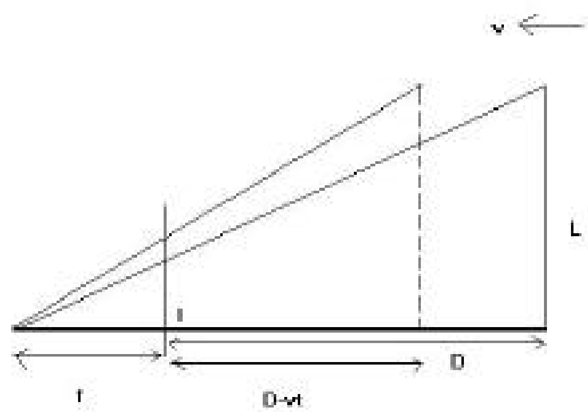
Motion

Example 1: Random dot sequences

By using two 2D arrays with random entries and pasting a displaced part of the first into the second gives motion perception.

Example 2: Estimating time to impact

Simple distance measurement based on information from sequences of images. When does the robot hit the wall?



Assumption: Relative speed robot-wall is constant.

$$l(t) = f \frac{L}{D} \qquad (1)$$

$$\frac{dl}{dt} = f \frac{D \frac{dL}{dt} - L \frac{dD}{dt}}{D^2} = f \frac{L v}{D^2} \qquad (2)$$

$$\frac{(1)}{(2)} = \frac{f \frac{L}{D}}{f \frac{L v}{D^2}} = \frac{D}{V} = \tau \qquad (3)$$

Since you can estimate the height of the point *l(t)* and its time derivative *l'(t)* you can estimate the time to impact without knowing the real world position of the point or tthe velocity of the robot.

Motion

Motion is a 2D to 3D problem since what you have got to use is the projection of the 3D world onto a sequence of 2D images.

The first fundamental assumption to facilitate the calculations is that all moving objects are rigid. This assumption implies that all points in an object have the same 3D motion. The 3D motion can be divided in two components: translational **T** and angular **W**.

Two sub-problems of motion

- Correspondence: Which elements of a frame correspond to which element of the next frame of the sequence. This problem differs from stereo in that disparities are small.
- Reconstruction: Given a number of corresponding elements, and possibly knowledge of the camera's intrinsic parameters, what can we say about the 3D motion and structure of the observed 3D world? Unlike stereo, in motion the relative 3D displacement between the viewing camera and the scene is not necessarily caused by a single 3D rigid transformation.

The 2D motion field of a 3D point

The speed of a point is calculated as follows

$$\vec{V} = -\vec{T} - \vec{W} \times \vec{P}$$

and has components

$$\vec{V} = [v_x, v_y, v_z]^T$$

The cross product is

$$\vec{W} \times \vec{P} = \det\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ w_x & w_y & w_z \\ X & Y & Z \end{vmatrix} = \vec{a}_x\left[Zw_y - Yw_z\right] - \vec{a}_y\left[Zw_x - Xw_z\right] + \vec{a}_z\left[Yw_x - Xw_y\right]$$

$$\left.\begin{aligned} v_x &= -T_x - w_y Z + Y w_z \\ v_y &= -T_y + w_x Z - X w_z \\ v_z &= -T_z - w_x Y + X w_y \end{aligned}\right\} \quad (4)$$

$$\vec{P} = [X, Y, Z]^T,\ \vec{p} = [x, y, f]^T$$

$$\vec{P} = \frac{f}{Z}\vec{p}$$

Take derivatives of both sides with respect to time

$$\frac{d\vec{P}}{dt} = \vec{v} = [v_x, v_y, 0]^T \Big\} LHS$$

$$\left. f \quad \frac{z\ \vec{V} - \vec{P}\ \ v_z}{Z^2} \right\} RHS$$

Plug (4) into RHS.

