ignored when calculating the limitation. Then the results come as below

$$\begin{aligned}\lim_{n\to\infty} Res(n) = \lim_{n\to\infty} \frac{r(w_0-2r)^2 - \frac{4}{3}\pi r^3}{4r(w_0-2r)} \\ = \lim_{n\to\infty} \frac{w_0^2 - 4rw_0 + (2-\frac{4}{3}\pi)r^2}{4w_0 - 8r} \\ = \lim_{n\to\infty} \frac{w_0 - 4r + (2-\frac{4}{3}\pi)\frac{r^2}{w_0}}{4 - 8\frac{r}{w_0}} \\ = \frac{w_0}{4} \\ = \frac{w}{4\log n}.\end{aligned} \tag{4-6}$$

The proof is achieved. □

4.5 Utilities and Significance

When deploying barrier sensor networks with i.i.d mobility model, the engineers are demanded with several parameters which are provided by the network investor.

- The barrier region's length l.
- How many sensors at most, also represent as n which is must at a sufficient large amount(say larger than 10^5), do the investors accept for the network deployment.
- What kinds of intruders are the investors primely detecting, and their minumum average speed v_{min}.

Without the speed resolution, the engineers can solely deploy the barrier who can only sense the high speed intruders, and with no acknowledgement of how small speed can the barrier tolerate. If the tolerance, in other words, the resolution is around $10m/s$ while the barrier is mostly desired to monitor human activities, then the barrier is nothing but a failure for human speed is mostly around $1m/s$. If equipped with speed resolution when designing a barrier sensor network, the engineer should follow the below steps to gain an effective design.

- Use n to calculate the CSR under i.i.d model, with the formula in 3.4.

- Use the estimation in proposition 4.2, which is $Res(n) = \frac{w}{4\log n} = v_{min}$. The design of barrier width can be any value satisfying $w \leq 4v_{min}\log n$.
- Multiple $R_\star$ by l and select proper the sensor type with radius larger than $lR_\star$ which producers can provide.
- Apply vehicles or airplanes to drop the sensors with constant speed and dropping amount per minute along the barrier region. By this step, the barrier sensor network's deployment is achieved.

4.6 Chapter Review

In this chapter, we first realized the in the i.i.d model's barrier coverage problem, only the CSR is not sufficient to direct a engineer to deploy a barrier sensor network. For in our fast motion model, we haven't give any script on intruder's speed. The word high speed is too vague in a research environment. So this motivated us to propose a new concept in barrier coverage, the speed resolution. In section 4.1, we give definitions related to speed resolution and models on approach to derive the expectation of speed resolution. And in section 4.2, we derived the expectation of speed resolution, in terms of the barrier length w and CSR r which are both in terms of n. The expectation has inner restrains that need to be recognized. In section 4.3 we analyzed the boundary effect of speed resolution and proposed the window function $g(n)$ to modify the expression of resolution. Afterwards, we derived properties of resolution in 4.4, which is mainly the limitation of resolution. Finally, we spot highlights on the speed resolution's utilities and practical significance when we need to deploy a barrier sensor network in realistic situation.

Chapter 5 Barrier Coverage Under Random Walk Model

Under the 1-dimensional random walk mobility model, the sensing process is time slotted and sensors use each time interval to move and sense. Here we use $\mathcal{H}^\tau$ to denote the event that $\mathbb{M}$ is covered in a given time slot τ, and $\mathbb{P}_\tau(\mathcal{H}^\tau)$ to denote the corresponding probability. Similarly, we define the CSR for 1-dimensional random walk model.

Definition 5.1. *$R_\diamond$ is the critical sensing range (CSR) for event $\mathcal{H}^\tau$ if*

$$\lim_{n\to\infty} \mathbb{P}_\tau(\mathcal{H}^\tau) = 1, \textit{if } r_\diamond \geq cR_\diamond \textit{ for any } c > 1;$$
$$\lim_{n\to\infty} \mathbb{P}_\tau(\mathcal{H}^\tau) < 1, \textit{if } r_\diamond \leq cR_\diamond \textit{ for any } 0 < c < 1.$$

5.1 Preliminary Barrier Coverage

We first come to focus on preliminary conditions of barrier coverage under 1-dimensional random walk mobility model. Remember that in this preliminary situation, the intruders move vertically and we make our efforts to analyze coverage in the line virtual grid $\mathbb{M}$.

5.1.1 Failure Probability of a Point in Grid $\mathbb{M}$

We use $\mathcal{F}_i$ to denote the event that point P_i in $\mathbb{M}$ is not covered by any sensor and let $\mathbb{P}(\mathcal{F}_i)$ be the failure possibility of point P_i in $\mathbb{M}$.

Firstly, we consider the probability of an arbitrary point P_i in $\mathbb{M}$ can be successfully covered by the projection of a sensor S_y in group G_y, and denote this probability as $\mathbb{P}_(i, y)$. Because of the vertical movement of S_y do not contribute any coverage to $\mathbb{M}$, we only take the horizontal movement into account.

Because that S_y choose to move left or right with equal possibility, we suppose S_y moves right. Initially, sensors are uniformly deployed and according to the 1-dimensional random walk mobility model, sensors are always uniformly distributed at each time slot τ in the operational region of the barrier. On the other hand, the line grid is randomly uniformed located in the barrier bound line.

