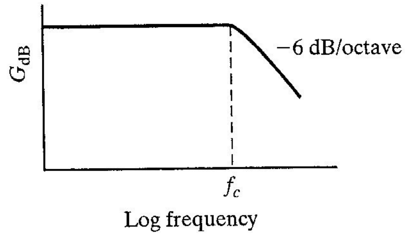


FIGURE 3.12
Frequency response of op-amp amplifier.

3.15

$$f_c = \frac{\text{GPB}}{G}$$

$$\phi = -\tan^{-1}\frac{f}{f_c}$$

3.16

$$\text{GBP}_{\text{inv}} = \frac{R_2}{R_1 + R_2}\text{GBP}_{\text{noninv}} \quad (3.18)$$

$$G = \left(1 + 2\frac{R_2}{R_G}\right)\left(\frac{R_2}{R_3}\right)$$

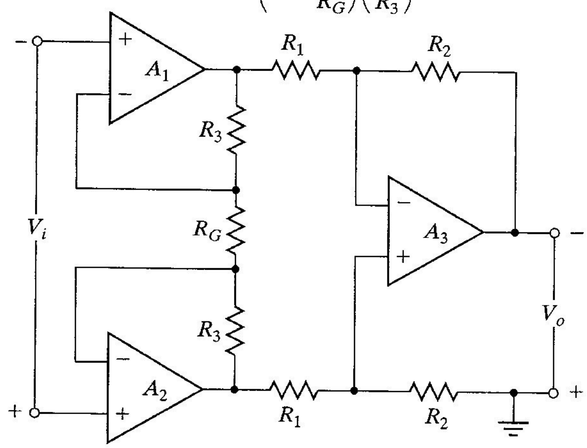
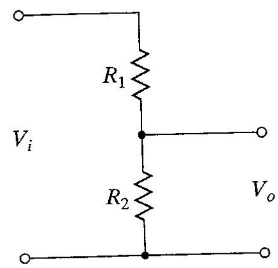


FIGURE 3.14
True differential input instrument amplifier.
(From Franco, 2002.)



$$V_o = V_i\frac{R_2}{R_1 + R_2} \quad (3.19)$$

FIGURE 3.15
Attenuation using dividing network.

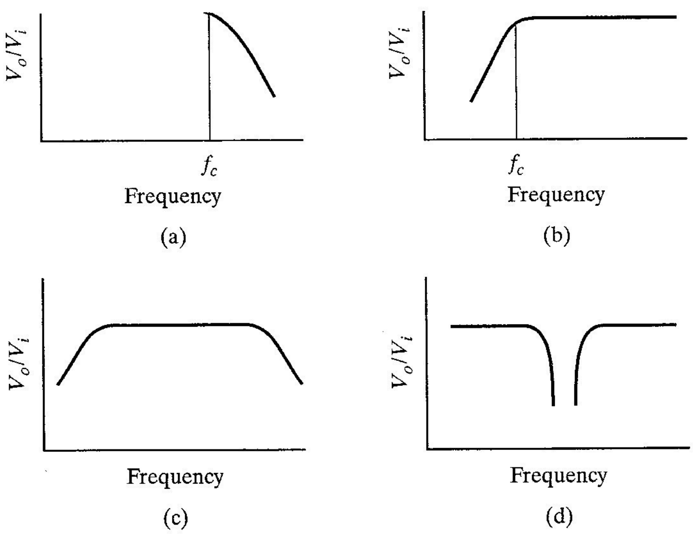


FIGURE 3.16
Categories of electrical filters: (a) lowpass; (b) highpass; (c) bandpass; (d) bandstop.

$$G = \frac{1}{\sqrt{1 + (f/f_c)^{2n}}}$$

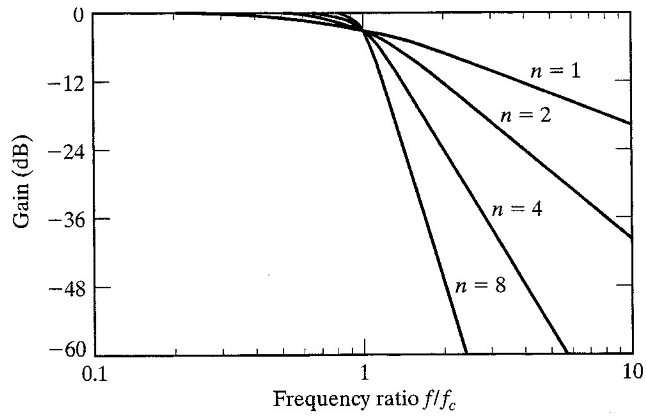


FIGURE 3.17

Gain of lowpass Butterworth filters as a function of order and frequency.

