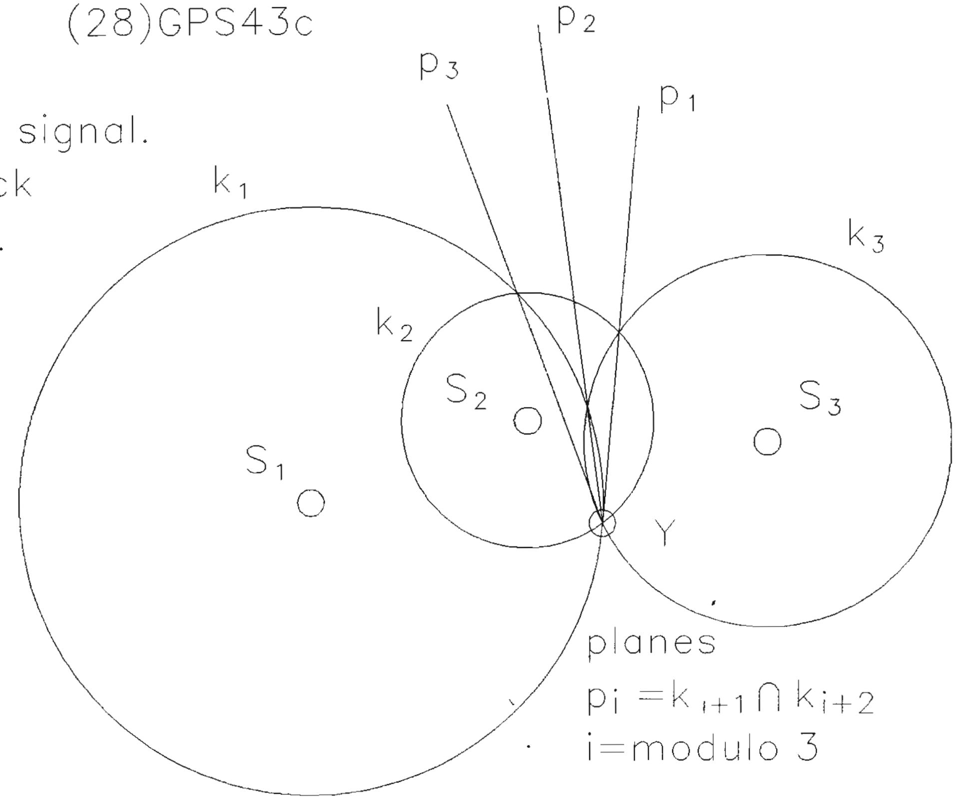
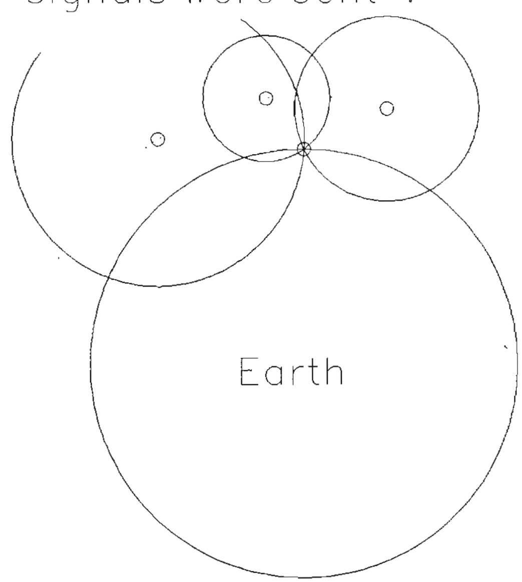
(28)GPS43c

1. Satellite S_i sends time signal.
2. You have accurate clock and can figure out ToF.
3. So you know r_i radius of sphere k_i.
4. With three satellites you get three radii.
5. You know where satellites were when signals were sent*.



planes
$p_i = k_{i+1} \cap k_{i+2}$
i=modulo 3



GPS
Satellite orbit

6. You find intersection of three spheres.
7. You know where you are.. (Find LoX of any two planes and intersect that line with any sphere.)

This would make a neat exam question.