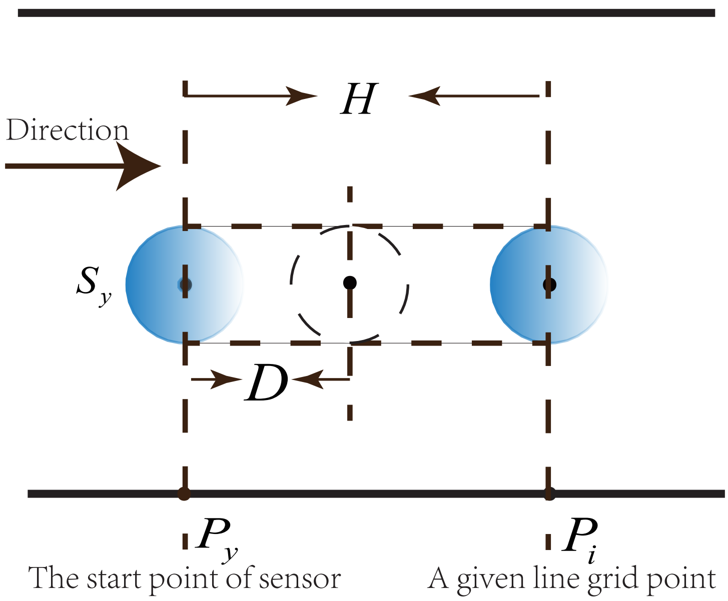


Figure 5.1 Coverage of a Line Grid Point

Since we consider the line coverage here, we build a number axis in the horizontal direction of figure 5.1 and denote the position of S_y and P_i as u_1 and u_2, respectively. Then we understand that u_1, u_2 are uniformly distributed from 0 to 1.

We ignore the vertical distance here, and only take the horizontal distance between S_y and P_i into account. On the horizontal dimension, S_y moves in certain direction and might have its projection cover P_i on its way, which leads the horizontal distance to be $H = |u_1 - u_2|$. Thus we can thereby have the p.d.f of H

$$f_H(h) = \begin{cases} 2(1-h) & 0 \leq h \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{5-1}$$

Point P_i can be successfully covered by S_y if and only if S_y can enter the sensing region with the radius r_y and be centered at the corresponding point who is the projection of P_i to S_y's movement trace line.

Lemma 5.1. *The possibility of P_i can be successfully covered by S_y is denoted as $\mathbb{P}(i, y)$. And $\mathbb{P}(i, y) = \frac{2}{3}$, this probability is fixed without being influenced by r_y.*

Proof. As figure 15 illustrates, we have

$$\begin{aligned}\mathbb{P}(i,y) &= \int\int_{h\leq d} 2(1-h)\,dh\,dd \\ &= \int_0^1 dd \int_0^d 2(1-h)\,dh \\ &= \int_0^1 2d - d^2\,dd \\ &= \frac{2}{3}\end{aligned} \tag{5-2}$$

The proof is achieved. □

5.1.2 Critical Conditions Under Preliminary Barrier Coverage

Using the above lemma, we may derive the following proposition concerning that the critical sensing radius under 1-dimensional random walk model does not exist and any given positive value can serve as $r_\diamond$.

Proposition 5.1. *Under 1-dimensional random walk mobility model, and with condition that the sensor is uniformly distributed.We have* $\forall r_\diamond \in [0,1)$, *if sensor number* $n- > \infty$, *the operational belt region is always barrier covered.*

Proof. Firstly, we come to analyze the calculation of $\mathbb{P}(\mathcal{F}_i)$. Suppose that the line grid is $m = n$ partitioned and we have $\mathbb{P}(\mathcal{F}_(i,y))$ to denote the probability that none sensor in group y ever covered pointP_i. Therefore,

$$\mathbb{P}(\mathcal{F}_(i,y)) = 1 - \mathbb{P}(i,y) = \frac{2}{3} \tag{5-3}$$

And since we have u groups, the probability of no sensor ever covered P_i, which is

$\mathbb{P}(\mathcal{F}_i)$, shall be

$$\begin{aligned}\mathbb{P}(\mathcal{F}_i) &= \lim_{n\to+\infty} m\prod_{y=1}^{u}[1-\mathbb{P}(\mathcal{F}_(i,y))]^{c_y n} \\ &= \lim_{n\to+\infty} m\prod_{y=1}^{u}\frac{1}{3}^{c_y n} \\ &= 0\end{aligned} \tag{5-4}$$

Thus, any value of $r_y(y \in [1,2,...,u])$ can satisfy that $\mathbb{P}(\mathcal{F}_i) \to 0$ when n is sufficiently large. And due to $r_\diamond = \sum_{y=1}^{u} c_y r_y$, we can select any given positive value as the CSR to achieve preliminary barrier coverage under this condition. In other words, no matter what value the sensors' radius is, if and only if the sensor amount $n \to \infty$, the preliminary barrier coverage is always satisfied. □

5.2 Towards General Barrier Coverage

We start analyze the general barrier coverage under 1-dimensional random walk mobility model. We here put forwards several weakness of preliminary barrier coverage under this condition.

- Hard to determine the exact number of sensors needed in a given detected region. If a expression of CSR is given, using numerical simulation, we can denote the corresponding sensor numbers n, which shall be provided more details in evaluation section. This way, to achieve sufficient barrier coverage, much more sensors will be applied than the actual demanded amount.

- Not always applicable. As indicated in I.I.D model condition's speed resolution section, to securely cover any intruders, we need to make analysis over the intruder's speed and our sensor network's sensibility. In 1-dimensional random walk mobility model, due to the non-existence of CSR's expression, it is impossible to achieve the sensor network's speed resolution. Without proper resolution to cover prospective intruder types, there is no security in barrier sensor networks at all.

- Unable to derive general barrier condition's CSR with basement of preliminary conditions. An alternative method should be utilized to access to the general barrier coverage in 1-dimensional random walk mobility model.

As these above flaws holds, we can not apply the partition approach of the integral barrier to achieve our general pattern. Therefore, we are here to start consider the full barrier coverage strategy, meaning to achieve full coverage in the narrow belt region.

5.3 Full Barrier Coverage

The full coverage problem in 1-dimensional random walk mobility model is already well-solved in Wang et al. (2011), with similar definitions in preliminary barrier coverage, we may utilize some of their results when deriving ours.

5.3.1 Dense Grid in Barrier Coverage

Similarly, as the operation area in Wang et al. (2011) is partitioned to $\mathbb{M} = \sqrt{m} \times \sqrt{m}$ dense grids, we apply here as $\mathbb{M} = n^{\alpha} \times \Phi(n)$. Here $\Phi(n) = n^{1-\alpha} \log n$, where α is determined by the ratio between unit length 1 and the barrier width w to make the dense grids well-distributed. And in Kumar et al. (2004), the lemma 3.1 and theorem 4.1 illustrate that if m is large enough(i.e., the grid is dense enough), the sensing radius that sufficient to ensure the asymptotic coverage of the grid points in $\mathbb{M}$ will grant an asymptotic coverage of the whole operational region as well. Meanwhile, the necessary sensing radius for $\mathbb{M}$ is also necessary for the whole barrier region to achieve full coverage. Therefore, we focus on the coverage on dense grid. We will put forward our derivation of CSR in full coverage barrier condition.

