MECH 261/262
Measurement Lab (& Statistics)

March 7, 2007

Designing a Force/Displacement Transducer(MECH261-2)FDXdc73g

1 Problem Statement

Design a cantilever beam type force and/or displacement transducer according to the following specifications.

- The beam will have a pair, top (tension) and bottom (compression), of strain gauges mounted on it a distance $C = 10$mm away from the "wall" into which it is anchored.
- The material will be steel strip, Young's Modulus $E = 2.07 \times 10^{11}$Pa, which is supplied $b = 10$mm wide and in various custom-ground thickness h.
- Your gauge is intended to deflect $\delta_{max} = 10$mm, full-scale or maximum, when the gauge strains will be $\epsilon_C = \pm 10^{-3}$, *i.e.*, $\pm 1000_{\mu\epsilon}$.
- This tip deflection occurs when a force $F_{max} = 1$N is applied at a distance L from the "wall".
- Which "design variables" are you required to determine?
- Which "design equations" in the article **End Loaded Cantilever**(26)FDXdc41n will you solve simultaneously to obtain the design variables?
- Specify the magnitude and units of these variables in your design.

2 Deflection Transducer Design with *Maple*

Here are the linear force and displacement transducer design equations that pertain to a strain gauged cantilever beam. Note that I in Maple is reserved as the imaginary square-root of -1.

```
> restart:bendingstress:=sigma-F*(L-C)*h/(2*Ix);
```

$$bendingstress := \sigma - \frac{F\,(L-C)\,h}{2\,Ix}$$

```
> deflection:=delta-F*L^3/(3*E*Ix);
```

$$deflection := \delta - \frac{F\,L^3}{3\,E\,Ix}$$

```
> linearstrain:=epsilon-F*(L-C)*h/(2*E*Ix);
```

$$linearstrain := \varepsilon - \frac{F\,(L-C)\,h}{2\,E\,Ix}$$

```
> force:=solve(linearstrain,F);
```

$$force := \frac{2\,\varepsilon\,E\,Ix}{(L-C)\,h}$$

```
> Deflection:=solve(subs(F=force,deflection),delta);
```

$$Deflection := \frac{2\,\varepsilon\,L^3}{3\,(L-C)\,h}$$

```
> Force:=subs(Ix=b*h^3/12,force);
```

$$Force := \frac{\varepsilon\,h^2\,E\,b}{6\,(L-C)}$$

To design a cantilever deflection transducer, use *Deflection* and make certain assumptions, *e.g.*, maximum strain $\varepsilon = 0.001$ and $C = 10h$. Then investigate some range of length L corresponding to some given values of strip thickness h, say, from 0.2, 0.5, 1 and 2mm. The equation *Deflection* is a cubic univariate in L. It is not surprising that each set of three solutions, all real, contains only one that is practical. Having substituted $\varepsilon = 0.001$, $\delta = 10$ and $C = 10h$, the following equation is solved for the four values of h.

$$3(L-10h)h - 0.002L^3 = 0$$

The first roots are all negative and the second roots are approximately the value of C. Since C is about half the length of a strain gauge placed as close as possible to the cantilever support, such a design length would have half of the strain gauge extending *past* the point of application of the concentrated load that produces the displacement that is to be measured.

$$-55.74612806,\ 2.002677391,\ 53.74345067$$
$$-89.00190611,\ 5.016835590,\ 83.98507052$$
$$-127.1977445,\ 10.06803668,\ 117.1297078$$
$$-182.4514878,\ 20.27793946,\ 162.1735483$$

Notice that force F and strip width (breadth) b play no part in the design equation of such a displacement transducer. Of course greater breadth or thickness will make the transducer stiffer and then greater force is necessary to achieve the desired maximum tip deflection. Fig. 1 describes the relevant geometric parameters of a cantilever force/linear displacement transducer.

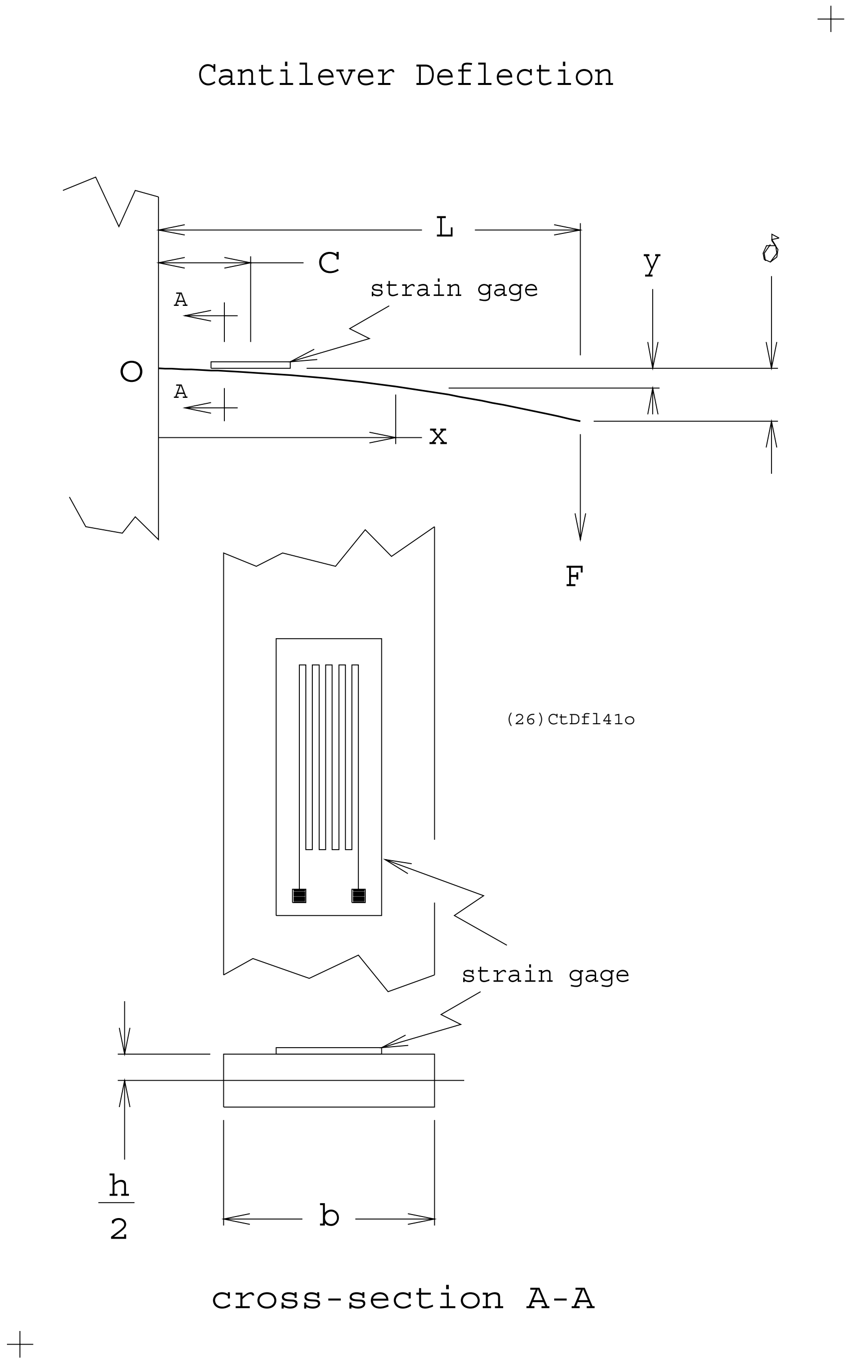

Figure 1: Cantilever and Strain Gauge