The resulting 2D motion field is:

$$v_x = \underbrace{\frac{T_z x - T_x f}{Z}}_{\text{Translational component}} \underbrace{- w_y f + w_z y + \frac{w_x xy}{f} - w_y \frac{x^2}{f}}_{\text{angular component}}$$

$$v_y = \underbrace{\frac{T_z y - T_y f}{Z}}_{v_y^T} \underbrace{+ w_x f - w_z x - \frac{w_y xy}{f} + w_x \frac{x^2}{f}}_{v_y^W}$$

The motion field can give rise to three different cases:

Case one and two
The angular component of the motion field *w* is zero.
The translational component T_z is not zero.

Let

$$\vec{p}_0 = [x_0, y_0]^T = \left[f \frac{T_x}{T_z}, f \frac{T_x}{T_z} \right]^T$$

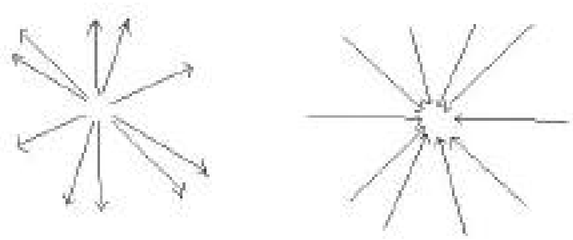
$$\frac{v_x}{T_z} = \frac{x - f \frac{T_x}{T_z}}{Z} = \frac{x - x_0}{Z} \qquad v_x = (x - x_0)\frac{T_z}{Z}$$

$$\frac{v_y}{T_z} = \frac{y - f \frac{T_y}{T_z}}{Z} = \frac{y - y_0}{Z} \qquad v_y = (y - y_0)\frac{T_z}{Z}$$

The magnitude of the motion field is therefore:

1. proportional to the distance between p and p_0.
2. proportional to 1/depth (i.e. 1/Z).

These two cases gives a radial motion field pointing from or towards p_0.



Case three

The angular component w is zero. The translational component T_z is also zero.

$$\left.\begin{aligned} v_x &= -\frac{T_x f}{Z} \\ v_y &= -\frac{T_y f}{Z} \end{aligned}\right\} \vec{v} = \frac{1}{Z}[-T_x f, -T_y f]$$

This case give rise to a parallel motion field.

Instantaneous epipoles

Let $P=[x\ y\ z]^T$ and $P_1=[x_1\ y_1\ z_1]^T$ be two different world points projected into p and p_1. The motion fields for the two latter points are the following:

$$\vec{v} = \begin{cases} v_x = v_x^T + v_x^w \\ v_y = v_y^T + v_y^w \end{cases}$$

$$\vec{v}_1 = \begin{cases} v_{1x} = v_{1x}^T + v_{1x}^w \\ v_{1y} = v_{1y}^T + v_{1y}^w \end{cases}$$

Due to the fact that p and p_1 are the same in this case the angular component of the

velocities are the same.
We define the relative motion field:

$$\left.\begin{aligned}\Delta v_x &= v_x^T - v_{1x}^T = (T_z x - T_x f)\left(\frac{1}{z} - \frac{1}{z_1}\right)\\ \Delta v_y &= v_y^T - v_{1y}^T = (T_z y - T_y f)\left(\frac{1}{z} - \frac{1}{z_1}\right)\end{aligned}\right\} \qquad \frac{\Delta v_y}{\Delta v_x} = \left(\frac{y - y_0}{x - x_0}\right)$$

Where $p_0=[x_0\ y_0]^T$ are the coordinates of the vanishing point of translation. The motion field points away from the vanishing point.

Optical flow

Brightness: *E(x,y,t)* where *t* is the time. The constant brightness assumption is that the derivative with respect to time of the brightness is zero.

- The aperture problem gives that the only component of the motion field you can recover is the normal (in the direction of the gradient of the brightness) component.

Problem description:
Given a time-varying sequence of images, find the apparent motion (the normal component of the motion) under the constant brightness assumption.

Taking the derivative of the brightness yields

$$\frac{dE}{dt} = \frac{\partial E}{\partial x}\frac{dx}{dt} + \frac{\partial E}{\partial y}\frac{dy}{dt} + \frac{\partial E}{\partial t} = 0$$

and the image brightness constancy equation is

$$\frac{dE}{dt} = (\nabla E)^T \vec{v} + \frac{\partial E}{\partial t} = 0$$

where $\vec{v} = \left[\frac{dx}{dt}, \frac{dy}{dt}\right]^T$ is the motion field.