In Kumar et al. (2004), the selection of proper value of $\mathbb{M}$ should guarantee that $\mathbb{M} = \Omega(n \log n)$. Since in our barrier region, the length is orderly greater than the width. Here we suppose that $\Phi(n) = n^{1-\alpha} \log n$, and the barrier dense grid shall be $\mathbb{M} = n \log n$ which is sufficiently large enough to obtain the grid's proper density.

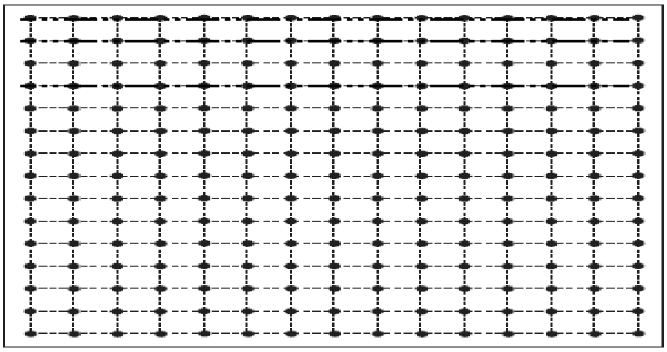


Figure 5.2 Dense Grid Within the Barrier Operational Region

5.3.2 Failure Probability of a Point in Grid $\mathbb{M}$

We use $\mathcal{F}_i$ to denote the event that point P_i in $\mathbb{M}$ is not covered by any sensor and let $\mathbb{P}(\mathcal{F}_i)$ be the failure possibility of point P_i in $\mathbb{M}$.

Firstly, we consider the probability of an arbitrary point P_i in $\mathbb{M}$ can be successfully covered by a sensor S_y in group G_y, and denote this probability as $\mathbb{P}_(i, y)$. Because of the symmetry of the topology, we only need to take care of the condition that S_y moves horizontally.

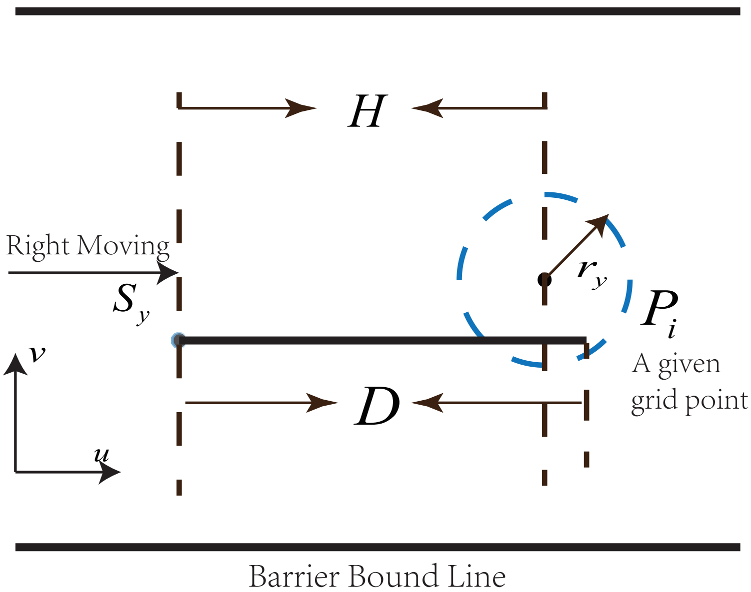


Figure 5.3 Coverage of a Dense Grid Point

Similar from previous analysis, we suppose S_y moves right. Initially, sensors are uniformly deployed and according to the 1-dimensional random walk mobility model, sensors are always uniformly distributed at each time slot τ in the operational region seen by the points in $\mathbb{M}$. On the other hand, the dense grid is randomly uniformed located in the area between barrier bound line.

We build a Cartesian coordinate system in the operational region and denote the position of S_y and P_i with (u_1, v_1) and (u_2, v_2), respectively. Then we know that u_1, u_2 are uniformly distributed from 0 to 1, while v_1, v_2 are uniformly distributed from 0 to w, which is the width of the operational region.

Since S_y does not move vertically, when considering the vertical distance between S_y and P_i which is denoted as G, we should understand that the upper boundary of the operational region is adjacent to the lower boundary. Therefore, we have

$$G = \min\left(|v_1 - v_2|, 1 - |v_1 - v_2|\right) \tag{5-5}$$

Meanwhile, S_y moves in a certain direction on horizontal dimension and might sense P_i on its way, which leads the horizontal distance to be

$$H = |u_1 - u_2| \tag{5-6}$$

At last, we can derive the p.d.f of G and H

$$f_G(h) = \begin{cases} \frac{2}{w} & 0 \leq g \leq \frac{w}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5-7}$$

$$f_H(h) = \begin{cases} 2(1-h) & 0 \leq h \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{5-8}$$

Point P_i can be successfully covered by S_y if and only if S_y can enter the circle with radius r_y centered at point P_i. Hence, we have the failure probability

$$\begin{aligned}\mathbb{P}(i,y) &= \mathbb{P}(G \le r_y)\cdot\mathbb{P}(H \le D)\\ &= \frac{2r_y}{w}\cdot\int\int_{h\le d} 2(1-h)\,dh\,dd\\ &= \frac{4r_y}{3w}\end{aligned} \tag{5-9}$$

Then $\mathbb{P}(\mathcal{F}_i)$ can be easily achieved.

5.3.3 Necessary Condition for Full Barrier Coverage

Make $\overline{\mathcal{G}^\tau}$ denote the event that the dense grid $\mathbb{M}$ is not fully covered in the given time slot τ. We have the following technical lemma.