* This info stored in ROM in the GPS unit you bought at Canadian Tire for about $500.

```
> restart:
> e1:=(x-xs1)^2+(y-ys1)^2+(z-zs1)^2-r1^2;e2:=(x-xs2)^2+(y-ys2)^2+(z-
  zs2)^2-r2^2;e3:=(x-xs3)^2+(y-ys3)^2+(z-zs3)^2-r3^2;
```

$$e1 := (x - xs1)^2 + (y - ys1)^2 + (z - zs1)^2 - r1^2$$

$$e2 := (x - xs2)^2 + (y - ys2)^2 + (z - zs2)^2 - r2^2$$

$$e3 := (x - xs3)^2 + (y - ys3)^2 + (z - zs3)^2 - r3^2$$

Three spheres:- e1, e2, e3, quadrics in x,y,z

```
> e4:=collect(e1-e2,[x,y,z],distributed);e5:=collect(e2-e3,[x,y,z],d
  istributed);
```

$$e4 := (-2\,xs1 + 2\,xs2)\,x + (-2\,ys1 + 2\,ys2)\,y + (-2\,zs1 + 2\,zs2)\,z + xs1^2 - r1^2 - xs2^2 + ys1^2 + zs1^2 - ys2^2 - zs2^2 + r2^2$$

$$e5 := (-2\,xs2 + 2\,xs3)\,x + (-2\,ys2 + 2\,ys3)\,y + (-2\,zs2 + 2\,zs3)\,z + xs2^2 - r2^2 - xs3^2 + ys2^2 + zs2^2 - ys3^2 - zs3^2 + r3^2$$

Find two planes e4, e5 formed by differences between e1-e2 and e2-e3. (e3-e1) would have served as one of the two, just as well.

```
> e6:=collect(resultant(e4,e5,z),[x,y],distributed);
```

$$e6 := (4\,xs1\,zs3 + 4\,xs2\,zs1 - 4\,xs1\,zs2 - 4\,xs2\,zs3 - 4\,xs3\,zs1 + 4\,xs3\,zs2)\,x + (-4\,ys3\,zs1 - 4\,ys2\,zs3 + 4\,ys3\,zs2 - 4\,ys1\,zs2 + 4\,ys1\,zs3 + 4\,ys2\,zs1)\,y + 2\,r2^2\,zs1 - 2\,r3^2\,zs1 + 2\,xs3^2\,zs1 - 2\,xs2^2\,zs1 - 2\,ys2^2\,zs1 - 2\,xs3^2\,zs2 - 2\,zs3^2\,zs2 + 2\,zs3^2\,zs1 - 2\,ys3^2\,zs2 + 2\,ys3^2\,zs1 - 2\,zs2^2\,zs1 + 2\,xs1^2\,zs2 - 2\,ys1^2\,zs3 + 2\,ys1^2\,zs2 + 2\,xs2^2\,zs3 + 2\,zs2^2\,zs3 - 2\,xs1^2\,zs3 + 2\,ys2^2\,zs3 - 2\,zs1^2\,zs3 + 2\,zs1^2\,zs2 - 2\,r1^2\,zs2 + 2\,r1^2\,zs3 - 2\,r2^2\,zs3 + 2\,r3^2\,zs2$$

Eliminate z between planes e4 and e5 to create another plane, e6 which is parallel to the z-axis; vertical if you wish.

```
> e7:=sort(collect(resultant(e1,e4,z),[x,y],distributed));
```

$$e7 := xs1^4 + 2\,xs1^2\,ys1^2 + 2\,xs1^2\,zs2^2 - 4\,xs1^2\,zs2\,zs1 + 2\,xs1^2\,r2^2 - 2\,xs1^2\,ys2^2 - 2\,xs1^2\,xs2^2 - 2\,xs1^2\,r1^2 + 2\,xs1^2\,zs1^2 + ys1^4 + 2\,ys1^2\,zs2^2 - 4\,ys1^2\,zs2\,zs1 + 2\,ys1^2\,r2^2 - 2\,ys1^2\,ys2^2 - 2\,ys1^2\,xs2^2 - 2\,ys1^2\,r1^2 + 2\,ys1^2\,zs1^2 + zs2^4 - 4\,zs2^3\,zs1 - 2\,zs2^2\,r2^2 + 2\,zs2^2\,ys2^2 + 2\,zs2^2\,xs2^2 - 2\,zs2^2\,r1^2 + 6\,zs2^2\,zs1^2 + 4\,zs2\,r2^2\,zs1 - 4\,zs2\,ys2^2\,zs1 - 4\,zs2\,xs2^2\,zs1 + 4\,zs2\,r1^2\,zs1 - 4\,zs2\,zs1^3 + r2^4 - 2\,r2^2\,ys2^2 - 2\,r2^2\,xs2^2 - 2\,r2^2\,r1^2 - 2\,r2^2\,zs1^2 + ys2^4 + 2\,ys2^2\,xs2^2 + 2\,ys2^2\,r1^2 + 2\,ys2^2\,zs1^2 + xs2^4 + 2\,xs2^2\,r1^2 + 2\,xs2^2\,zs1^2 + r1^4 - 2\,r1^2\,zs1^2 + zs1^4 + (4\,xs1^2 - 8\,xs1\,xs2 + 4\,zs2^2 - 8\,zs2\,zs1 + 4\,xs2^2 + 4\,zs1^2)\,x^2 + (8\,xs1\,ys1 - 8\,xs1\,ys2 - 8\,ys1\,xs2 + 8\,ys2\,xs2)\,x\,y + (4\,ys1^2 - 8\,ys1\,ys2 + 4\,zs2^2 - 8\,zs2\,zs1 + 4\,ys2^2 + 4\,zs1^2)\,y^2 + (-4\,xs1^3 + 4\,xs1^2\,xs2 - 4\,xs1\,ys1^2 - 4\,xs1\,zs2^2 + 8\,xs1\,zs2\,zs1 - 4\,xs1\,r2^2 + 4\,xs1\,ys2^2 + 4\,xs1\,xs2^2 + 4\,xs1\,r1^2 - 4\,xs1\,zs1^2 + 4\,ys1^2\,xs2 - 4\,zs2^2\,xs2 + 8\,zs2\,xs2\,zs1 + 4\,r2^2\,xs2 - 4\,ys2^2\,xs2 - 4\,xs2^3 - 4\,xs2\,r1^2 - 4\,xs2\,zs1^2)\,x + (-4\,xs1^2\,ys1 + 4\,xs1^2\,ys2 - 4\,ys1^3 + 4\,ys1^2\,ys2 - 4\,ys1\,zs2^2 + 8\,ys1\,zs2\,zs1$$