$\vec{v}_n$, which we can find, is the component of $\vec{v}$ in the direction of ∇E^T.

$$\vec{v}_n = \frac{(\nabla E)^T}{\|\nabla E\|}\vec{v} = -\frac{\frac{\partial E}{\partial t}}{\|\nabla E\|}$$

The equation for the brightness of a surface with normal $\vec{n}$ is

$$\vec{E} = \rho\ \vec{I}^T \vec{n}$$

Again $\vec{V} = -\vec{T} - \vec{W} \times \vec{P}$

At $t = 0$, the normal is $\vec{n}_0$.

At $t = \Delta t$, the normal is $\vec{n}_{\Delta t} = \vec{n}_0 - \vec{w} \times \vec{n}_0$.

$$\frac{\vec{n}_0 - \vec{n}_{\Delta t}}{\Delta t} = \frac{\vec{w} \times \vec{n}_0}{\Delta t} \underbrace{}_{\Delta t \to 0} \frac{d\vec{n}}{dt} = \vec{w} \times \vec{n}_0$$

$$\frac{dE}{dt} = \rho \vec{I}^T (\vec{w} \times \vec{n}) = (\ E)^T \vec{v} + \frac{\partial E}{\partial t} \to \frac{\frac{\partial E}{\partial t}}{\|\ E\|} = \frac{(\ E)^T \vec{v} - \rho \vec{I}^T (\vec{w} \times \vec{n})}{\|\ E\|} \qquad (5)$$

$$\vec{v}_n = \frac{(\ E)^T \vec{v}}{\|\ E\|}$$ (6) This is the estimated normal velocity

The difference between (5) and (6) is described by

$$\underbrace{|\Delta v|}_{\text{The difference between (5) and (6)}} = \frac{\rho \vec{I} (\vec{w} \times \vec{n})}{\|\ E\|}$$

When is this error zero?

Observations:
In general, under the constant brightness assumption and the Lambertian model, it only reveals the "true" v_n when $w = 0$ (pure translation) or when $\vec{I}^T$ is perpendicular to $(\vec{w} \times \vec{n})$.
When the gradient of E is high you get better estimates.

Different approaches to the estimation of the motion field.

Differential approaches

- use derivatives
- dense
- numerically

Feature based approaches

- tracking features
- Kalman filters (estimation techniques)
- sparse methods

Window based approach

Assumptions:

1. Constant brightness assumption holds
2. Over a window or a patch there is a single dominant motion.

Q is the N x N patch

$$\Psi(v) = \sum_{p_i \in Q} \left(\frac{\partial E}{\partial t} + (\ E)^T \vec{v} \right)^2$$

where $\vec{p}_i, i = 1, \ldots, N \times N$, is the points in patch Q.

This is a standard least squares problem. Find the ν which minimizes Ψ(ν).

The solution is to solve for $\vec{v}_n$.

$$\underbrace{A^T A}_{2\times 2}\underbrace{\vec{v}}_{2\times 1} = \underbrace{A^T}_{2\times N^2}\underbrace{\vec{b}}_{N^2\times 1} \qquad \vec{v} = (A^T A)^{-1} A^T \vec{b}$$

$$A = \begin{bmatrix} E(\vec{p}_1) \\ E(\vec{p}_2) \\ : \\ : \\ E(\vec{p}_{N\times N}) \end{bmatrix}_{N^2\times 2} \qquad \vec{b} = \begin{bmatrix} \frac{\partial E}{\partial t}(\vec{p}_1) \\ \frac{\partial E}{\partial t}(\vec{p}_2) \\ : \\ : \\ \frac{\partial E}{\partial t}(\vec{p}_{N\times N}) \end{bmatrix}_{N^2\times 1}$$

The structure of AA^T is numerically important since it could be singular, the problem could be ill-posed.