Lemma 5.2. *If grid density $m = n\log n$ and $r_\diamond = \frac{3w(\log n+\log\log n+\eta)}{4n}$, then for any fixed $\beta < 1$,*

$$m\prod_{y=1}^{u}(1-\frac{4}{3w}r_y)^{c_y n} \ge \beta e^\eta, \tag{5-10}$$

holds for any sufficient large n.

Proof. Using the same technique presented in the proof of Lemma 3.4, the result follows. We also provide appendix to support our proof on this. □

Then we present the following proposition regarding the necessary condition on sensing radius.

Proposition 5.2. *In mobile heterogeneous WSNs with 1-dimensional random walk mobility model, if $r_\diamond = \frac{3w(\log n+\log\log n+\eta(n))}{4n}$ and the density of the dense grid $\mathbb{M}$ is $m = n\log n$, then*

$$\liminf_{n\to\infty}\mathbb{P}_\tau(\overline{\mathcal{G}^\tau}) \ge e^{-\eta}-e^{-2\eta}. \tag{5-11}$$

where $\eta = \lim_{n\to\infty}\eta(n)$.

Proof. The methodology in this proof is similar to that utilized in the approach of

Proposition 3.1 and we omit here. The steps can be found in the appendix for support. □

From Proposition 5.2, we know that $r_\diamond \geq \frac{3w(\log n+\log\log n)}{4n}$ is necessary to achieve full barrier coverage of $\mathbb{M}$ in the 1-dimensional random walk mobility model.

5.3.4 Sufficient Condition for Full Barrier Coverage

If $r_\diamond = cr_\diamond$, where $c > 1$, then

$$\begin{aligned}\mathbb{P}_\tau(\bigcup_{i=1}^{m}\mathcal{F}_i) &\leq \sum_{i=1}^{m}\mathbb{P}_\tau(\mathcal{F}_i)\\ &= n\log n\prod_{y=1}^{u}(w-\frac{4}{3w}r_y)^{c_y n}\\ &= n\log n w^n e^{-\frac{4}{3w^2}nr_\diamond}\\ &= \frac{1}{(n\log n)^{c-1}}.\end{aligned} \tag{5-12}$$

This probability approaches to 0 as $n \to \infty$. Thus, $r_\diamond \geq \frac{3w(\log n+\log\log n)}{4n}$ is sufficient to guarantee the full coverage of dense grid $\mathbb{M}$.

5.3.5 CSR for full barrier coverage

The analysis towards CSR is alike in the section of i.i.d mobility model, and the following theorem is at corner.

Theorem 5.1. *Under random uniform deployment scheme with 1-dimensional random walk mobility model, the CSR(critical sensing radius) for mobile heterogeneous WSNs to achieve asymptotic full barrier coverage is* $R_\diamond = \frac{3w(\log n+\log\log n)}{4n}$.

5.4 chapter review

In this section, we cast analysis over barrier coverage under random walk mobility model. Random walk is a strong mobility model which can evidently increase network performance as is informed in Wang et al. (2011). In section 5.1, we give analysis over preliminary barrier coverage under random walk mobility model. Because random walk mobility model it self already provides line-coverage which is sufficient to serve as barrier coverage, the CSR doesn't exist. Any value of sensing radius can achieve a effective barrier coverage. Then in section 5.2, we give our reasons for applying full barrier coverage under random walk mobility model as well as the flaws in general barrier coverage under this very situation. Finally, in section 5.3 we derived the critical conditions of CSR to achieve full barrier coverage both sufficiently and necessarily.

Chapter 6 Evaluation and Comparison

In this section, we compare our results (CSR) with previous works and evaluate our results' rationality.

6.1 Barrier Coverage under I.I.D mobility model

In this section, we give numerical simulations on barrier coverage's CSR under i.i.d mobility model. And we provide analysis towards inner correlations between general and preliminary barrier coverage, which may indicate its rationality and convincement. Finally, we compared our results with other famous works on coverage problem under i.i.d mobility model.

6.1.1 Numerical simulation

Figure 6.1 illustrates the relationship between CSR and the total number of sensors. When n goes to infinity, $R_\star \to 0$. This consists with the instinct that only smaller

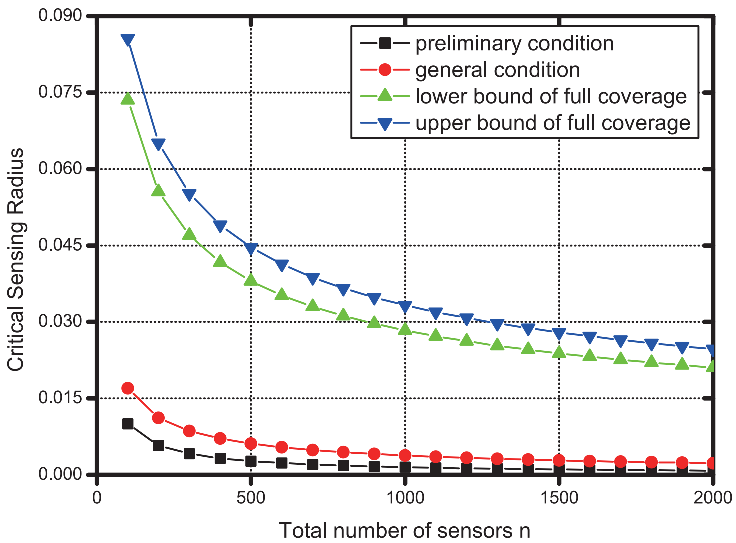


Figure 6.1 Simulation for coverage CSRs in I.I.D mobility model

sensing radius is needed to achieve barrier coverage if the total number of sensors increases. It is relatively clear that the CSR under preliminary condition is much smaller than that under general condition when n is relatively small, as n becomes larger, the gap becomes smaller. This results shows that when n is relatively large, the increasing number of sensors (density) offsets the gap between two conditions, therefore, two CSRs come much closer.