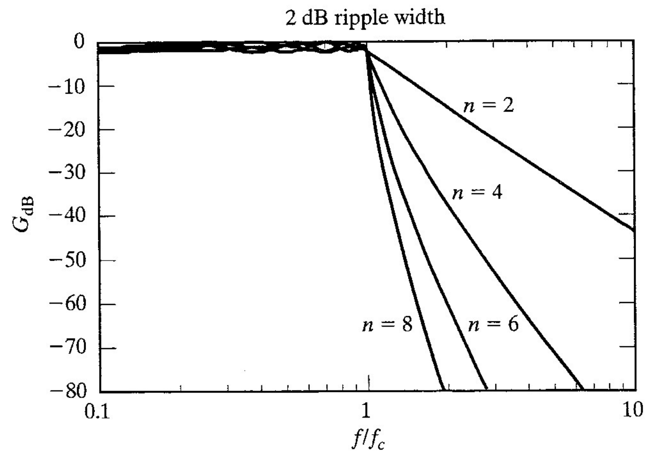


FIGURE 3.18

Gain of lowpass Chebyshev filters as a function of order and frequency.

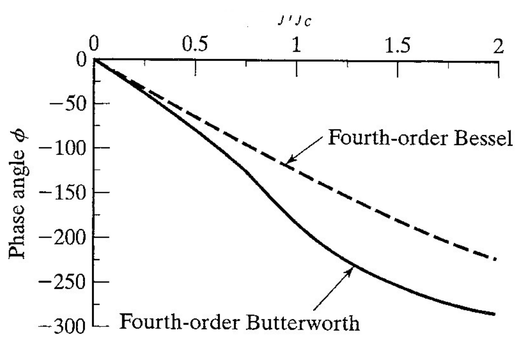


FIGURE 3.19

Comparison of Butterworth and Bessel phase-angle variation with frequency.

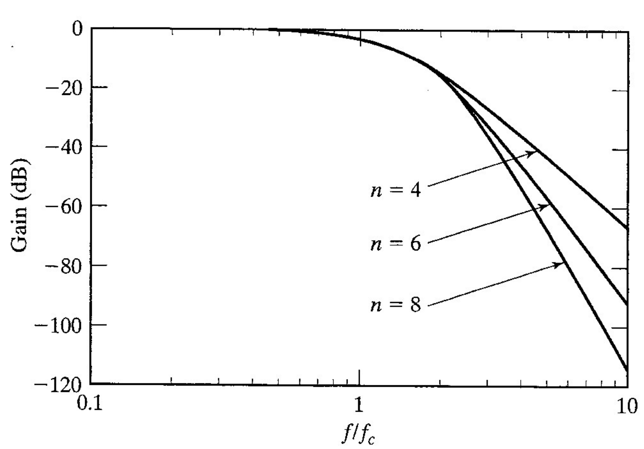


FIGURE 3.20

Gain of lowpass Bessel filters as a function of order and frequency.

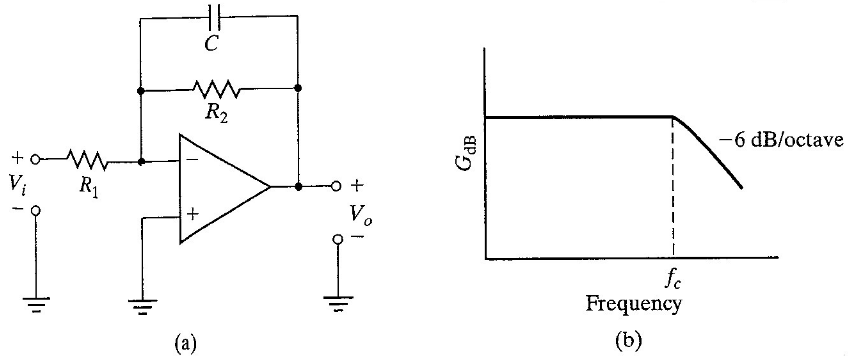


FIGURE 3.21

Lowpass Butterworth filter using op-amp: (a) op-amp circuit; (b) frequency response.