$$- 4\, ys1\, r2^2 + 4\, ys1\, ys2^2 + 4\, ys1\, xs2^2 + 4\, ys1\, r1^2 - 4\, ys1\, zs1^2 - 4\, zs2^2\, ys2 + 8\, zs2\, ys2\, zs1$$

$$+ 4\, r2^2\, ys2 - 4\, ys2^3 - 4\, ys2\, xs2^2 - 4\, ys2\, r1^2 - 4\, ys2\, zs1^2)\, y$$

Eliminate z between sphere e1 and plane e4 to create a vertical cylinder whose axis is parallel to the z-axis. Careful, this is not a circular cylinder!

```
> e8:=resultant(e6,e7,y):
```

A plane *pair* e8, both normal to the x-axis, is created by eliminating y between the vertical plane e6 and the cylinder e7. This gives us two x-coordinates in the solution for the desired point(s) of intersection among the three spheres, e1,e2,e3. The corresponding y-coordinates are found with these two and any one of the planes, say, e4. This result is not shown because, symbolically, it's a mess. With actual numbers it's no big deal.

```
> e9:=collect(resultant(e4,e5,y),z);
```

$$e9 := -2\, ys1\, xs2^2 - 2\, ys1\, ys2^2 - 2\, ys1\, zs2^2 + 2\, ys1^2\, ys2 + 2\, xs1^2\, ys2 + 2\, ys2\, zs1^2 + 2\, ys1\, r2^2$$

$$- 2\, ys2\, r1^2 + (4\, ys1\, zs2 - 4\, zs2\, ys3 - 4\, ys2\, zs1 - 4\, ys1\, zs3 + 4\, zs1\, ys3 + 4\, ys2\, zs3)\, z - 2\, r2^2\, ys3$$

$$- 2\, r3^2\, ys1 + 2\, r1^2\, ys3 + 2\, r3^2\, ys2 + 4\, x\, xs2\, ys1 - 4\, x\, xs1\, ys2 + 2\, zs2^2\, ys3 + 4\, x\, xs1\, ys3$$

$$- 2\, ys1^2\, ys3 + 2\, ys2^2\, ys3 + 2\, xs2^2\, ys3 - 2\, zs1^2\, ys3 - 4\, x\, xs3\, ys1 - 2\, xs1^2\, ys3 + 2\, zs3^2\, ys1$$

$$- 2\, ys3^2\, ys2 + 2\, xs3^2\, ys1 - 2\, xs3^2\, ys2 - 2\, zs3^2\, ys2 + 2\, ys3^2\, ys1 - 4\, x\, xs2\, ys3 + 4\, x\, xs3\, ys2$$

See how to get the two values of z with this special horizontal plane? The difficulty and confusion you might experience when reading all this comes about because the algebra we learn in school does not handle the underlying geometry very neatly. Go ahead and try it with three actual intersecting spheres, with numbers. You'll see what I mean.

(28)X3sph43c.mws 04-03-03