Table 6.1 Coverage Problems and CSRs

Coverage Problem		CSR's upper bound	CSR's lower bound
Full Coverage		$\sqrt{\frac{\log n+\phi(n)+\log\log n}{\pi n}}$	$\sqrt{\frac{\log n-\phi(n)-\log\log n}{\pi n}}$
Weak Barrier Coverage		$\frac{\log n+\phi(n)}{2n}$	$\frac{\log n-\phi(n)-\log\log n}{2n}$
Barrier Coverage	Preliminary Condition	$\frac{\log n}{2n}$	$\frac{\log n}{2n}$
	General Condition	$\frac{\log n(\log n-\log\log n)}{2n}$	$\frac{\log n(\log n-\log\log n)}{2n}$

6.1.2 Preliminary condition and general condition

Under preliminary condition, we assume that the intruding path is a straight line and we further get the corresponding CSR: $R_\star = \frac{\log n}{2n}$. However, this straight-line restrict is not realistic in the real-time application, so we further consider a general condition which allows the crossing path to be an arbitrary curve line, and we have the result of CSR: $R_\star = \frac{\log n(\log n-\log\log n)}{2n}$.

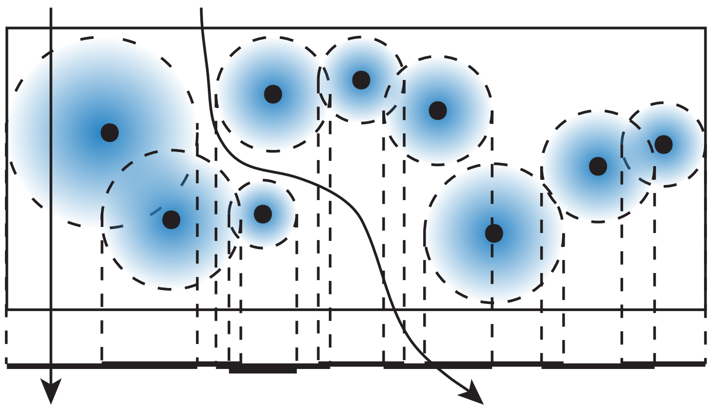


Figure 6.2 Straight line intruding and general intruding

Comparing these two CSRs, intuitively, the later one comes larger. This is quite obvious because when all the sensors' projections cover the entire bottom boundary of the belt region, the barrier coverage for straight-line intruding is achieved while the general intruding condition may not be guaranteed. Intruders can utilize the space between the sensing area and escape from the belt region successfully, as is illustrated in the figure above.

Therefore, we must raise the requirement for general condition, the later CSR is $(\log n - \log\log n)$ times more than the previous CSR, and this factor, compensates for the arbitrary curve crossing line instead of straight line.

6.1.3 Compared with other coverage problems

In table 6.1, we list several coverage problems and the corresponding CSRs. We will give some insightful comparisons and reasonable analysis based on the table.

As is presented in the table, only our work has a non-gap CSR, while the other two CSRs both have gaps between the upper bound and corresponding lower bound. S. Kumar Kumar et al. (2004) studied asymptotic k-coverage in a mostly sleeping stationary sensor network. A parameter $c(n) = \frac{np\pi r^2}{\log(np)}$ was defined to evaluate the critical condition. It has been proven that when $c(n) > 1 + \frac{\phi(np)+k\log\log(np)}{\log(np)}$ ($\phi(np)$ is a slowly growing function), the sensing radius is sufficiently large to ensure the whole network's full coverage. Therefore, we can convert the result to CSR from the equation $\frac{n\pi r^2}{\log n} = 1 + \frac{\phi(n)+\log\log n}{\log(n)}$ (let $k = 1$ and $p = 1$). Also when $c(n) < 1 - \frac{\phi(np)+\log\log(np)}{\log(np)}$, there always exists a point that cannot be covered. So the CSR's upper bound and lower bound for ensuring full area coverage should be $\sqrt{\frac{\log n+\phi(n)+\log\log n}{\pi n}}$ and $\sqrt{\frac{\log n-\phi(n)-\log\log n}{\pi n}}$, respectively. This CSR is much larger than that of barrier coverage, since barrier coverage doesn't necessarily demand full coverage. Therefore, when we come to some barrier coverage applications, using full coverage CSR is quite a waste of energy. While our results save energy consumption considerably by reducing the CSR to a relatively small level but still guarantees barrier coverage. The comparison between barrier and full coverage inspires us that the barrier coverage problem is a local coverage problem while the full coverage is a global one. Thus, it's no wonder that the requirement for barrier coverage is much weaker.

Another related result we would like to mention is the CSR for weak barrier coverage under poisson deployment. In this weak barrier coverage, intruder's crossing lines are orthogonal, and the coverage problem means to cover these orthogonal lines in a belt region. Study in Kumar et al. (2005) used the same method in Kumar et al. (2004), by carefully scrutinizing this result we can find out that this CSR's upper bound has the same order as our result. This is not hard to understand because the weak barrier coverage is a specific example of our preliminary condition (we allow multi-angle intruding). And we believe that this CSR's gap results from their specific poisson deployment and the belt region implementation.

6.2 Simulations on Speed Resolution

In this section, we run simulations on speed resolution concerning chapter 4. We first provide numerical simulation and give analysis to its behaviors.

6.2.1 Performance of $g(n)$

In section 4.3, we give the formula on resolution 4-2 a practical amend by filtering it with a window function $g(n)$. As is

$$g(n) = \begin{cases} 0 & 0 < w_0 < 2r \\ -1 & 2r < w_0 < 4r \\ 1 & w_0 \geq 4r \end{cases} \tag{6-1}$$

We can see $g(n)$ is a function of n, but the two boundaries within it, say $w_0 = 2r$ and $w_0 = 4r$, are not equations directly related to n. The first boundary is the solution n_1 of the below equation.

$$\frac{w}{\log n} = \frac{\log n(\log n - \log\log n)}{n} \tag{6-2}$$

And the second boundary is the solution n_2 of the following equation.

$$\frac{w}{\log n} = 2\frac{\log n(\log n - \log\log n)}{n} \tag{6-3}$$

Afterwards we have derived the window function of speed resolution, a numerical simulation is desired to make it more appealing and convincing. In Kumar et al. (2005), Kumar holds that the width of barrier region is normally less than 1 percent of its length, and here we assign $w = 0.01$ for simplifying our simulation. Keep in mind that the length is uniformed to 1 as well as sub barrier's width being $w_0 = \frac{w}{log(n)}$, we can start our simulation with independent variable n (the total sensor number).