To determine the frequency response, we will use ac circuit analysis to examine the behavior of the circuit when a single-frequency sinusoidal signal is input. This input signal takes the form $V_{mi}\, e^{j2\pi ft}$, where V_{mi} is the sine-wave amplitude and f is the frequency. In deriving Eq. (3.17), the expression for the gain on an inverting amplifier, we used resistances R_1 and R_2 in the feedback loop. If instead we had used complex impedances Z_1 and Z_2, the resulting equation for gain G would have been

$$G = -\frac{Z_2}{Z_1} \tag{3.21}$$

We can apply this equation to the filter in Figure 3.21(a). Z_2 consists of C and R_2 in parallel. Since the impedance of a capacitor is $1/j2\pi fC$, the value of Z_2 is found to be

$$Z_2 = \frac{1}{\left(\frac{1}{R_2}\right) + j2\pi fC} = \frac{R_2}{1 + j2\pi fCR_2} \tag{3.22}$$

Z_1 is simply R_1, so the expression for G, Eq. (3.21), becomes

$$G = -\frac{1}{R_1}\frac{R_2}{1 + j2\pi fCR_2} = G_o\frac{1}{1 + j2\pi fCR_2} = \frac{V_o}{V_i} \tag{3.23}$$

where G_o, the low-frequency gain $(-R_2/R_1)$, is the same as that of the simple inverting amplifier (without the presence of capacitor C). It should be noted that the gain G is represented by a complex number.

If we take the absolute value of the terms in Eq. (3.23), we obtain the following expression:

$$\left|\frac{G}{G_o}\right| = \left|\frac{1}{1 + j2\pi fCR_2}\right| = \frac{1}{\sqrt{1 + (2\pi fCR_2)^2}} \qquad (3.24)$$

The corner frequency, f_c, for a Butterworth filter is defined as the frequency where the magnitude of the gain is reduced 3 dB from its low-frequency value, G_o. Equation (3.2) shows that 3 dB corresponds to a reduction in gain, G, by a fraction equal to 0.707. Substituting the value $G = 0.707G_o$ into Eq. (3.24) gives

$$\left|\frac{0.707G_o}{G_o}\right| = 0.707 = \frac{1}{\sqrt{1 + (2\pi fCR_2)^2}} \qquad (3.25)$$

This can be solved for the corner frequency f_c:

$$f_c = \frac{1}{2\pi CR_2} \qquad (3.26)$$

It should be noted that the corner frequency for a lowpass filter computed by Eq. (3.26) cannot be any higher than the cutoff frequency for the inverting amplifier itself.

Equation (3.24) can be used to determine the roll-off rate as frequency becomes high relative to the corner frequency. At high frequencies, $2\pi fCR_2$ is large compared to 1, and Eq. (3.25) reduces to

$$\left|\frac{G}{G_o}\right| = \frac{1}{2\pi fCR_2} \qquad (3.27)$$

This shows that if the frequency is doubled (a change of 1 octave), the gain will be reduced by a factor of 1/2. Based on Eq. (3.2), a reduction in gain G by 1/2 corresponds to −6 dB. Thus the roll-off is 6 dB per octave.

Equations (3.23) and (3.26) can be combined to give an expression for gain in terms of f_c:

$$G = \frac{V_o}{V_i} = G_o\frac{1}{1 + j(f/f_c)} \qquad (3.28)$$

Equation (3.28) can be used to evaluate the phase angle of V_o relative to V_i. The phase of V_o relative to V_i is then the phase of $1/[1 + j(f/f_c)]$:

$$\phi = \tan^{-1}\frac{-f}{f_c} = -\tan^{-1}\frac{f}{f_c} \qquad (3.29)$$

While Eq. (3.16) has the same form as Eq. (3.29), Eq. (3.16) results from characteristics of the op-amp itself, whereas Eq. (3.29) is a consequence of the capacitor in the feedback loop.

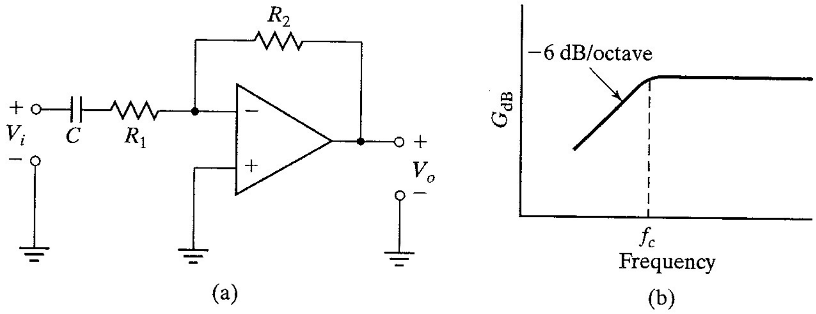


FIGURE 3.22

First-order Butterworth highpass filter using an op-amp: (a) op-amp circuit; (b) frequency response.