We present the numerical simulation of $g(n)$ as below figure 6.3. The figure of

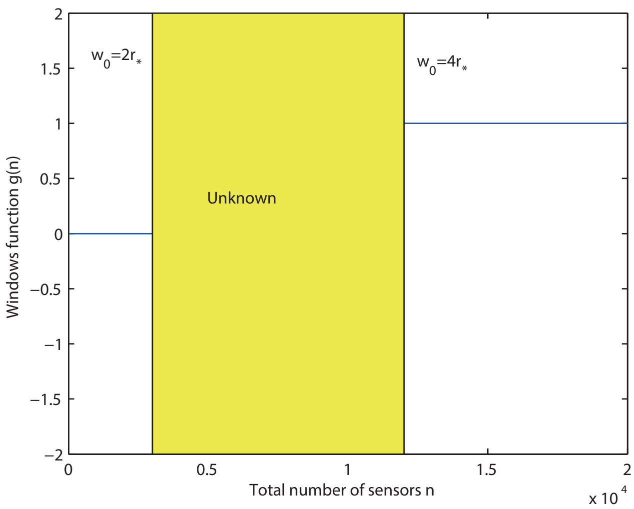


Figure 6.3 Boundary effect of resolution

boundary effect illustrate that the the positions where boundaries locates. The second boundary of n_2 is vital in practical deployment for the actual number of sensors shall be larger than n_2 to make sure the speed resolution won't be influenced by boundary turmoil. After the second boundary, as the figure 6.3 indicates, the window function has no effect on the resolution expression.

6.2.2 Performance of Resolution

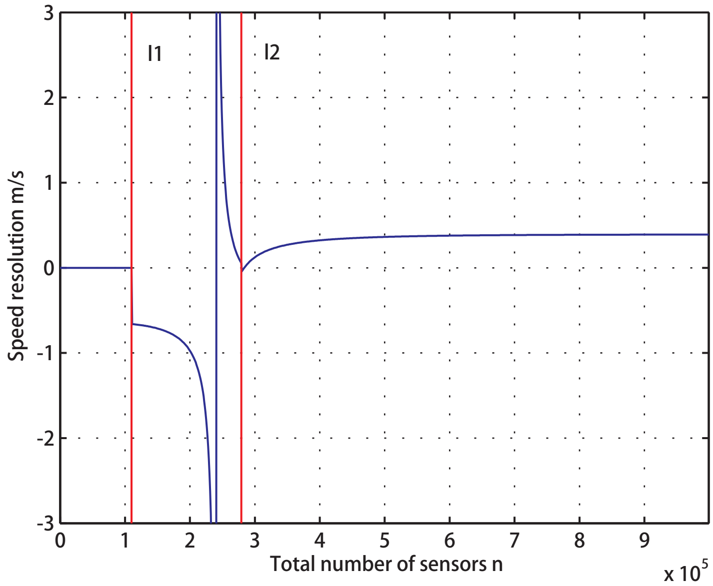
Finally, we simulate the function $f(n) = g(n)\mathbb{E}[D(O, i)]$ to properly indicate the properties of speed resolution. The result is as below.

For Short Barrier

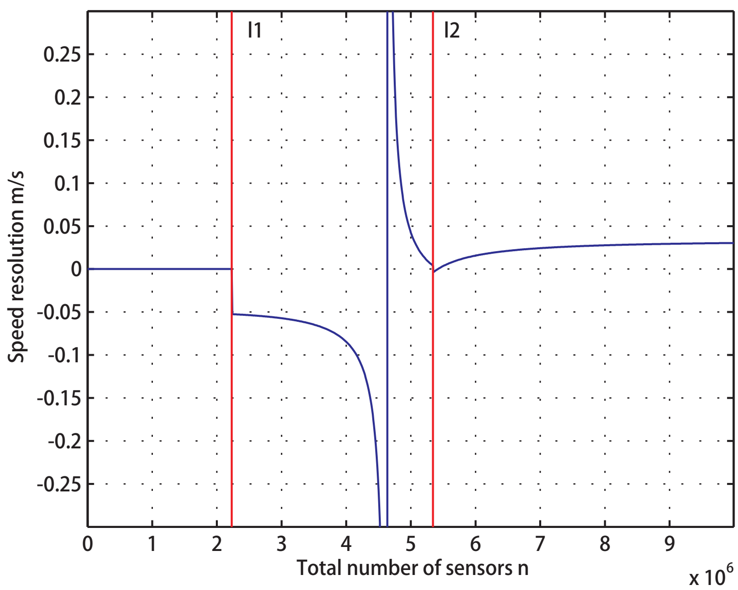
Firstly, we analyze conditions when the barrier length is relatively small, say around several hundreds kilometers which is quite alike the boundary of city Shanghai. We here apply boundary length $l = 10^5 m$. To evaluate the effect of barrier width towards the speed resolution, we apply two barrier regions whose width is $10m$ and $1m$, respectively. The simulation results is presented as the below two graphs.

Let $l1, l2$ represents the two boundary $w_0 = 2r$ and $w_0 = 4r$ respectively. We see directly in figure 6.4, the resolution in region between the two cursor is quite chaotic. Before the first cursor$l1$, the resolution of zero is quite the property of strong barrier coverage. And after $l2$, the simulation is more applicable as the general barrier condition. And we see in both figures in 6.4, the speed resolution is arriving at the relative fixed value as $n \to \infty$. We can this value the speed resolution of this very barrier sensor network.

As $n \to \infty$, the speed resolution when $w = 10m$ is mostly $0.4m/s$, which meaning a $100km$ long barrier can have its resolution almost as $0.4m/s$. While the speed resolution of barrier sensor network whose width is $1m$ and length is $100km$can achieve a orderly lower value of $0.03m/s$. The narrower the width is, the resolution is more precise and sensitive. We believe the barrier length of $10m$ is enough to cover most human intruders even they are slowly walking, as is shown in figure 6.4(a). While if some intruders are specifically annoying and nasty who move much slower than human beings, we can apply the narrow barrier region, as is in figure 6.4(b), to reach a more sensitive speed resolution though we have to apply more delicate sensors whose sensing is regionally small but precise. To make the resolution even lower, the width of barrier region is the critical contribution. Deducing it can strongly make the speed resolution more sensitive. Meanwhile, we found that to apply an successful barrier coverage



(a) $w = 10m, l = 10^4m$



(b) $w = 1m, l = 10^4m$

Figure 6.4 Simulation of speed resolution for short barrier

whose speed resolution is sensitive enough, we need an sufficient amount of sensors. The narrower the width of barrier is, the number of sensor demanded is larger. As is indicated in figure 6.4(a), the number of sensor demand to achieve speed resolution is $k \times 10^5$ when the barrier width is 0.01 percent of barrier length. While the number of sensor demand to achieve speed resolution is $k \times 10^6$, which is orderly larger than previous one when the barrier width is 0.001 percent of barrier length, which is shown in figure 6.4(b).

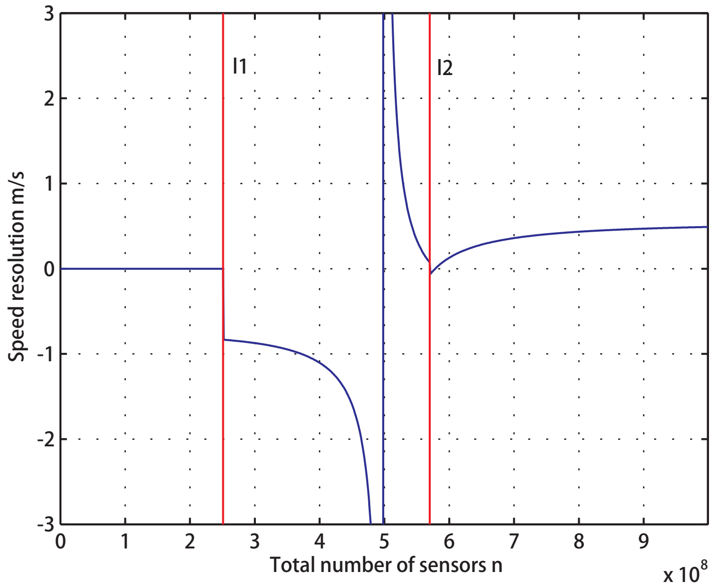
For Long Barrier

Secondly, we give simulations on long barrier region, whose length may be around our national boundaries, say at the order of $10^7 m$.

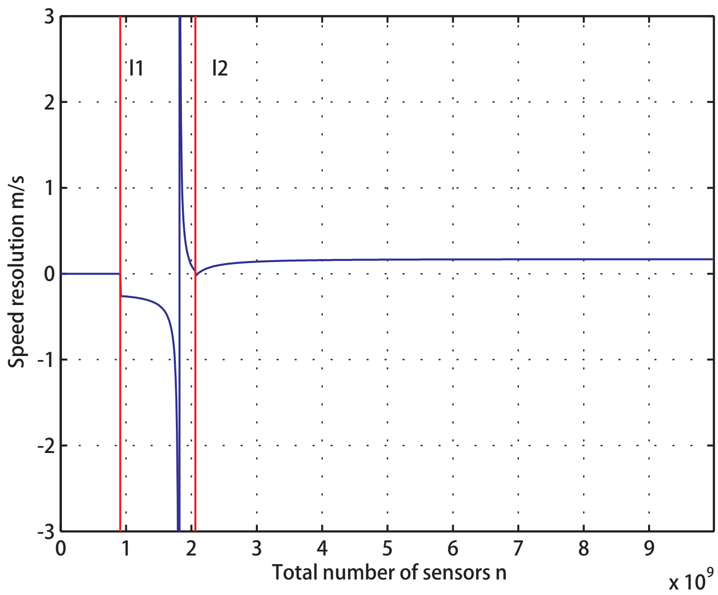
We here apply boundary length $l = 10^7 m$. Again, to evaluate the effect of barrier width towards the speed resolution, we apply two barrier regions whose width is $\log l$ and $3 \log l$, respectively. The simulation results is presented as the below two graphs.

Let $l1, l2$ represents the two boundary $w_0 = 2r$ and $w_0 = 4r$ respectively. We see directly in figure 6.5, the resolution in region between the two cursor is quite chaotic. Before the first cursor$l1$, the resolution of zero is quite the property of strong barrier coverage. And after $l2$, the simulation is more applicable as the general barrier condition. And we see in both figures in 6.5, the speed resolution is arriving at the relative fixed value as $n \to \infty$. We can this value the speed resolution of this very barrier sensor network.

As $n \to \infty$, the speed resolution when $w = 3 \log l$ is mostly $0.4 m/s$, which meaning a $10000 km$ long barrier can have its resolution almost as $0.4 m/s$. While the speed resolution of barrier sensor network whose width is $\log l$ and length is $10000 km$ can achieve a quite lower value of $0.1 m/s$. The narrower the width is, the resolution is more precise and sensitive. We believe the barrier length of $3 \log l$ is enough to cover most human intruders even they are slowly walking, as is shown in figure 6.5(a). While if some intruders are specifically annoying and nasty who move much slower than human beings, we can apply the narrow barrier region, as is in figure 6.5(b), to reach a more sensitive speed resolution though we have to apply more delicate sensors whose



(a) $w = 3\log l, l = 10^7 m$



(b) $w = \log l, l = 10^7 m$

Figure 6.5 Simulation of speed resolution for long barrier

sensing is regionally small but precise. To make the resolution even lower, the width of barrier region is the critical contribution. Deducing it can strongly make the speed resolution more sensitive. Meanwhile, we found that to apply an successful barrier coverage whose speed resolution is sensitive enough, we need an sufficient amount of sensors. The narrower the width of barrier is, the number of sensor demanded is larger. As is indicated in figure 6.5(a), the number of sensor demand to achieve speed resolution is $k \times 10^8$ when the barrier width is $3 \log l$. While the number of sensor demand to achieve speed resolution is $k \times 10^9$, which is orderly larger than previous one when the barrier width is $\log l$, which is shown in figure 6.5(b).