Figure 3.22 shows an op-amp circuit and frequency response for a first-order high-pass Butterworth filter. The formula for the cutoff frequency, which can be derived in a manner similar to that for the low-pass filter, is given by

$$f_c = \frac{1}{2\pi R_1 C} \tag{3.30}$$

It should also be noted that highpass filters using op-amps are in fact bandpass filters since the filter amplifier also has a high-frequency cutoff.

Figure 3.23 shows an op-amp circuit and frequency response for a first-order Butterworth bandpass filter. The upper and lower cutoff frequencies for the bandpass filter are given by

$$f_{c1} = \frac{1}{2\pi R_1 C_1} \qquad f_{c2} = \frac{1}{2\pi R_2 C_2} \tag{3.31}$$

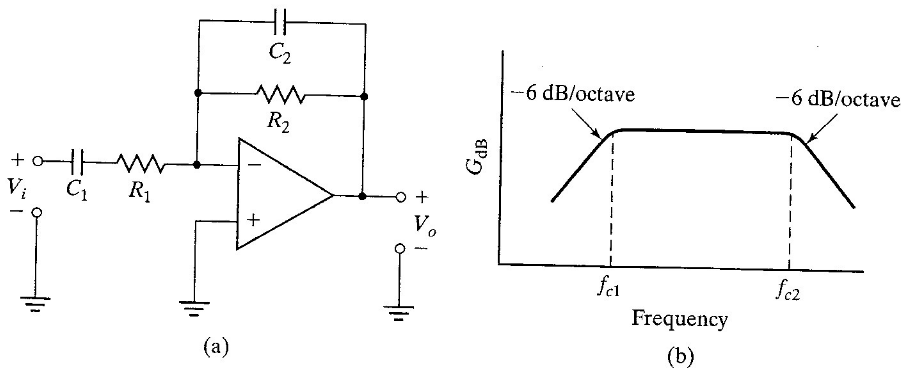


FIGURE 3.23

Bandpass filter using op-amp: (a) op-amp circuit; (b) frequency response.

$$V_o(t) = -\frac{1}{RC}\int_0^t V_i(t)\,dt + V_o(0) \quad (3.32)$$

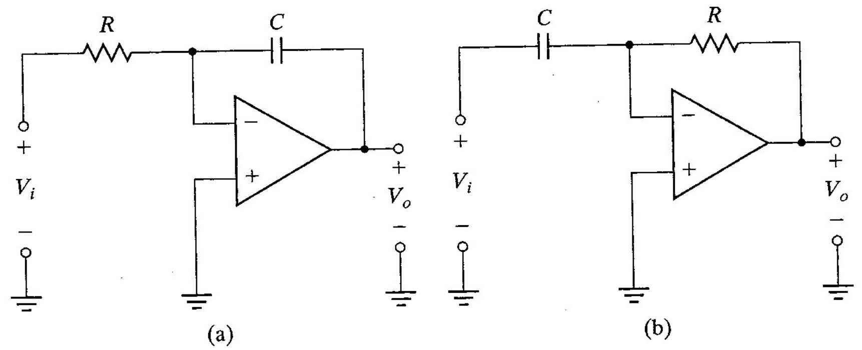


FIGURE 3.24
Op-amp circuits for (a) integration and (b) differentiation.

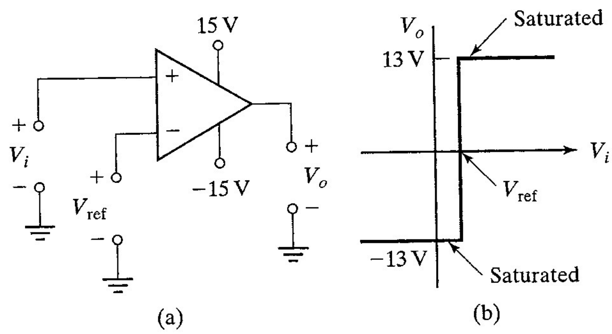


FIGURE 3.25
Op-amp comparator: (a) circuit; (b) output voltage.

For the differentiator circuit shown in Figure 3.24(b), the output voltage is the time derivative of the input voltage:

$$V_o(t) = -RC\frac{dV_i(t)}{dt} \quad (3.33)$$