Conclusion towards evaluation on speed resolution

In conclusion, according to the two practical simulations for long barrier sensor network and short barrier sensor network, and after thoroughly analysis over our evaluation results shown in both figure 6.4 and 6.5, we have the below conclusion: both the barrier width w and sensor number n have vital influence on speed resolution of a given barrier sensor network. The smaller the barrier width w is, the larger the sensor number n becomes, the smaller and more sensitive the speed resolution achieves. And this conclusion meets our theoretical derivation of the limitation of speed resolution, as is provided in section 4.4:

$$Res(n) = \frac{w}{4 \log n} \tag{6-4}$$

At this point, we can have confidence that our result is warranted and reasonable for pragmatic concerns.

6.3 Barrier Coverage under Random Walk Model

In this section, we give numerical simulations on barrier coverage's CSR under random walk mobility model. And we provide analysis towards inner correlations between barrier coverage and full coverage, which may indicate its rationality and convincement.

6.3.1 Numerical simulation

Figure 6.6 illustrates the relationship between CSR and the total number of sensors. When n goes to infinity, $R_\star \to 0$. This consists with the instinct that only smaller sens-

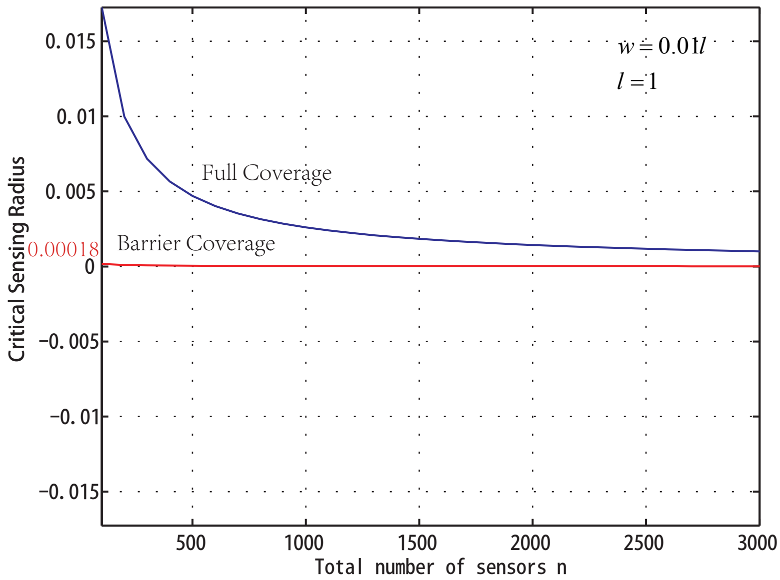


Figure 6.6 Simulation for coverage CSRs in I.I.D mobility model

ing radius is needed to achieve barrier coverage if the total number of sensors increases. It is relatively clear that the CSR of barrier coverage condition is much smaller than that under full coverage condition when n is relatively small, as n becomes larger, the gap becomes smaller. This results shows that when n is relatively large, the increasing number of sensors (density) offsets the gap between two conditions, therefore, two CSRs come much closer.

Table 6.2 Coverage Problems and CSRs

Coverage Problem	Critical Sensing Radius
Full Coverage	$\frac{3(\log n+\log\log n)}{4n}$
Barrier Coverage	$\frac{3w(\log n+\log\log n)}{4n}$

6.3.2 Compared with Full Coverage

In table 6.2, we list CSRs of full coverage and barrier coverage under 1-dimensional random walk mobility model. We will give some insightful comparisons and reasonable analysis based on the table.

As is presented in the table, both two works have a non-gap CSR, while the there are two CSRs in table 6.1 both have gaps between the upper bound and corresponding lower bound. The critical sensing radius of full coverage $\frac{3(\log n+\log\log n)}{4n}$ shall be scaling larger than the critical sensing radius of barrier coverage, say as $\frac{1}{w}$. Here w is the width of barrier region, which is at most 1 percent of barrier length, which is mostly assigned as 1. As is presented in Wang et al. (2011), the strong mobility of sensors save much energy and sensing radius demand to achieve successful coverage. And here in our barrier coverage, the narrow belt region of coverage problem makes our CSR w times smaller than full coverage, whose operational region is unified to 1. Due to this property, the CSR of barrier coverage under random walk mobility model is extremely small compared to full coverage under random walk mobility model.

As is indicated in our simulation result, with parameter $w = 0.01l, l = 1$, the CSR for full coverage is around 100 times to the CSR for barrier coverage. This multiple relationship between the two CSR remains the same however large n goes. But as $n \to \infty$, the relative difference between full coverage' CSR and barrier coverage' CSR fades to 0. This simulation results provide support to barrier coverage's high practical value to largely reduce CSR requirement and energy consumption.

Chapter 7 Conclusion

Barrier coverage is always a fundamental property that military application and security networks should consider, although researchers have investigated several deployment strategies and algorithms to implement the barrier coverage, the critical condition to achieve barrier coverage still remains unsolved.

In this paper, we analyze the critical conditions for barrier coverage under prevail mobility models both necessarily and sufficiently from the perspective of critical sensing radius (CSR). When the sensor's number gets sufficient large, we believe that CSR is a basic requirement to ensure barrier coverage and this result also provides a judgment for a successful barrier coverage. Meanwhile, our result shows guidance to networking engineers for their deployment of barrier sensor network, which is utilizing n to determine CSR, and utilize w and CSR to determine speed resolution for i.i.d mobility condition while utilizing n, w to determine the CSR for random walk mobility condition. The reasonability and convincement of our result is supported with numerical evaluations in section 6. Our work provide fundamentals theories towards critical area surveillance and intruder detecting, which can be widely applied to national boundary defend, battle field control, community security guarantee and vital resource management.

Our future work should incorporate most sleeping sensor network and collaboration to lower the requirement of achieving barrier coverage. We will analyze connected barrier coverage which is different to the strong barrier coverage whose sensor is connected one by one in Liu et al. (2008). And we should extend our results to a more realistic and efficient version. In addition, we may focus more on system simulation on NS2 and practical implementation.

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